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A Theoretical Model for Meaning Construction through Constructivist Concept Learning

A Conceptual, Terminological, Logical and Semantic Study within Human-Human-Machine Interactions

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A THEORETICAL MODEL FOR MEANING CONSTRUCTION THROUGH CONSTRUCTIVIST CONCEPT LEARNING

A CONCEPTUAL, TERMINOLOGICAL, LOGICAL AND SEMANTIC STUDY WITHIN HUMAN-HUMAN-MACHINE INTERACTIONS

BY
FARSHAD BADIE

DISSERTATION SUBMITTED 2017

AALBORG UNIVERSITY
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DENMARK

Dissertation submitted 2017

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This dissertation is based on sixteen scientific papers; twelve published, two accepted for publication (in press), and two submitted for publication. Farshad Badie is the sole author of all sixteen papers. Some parts of the articles are either directly or indirectly used in the 'Summary' and 'Outlooks' sections.

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CV

Farshad Badie holds a M.Sc. certificate (graded with honour; GPA: 4.82 / 5) in Software Information Technology and Computer Science, specialising in Information Systems from University of Debrecen, Hungary (2013). During his Master studies, he was a Teaching Assistant and Seminar Leader in Mathematical Logic in Computer Science. Farshad continued his researches in the areas of Formal Logics, Semantics, and Knowledge Representation Systems. He worked as a Research Assistant at the Agile Knowledge Engineering and Semantic Web (AKSW) research group, Institute of Applied Informatics, University of Leipzig, Germany. In AKSW, his central focus was on Description [and Fuzzy Description] Logics in Knowledge Representation Systems and on Inductive Concept Learning paradigm in Ontology Engineering. Farshad continued his work as a PhD researcher in HCCI (Human Centered Communication and Informatics) doctoral program at the Department of Communication at Aalborg University in 2014. He has been affiliated with the Center for Linguistics (CfL). His research interests are in the fields of Formal logics and Semantics, Formal Ontologies, Knowledge Processing, Logic and Cognition, Logic and Language, Logic and Learning, and Philosophy of Education. Farshad has had a special attention to educational and pedagogical systems; he has believed that educational systems could provide proper contexts for his researches in the area of Logic and Cognition. Farshad has published multiple articles in books, conference proceedings and journals. During his Ph.D. studies, he has received the best paper award at the International Conference on Education and Psychological Sciences, Florence, Italy (February 2016). Also, he has received a scholarship grant from the North American Summer School on Logic, Language, and Information (NASSLLI), hosted by Rutgers, the State University of New Jersey, New Brunswick, USA (July 2016). Farshad has been invited as technical committee member, reviewer, editorial board member, and sessions' chairman of a number of high-level conferences in Learning Sciences, Psychological Sciences, Human-Machine Interactions, Artificial Intelligence, Machine Intelligence, and Emerging Technologies for Education.

Constructivist Concept Learning is one's meaningful understanding production based on her/his mental constructions of her/his own conceptions of the world.

Farshad Badie

Painting by Vladimir Volegov, © volegov.com

ENGLISH SUMMARY

The central focus of this Ph.D. research is on ‘Logic and Cognition’ and, more specifically, this research covers the quintuple (Logic and Logical Philosophy, Philosophy of Education, Educational Psychology, Cognitive Science, Computer Science). The most significant contributions of this Ph.D. dissertation are conceptual, logical, terminological, and semantic analysis of Constructivist Concept Learning (specifically, in the context of humans’ interactions with their environment and with other agents). This dissertation is concerned with the specification of the conceptualisation of the phenomena of ‘learning’, ‘mentoring’, and ‘knowledge’ within learning and knowledge acquisition systems. In this research, the phenomena of ‘learning’ and ‘mentoring’ are interpreted as active processes of ‘knowledge construction’. Accordingly, this interpretation has provided the most considerable presupposition of further developments. Constructivism as an epistemology and as a model of knowing and, respectively as a theoretical model of learning builds up the central framework of this research.

The most significant question that I have tried to focus on is “How one’s constructed concepts could be followed by her/his own constructed meanings and, accordingly, by her/his meaningful understandings?”. Consequently, relying on the framework of constructivism, the major objectives of this dissertation are as follows:

- Conceptualisation and characterisation of concepts in humans’ minds and in machines’ knowledge bases.
- Conceptualisation of humans’ concept construction processes and, correspondingly, of logical descriptions and of formal analysis of concept constructions.
- Conceptualisation of humans’ conceptions that are produced in order to express their constructed concepts.
- Conceptual, logical, and terminological analysis of concept representation in both humans’ minds and in machines’ knowledge bases.
- Conceptual, logical, and terminological analysis of human concept understanding, or equivalently, understandings over constructed concepts and produced conceptions.
- Conceptualisation, logical, and terminological description/analysis and formal[-semantic] analysis of concept learning (by human beings and by machines metaphorically understood as learners) and their interconnections.

Another important question that I have tackled to focus on is “How knowledge may reasonably and logically be assumed to be constructed by a learner in the framework of constructivism and, also, in the context of her/his interactions?”. This question is highly relevant to the learners who are at their elementary, high school, and undergraduate levels of studies. Note that the HowNess and the quality of knowledge construction varies from learner to learner. This attribute is heavily concerned with any individual learner’s mental models of her/his knowings. Accordingly, it shall be

stressed that there is a strong dependency between the phenomenon of ‘knowledge’ and ‘any individual learner’s mental models of knowings’.

My proposed theoretical model is strongly dependent on my ideas of the concepts of ‘classification’ and ‘induction’. It means that my theoretical model expresses that learners’ reasoning processes are mainly structured over their mental abilities of classification and induction. Subsequently, the expression ‘concept learning’ will be analysed based upon classification and induction.

According to this dissertation, any human being becomes concerned with her/his specification(s) of her/his own conceptualisation through concept learning processes. It shall be stressed that there are, obviously, no compelling reasons to claim that concept learning must necessarily be structured based on the processes supported by constructivism. I do, though, strongly believe that there is an epistemological junction between ‘constructivism’ and ‘concept learning’. In fact, in my opinion,

- constructivism could provide a proper supportive base for description of concept construction processes, and
- constructivism, as an epistemology, could support so-called ‘constructivist concept learning’, if it is seen as an individual’s conditional reasoning in a learning and a pedagogical context.

It shall be emphasised that this research has mainly focused on constructivist concept learning by ‘human beings’. A few of my publications—which are included in this thesis—have had contributions in machine learning, artificial intelligence, and human-machine interactions, but it shall be drawn into consideration that those publications’ central focuses have been on human beings. Actually, their contributions have been on the terminological, logical, and semantic analysis of constructivist ‘training’ with regard to humans’ conceptions of their mental constructed concepts. In more technical words, this dissertation has not dealt with the HowNess of adaptation and conformation of machines and machines’ knowledge bases to human beings and their minds. It has, though, been concerned with the transformation of humans’ conceptions from their minds into machines’ knowledge bases as well as ontological descriptions in information systems.

DANSK RESUME[1]

Denne ph.d.-afhandlings fokus er ‘logik og kognition’, og derudover kan man mere specifikt sige at forskningsfeltet dækker fem faglige områder: logik og logikkens filosofi, uddannelsesfilosofi, uddannelsespsykologi, kognitionsvidenskab og computervidenskab. Afhandlingens mest betydningsfulde bidrag omhandler begrebslig, logisk, terminologisk og semantisk analyse af ‘Constructivist Concept Learning’, nærmere bestemt i konteksten ‘menneskers interaktion med omgivelserne og med andre handlende subjekter’. Afhandlingen drejer sig om bestemmelsen af begrebsliggørelsen af fænomenerne ‘læring’, ‘mentorering’ og ‘viden’ i forbindelse med læring og videnstilegnelses-systemer. Indenfor dette forskningsfelt forstås fænomenerne ‘læring’ og ‘mentorering’ som aktive processer af ‘videnskonstruktion’. Som følge heraf fungerer denne forståelse som den mest betydningsfulde antagelse som grundag for den videre behandling. Hvad der opbygger den centrale rammeforståelse i dette forskningsbidrag er Konstruktivisme set som epistemologi og som videns-model, og herudover som teoretisk lærings-model.

Det vigtigste spørgsmål som jeg har prøvet at fokusere på er “hvordan en persons konstruerede begreber vil kunne opfølges af personens egne konstruerede betydninger og, således, af personens meningsfulde forståelse”. På grundlag af Konstruktivisme er, som følge heraf, hovedmålene for denne afhandling følgende:

- Begrebsliggørelse og karakteristik af menneskers begreber som de forekommer i menneskers tankeverdener (‘minds’) og i maskiners vidensbaser.
- Begrebsliggørelse af menneskers begrebskonstruktions-processer og, således, af logiske beskrivelser og formale analyser af begrebskonstruktioner.
- Begrebsliggørelse af menneskers konceptioner som bliver skabt for at udtrykke de konstruerede begreber.
- Begrebslig, logisk og terminologisk analyse af begrebs-repræsentation i både menneskers tankeverdener (‘minds’) og i maskiners vidensbaser.
- Begrebslig, logisk og terminologisk analyse af menneskers begrebsforståelse, eller, anderledes udtrykt, forståelsen af konstruerede begreber og konceptioner.
- Begrebsliggørelse og logisk og terminologisk beskrivelse/analyse og formal[semantisk] analyse af begrebs-læring (foretaget af mennesker og af maskiner forstået som metaforisk som ‘learnere’) og forbindelserne mellem disse.

Et andet vigtigt spørgsmål som jeg har forsøgt at fokusere på er “hvordan man rimeligvis og logisk kan antage at viden konstrueres af en ‘learner’ indenfor Konstruktivismens ramme, og i ‘learnerens’ interakstions-sammenhæng”. Dette

[1] I would like to thank Dr. Hans Götzsche for translating and proofreading this section.

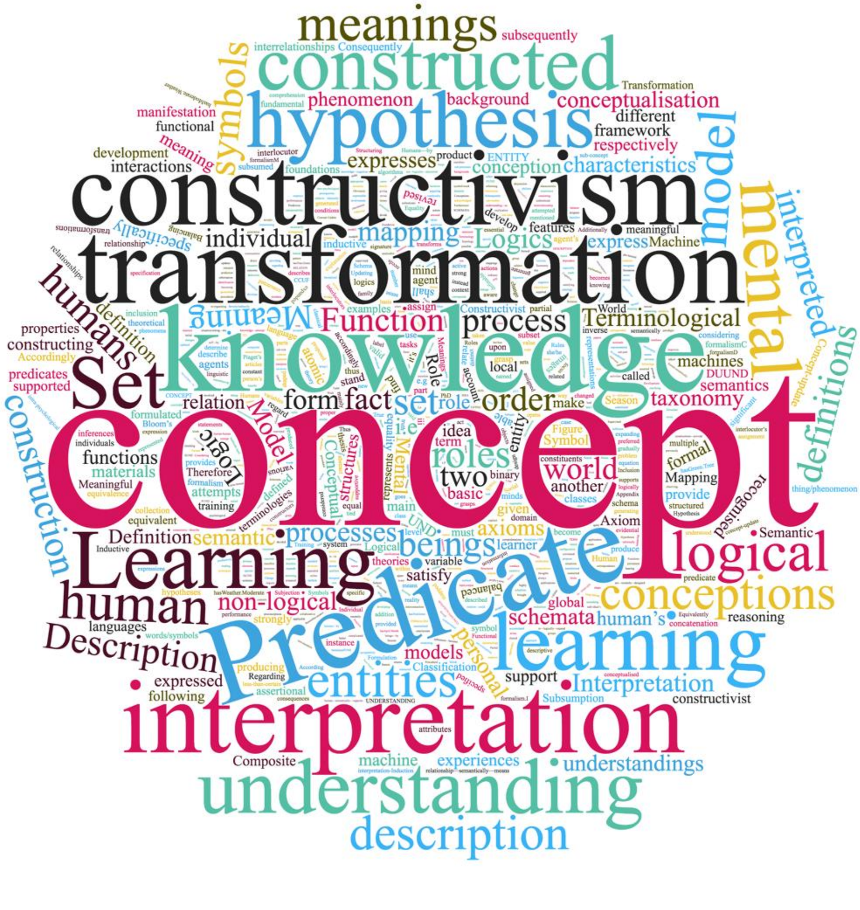
spørgsmål er i høj grad relevant for 'learnere' som befinder sig på skole- eller uddannelsesniveau. Man skal her bemærke at 'hvordan-heden' ('HowNess') og kvaliteten af videnskonstruktionen varierer fra enkeltmenneske til enkeltmenneske. Det drejer sig her afgørende om hver enkelt 'learners' mentale modeller af vedkommendes vidensindhold. Det skal derfor understreges at der er en stærk afhængighed mellem videns-fænomenet og den enkelte 'learners' mentale modeller af vidensindholdet.

Den af mig forelåede teoretiske model er éntydigt baseret på mine idéer om begreberne 'klassifikation' og 'induktion'. Det medfører at min teoretiske model udtrykker den opfattelse at en 'learners' tankeprocesser ('reasoning processes') hovedsageligt er struktureret over vedkommendes mentale forudsætninger ('capacities') hvad angår klassifikation og induktion. Følgelig vil udtrykket 'begrebs-læring' ('concept learning') blive analyseret ud fra klassifikation og induktion.

Ifølge opfattelsen i denne afhandling så vil ethvert menneske blive optaget af hvorledes vedkommende bestemmer ('her/his specification(s) of') sin egen begrebsliggørelse igennem begrebslærings-processer. Det skal understreges at der ikke er tvingende grunde til at hævde at begrebslæring nødvendigvis struktureres baseret på de processer som fremhæves af Konstruktivismen. Men jeg tror selv afgjort på en epistemologisk forbindelse mellem 'Konstruktivisme' og 'begrebslæring'. Det er således min opfattelse at

- Konstruktivisme vil kunne bidrage med en ægte underbyggelse af hvorledes begrebskonstruktions-processer bør beskrives, og
- Konstruktivisme vil, som epistemologi, kunne underbygge såkaldt 'konstruktivistisk begrebslæring' hvis det opfattes som den enkeltes betingede ('conditional') tankeprocesser ('reasoning') i en lærings- og pædagogisk sammenhæng.

Det skal understreges at disse forskningsresultater hovedsageligt har fokuseret på læring i forbindelse med 'mennesker'. Enkelte af mine publikationer, som er med i denne afhandling, har indeholdt bidrag indenfor områderne maskinlæring, kunstig intelligens og menneske/maskine-interaktion, men det skal fastholdes at disse udgivelser faktisk har fokuseret på mennesker. Således har nævnte udgivelser handlet om terminologiske, logiske og semantiske analyser af 'konstruktivistisk' træning hvad angår menneskers begrebsligørelse af deres konstruerede begreber. Udtrykt mere teknisk så handler denne afhandling ikke om 'hvordan-heden' ('HowNess') i hvorledes maskiner og maskiners vidensbaser tilpasses mennesker og deres tankeverden. Den handler også om hvorledes menneskers begrebsliggørelser transformeres til maskiners vidensbaser og ontologiske beskrivelser i informationssystemer.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ACKNOWLEDGEMENTS

First and foremost, I would like to express my special gratitude to my supervisor (and friend), Hans Götzsche, for granting me the freedom to pursue the research ideas that have been led to this dissertation. Hans has inspired me with his brilliant suggestions for directions of future work during my affiliation with Center for Linguistics. Especially his book *Deviational Syntactic Structures* has been one of my most precious references. I also would like to thank Hans most sincerely for editing my articles and for revising my conceptions by his comments.

I shall express my high appreciation to Mikael Vetner for listening to me, respecting me, believing in me, and for all his kind support. I shall extend my special gratitudes to Peter Øhrstrøm for supporting me with his brilliant knowledge and supporting me with his caring personality. Peter encouraged me to be a member of the Scandinavian Logic Society and the Copenhagen Association for Dynamics Interaction Logic Language and Computation, and this was very productive in my achievements in logic. My special gratitudes also goes to Jens Allwood for his compassionate hospitality over my stay in Gothenburg, Sweden, while I visited the Interdisciplinary Research Center SCCIIL at the department of Applied Information Technology, University of Gothenburg. Jens highly expanded my horizons with his brilliant expertises. I shall also thank Martin Prior for supporting me with his brilliant knowledge in both logics and linguistics. Martin’s shining personality has been so caring. I shall express my deep appreciation to Niels Jakob Buch for listening to me, respecting my ideas, supporting me with his professional advices, and for his kind encouragements in order to make myself concerned with the industry.

I highly appreciate the assistance of Charlotte Hyldgaard, Hanne Porsborg Clausen, Anne Kubel Teilskov, Susanne Gottlieb Togeby, Bente Nielsen, Jens Nielsen, and of all other members of the administrative staff for their kind helps and support. Many thanks go to my dear friends, Martin Mølholm, David Jakobsen, Elizabeth Jochum, and Evgenios Vlachos for making our department a pleasant environment.

I would like to express my special gratitude to my late M.Sc. supervisor at the university of Debrecen, Katalin Bognár, who showed me the way. In addition, I really appreciate all of the teachers and mentors who I have met during my education; they all enlightened me.

I would like to thank my dear uncle, Kambiz Badie, who has always inspired me. Kambiz’s involvement and efforts have caused bright and appropriate results in my life. I would like to express my sincere gratitude to my dear father and mother, Farhad Badie and Ladan Parviz, for their real love and their endless support. I shall thank them for everything they have done for me. I shall thank my lovely brother, Shahin, without whom the world would have been dark to me. I shall thank my beloved Sara for being my life’s sunshine, and for her unconditional love and pure kindness.

Finally, I would like to express my deepest love to my late grandfather, Naser Badie, and my late grandmother, Homa-Dokht Ghadimi, who are resting in peace in heaven. My warm memories of them are still fresh in my heart.

Farshad Badie
Aalborg, Denmark, April 2017

To my mother, Ladan, who has taught me how to love unconditionally,
and to my father, Farhad, who has taught me to be a true human being.

The fish will be the last to discover water.

Jerome Bruner

THESIS DETAILS

This Ph.D. dissertation consists of three main sections and two appendices.

SECTION I. THE OUTLOOKS

This section is structured in two parts:

- Constructivism, and
- Constructivist Concept Learning.

This section focuses on describing and specifying the fundamental expressions in order to provide a supportive background for section II. Note that some pieces and products of section II are either directly or indirectly used in this section.

SECTION II. ARTICLES

This part is structured based on sixteen articles (A) – (P) in a revised layout. Twelve of the articles (A, B, C, D, E, F, G, H, I, J, K, M) are already published, two (L, N) are accepted for publication, and two (O, P) are submitted for publication. The included figures and diagrams have been, graphically and not contextually, improved and updated. I have obtained permission from all the publishers to use their copyrighted material. The articles are mainly presented in chronological order. I have been the sole author of all of the included publications in this dissertation. I shall, though, emphasise that all articles have been submitted and published under the direct supervision of my supervisor, Dr. Hans Götzsche. In addition, Hans has supervised me through editing processes of all articles. The articles are as follows:

[A] Farshad Badie (2015): "A Semantic Basis for Meaning Construction in Constructivist Interactions". Proceedings of the 12th International Conference on Cognition and Exploratory Learning in Digital Age (Maynooth, Greater Dublin, Ireland). ISBN: 978-989-8533-43-2. International Association for Development of the Information Society (IADIS). Pages: 369–376.

[B] Farshad Badie (2015): "Towards a Semantics-based Framework for Meaning Construction in Constructivist Interactions". Proceedings of the 8th International Conference of Education, Research and Innovation (Seville, Spain). ISBN: 978-84-608-2657-6. ISSN: 2340-1095. International Association of Technology, Education and Development (IATED). Pages: 7995–8002.

[C] Farshad Badie (2017): "A Conceptual Mirror: Towards a Reflectional Symmetrical Relation Between Mentor and Learner". International Journal of Information and Education Technology: IJIET 2017, Vol.7(3). ISSN: 2010-3689.

Proceedings of the 3rd International Conference on Education and Psychological Sciences (Florence, Italy, 2016). Pages: 199–203 (*This paper has received the best paper of session award*)

[D] Farshad Badie (2016): "A Semantic Representation of Adult Learners' Developing Conceptions of Self Realisation through Learning process". Proceedings of the 10th Annual International Technology, Education and Development Conference (Valencia, Spain). ISBN: 978-84-608-5617-7. ISSN: 2340-1079 International Association of Technology, Education and Development (IATED). Pages: 5348–5353.

[E] Farshad Badie (2016): "Concept Representation Analysis in the Context of Human-Machine Interactions". Proceedings of the 14th International Conference on e-Society (Algarve, Portugal). ISBN: 978-989-8533-48-7. International Association for Development of the Information Society (IADIS). Pages: 55–62.

[F] Farshad Badie (2016): "Logical Characterisation of Concept Transformations from Human into Machine Relying on Predicate Logic". Proceedings of ACHI 2016: 9th International Conference on Advances in Computer-Human Interaction (Venice, Italy). ISSN: 2308-4138. ISBN: 978-1-61208-468-8. International Academy, Research and Industry Association (IARIA). Pages: 376–379.

[G] Farshad Badie (2016): "Towards Semantic Analysis of Training-Learning Relationships within Human-Machine Interactions". Proceedings of ACHI 2016: 9th International Conference on Advances in Computer-Human Interaction (Venice, Italy). ISSN: 2308-4138. ISBN: 978-1-61208-468-8. International Academy, Research and Industry Association (IARIA). Pages: 323–326.

[H] Farshad Badie (2016): "A Conceptual Framework for Knowledge Creation Based on Constructed Meanings within Mentor-Learner Conversations". In Smart Education and e-Learning 2016, Springer International Publishing. Volume 59 of the series Smart Innovation, Systems and Technologies. ISBN: 978-3-319-39690-3. Pages:167–177.

[I] Farshad Badie (2016): "Towards Concept Understanding relying on Conceptualisation in Constructivist Learning". Proceedings of the 13th International Conference on Cognition and Exploratory Learning in Digital Age (Mannheim, Germany). ISBN: 978-989-8533-55-5. International Association for Development of the Information Society (IADIS). Pages: 292–296.

[J] Farshad Badie (2017): "A Conceptual Framework over Contextual Analysis of Concept Learning within Human-Machine Interplays". In Emerging Technologies for Education. Lecture Notes in Computer Science, vol 10108. Springer International Publishing. ISBN: 978-3-319-52836-6. Pages: 65–74 (Proceedings of International Symposium on Emerging Technologies for Education. Rome, Italy)

[K] Farshad Badie (2017): "Towards Semantic Analysis of Mentoring-Learning Relationships within Constructivist Interactions". In Emerging Technologies for Education. Lecture Notes in Computer Science, vol 10108. Springer International Publishing. ISBN: 978-3-319-52836-6. Pages: 107–116 (Proceedings of International Symposium on Emerging Technologies for Education. Rome, Italy)

[L] Farshad Badie (Accepted Chapter, 2017): "Knowledge Building Conceptualisation in Smart Constructivist Learning Systems". Book title: "Smart Universities: Concepts, Systems, Research". The Smart Innovation, Systems and Technologies book series. ISSN: 2190-3018. Springer International Publishing AG.

[M] Farshad Badie (2017): "A Formal Semantics for Concept Understanding Relying on Description Logics". In Proceedings of the 9th International Conference on Agents and Artificial Intelligence (Porto, Portugal). Volume 2: ICAART. ISBN 978-989-758-220-2. Pages 42–52 (This paper has been selected to be included in 'Lecture Notes in Artificial Intelligence', Springer)

[N] Farshad Badie (Accepted Article, 2017): "On Logical Characterisation of Human Concept Learning based on Terminological Systems". Journal of Logic and Logical Philosophy. ISSN: 2300-9802. Nicolaus Copernicus University. Poznań, Poland.

[O] Farshad Badie (submitted, 2017): "Logical Foundation of Inductive Meaning Constructing in Constructivist Interactions".

[P] Farshad Badie (submitted, 2017): "From Concepts to Predicates within Constructivist Epistemology".

Figure *a* represents the logical relationships between the included papers and their significant products in different areas. In this network, all dashed arrows are expressing 'supports', e.g., Paper A supports Paper B.

SECTION III. TERMINOLOGY

This section proposes a covering terminology for the dissertation. Most of the terms are either defined by me or re-described in a revised version relevant to this research.

APPENDICES

This section is structured in two parts:

- Appendix I (Clarifications) that clarifies some possible confusions.
- Appendix II (Revisions) that reports some minor [contextual] changes in included articles.

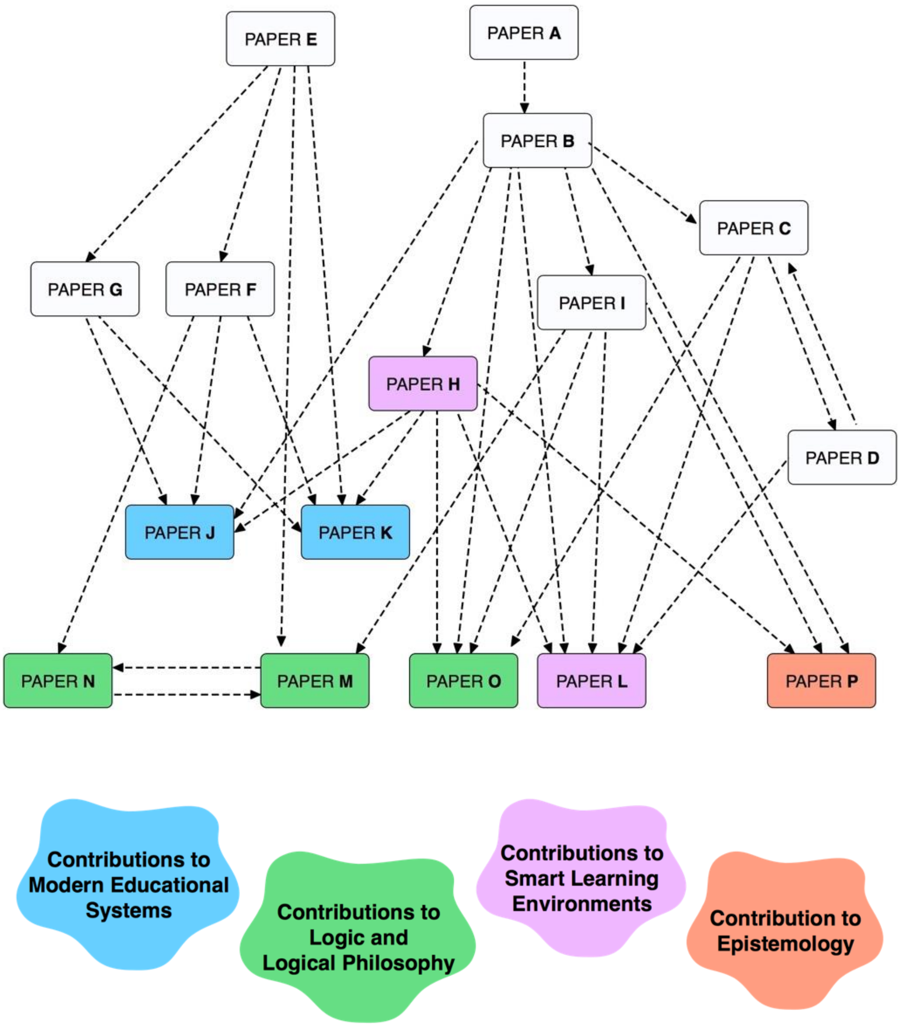

Figure a. *The Conceptual Relationships between Papers and their Final Contributions*

PH.D. ACTIVITIES

ONE

This Ph.D. research has been carried out in the period October 2014 – April 2017 at the Department of Communication and Psychology, Faculty of Humanities at Aalborg University. The research was conducted alongside with obligatory Ph.D. courses that have been equal to 30 ECTS.

TWO

In parallel with my Ph.D. progress and publications, that are included in this dissertation, I have had fruitful collaborations with Hans Götzsche in our projects, (i) New Perspective on Semantics and (ii) Meaning of Language. These collaborations have been summarised in the following abstracts and papers. In these researches, I have focused on developing a formal logic which has been supported by Götzsche's alternative theory of natural language syntax in Deviational Syntactic Structures under the label EFA(X)3 or Epi-Formal Analysis in Syntax, see (Götzsche, Hans (2013): Deviational Syntactic Structures. London / New Delhi / New York / Sydney: Bloomsbury Studies in Theoretical Linguistics). These collaborations have made interesting junctions between linguistics and logic.

- Farshad Badie and Hans Götzsche (2015): "Towards a Formal Occurrence Logic based on Predicate Logic". Book of Abstracts – The 15th Congress of Logic, Methodology and Philosophy of Science, and Logic Colloquium 2015, Helsinki, Finland.
- Farshad Badie and Hans Götzsche (2016): "Towards Logical Analysis of Occurrence Values in Truth-Functional Independent Logic". Logic Colloquium 2016. Bulletin of Symbolic Logic (by Association of Symbolic Logic). Leeds, UK.
- Farshad Badie (Accepted Chapter, 2017): "Towards a Formal Symbolic Occurrence Logic". In Meaning of Language. Cambridge Scholar Publishing. Proceedings of the 26th Scandinavian Conference of Linguistics, Aalborg, Denmark.
- Farshad Badie and Hans Götzsche (Submitted, 2017): "Towards a Predicate-Occurrence Logic".
- Farshad Badie and Hans Götzsche (Submitted, 2017): "A Combinatorial Model based on a Formal Occurrence Logic for Analysing Strings within Linguistic Expressions".

THREE

In the period spanning from May 2015 to November 2015, I was invited (by Prof. Peter Øhrstrøm) as a member of Prior meetings. The main task was reviewing and analysing "Past, Present and Future" by A. N. Prior, Oxford University Press.

FOUR

During July 2015, I attended the Scandinavian Logic Society (SLS) Summer School in Logic, Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland.

FIVE

During November 2016, I had a stay in Gotheburg, Sweden as a visiting researcher at the Research Center SCCIIL, Department of Applied Information Technology, University of Gothenburg. SCCIIL is an interdisciplinary research center (in the intersection of the Concepts Language, Semantics, Cognition, Communication, Information, and Interaction). Prof. Jens Allwood is the director of SCCIIL.

SIX

During my Ph.D. studies, I have presented my researches at the following conferences, symposiums, and workshops:

- Farshad Badie and Hans Götzsche (2015): “Logical Characterisation of Ontology Construction using Fuzzy Description Logics”, Revealing Hidden Knowledge Workshop, Copenhagen Business School, Denmark.
- Farshad Badie (2015): “Towards a Formal Occurrence Logic based on Predicate Logic”, Logic Colloquium 2015, University of Helsinki, Finland.
- Farshad Badie (2015): “Towards Symbolic Occurrence Logic”, 26th Scandinavian Conference of Linguistics, Aalborg University, Denmark.
- Farshad Badie (2015): “A Semantic Basis for Meaning Construction in Constructivist Interactions”, 12th International Conference on Cognition and Exploratory Learning in Digital Age. Session for Technology and Mental Models. Maynooth University, Ireland.
- Farshad Badie (2015): “Towards a Semantics-based Framework for Meaning Construction in Constructivist Interactions” (Virtual Talk). 8th Annual International Conference of Education, Research and Innovation. Session for Research on Technology in Education. Seville, Spain.
- Farshad Badie (2015): “A Component-based Model for Inductive Concept Learning through Human-Robot Interaction”, 4th Annual Aalborg University Robotics Workshop. Special Panel for Human-Robot Interaction. Aalborg, Denmark.
- Farshad Badie (2016): “A Semantic Representation of Adult Learners’ Developing Conceptions of Self Realisation through Learning process” (Virtual Talk), 10th Annual International Technology, Education and Development Conference. Session for Research on Technology in Education. Valencia, Spain.
- Farshad Badie (2016): “A Conceptual Mirror: Towards a Reflectional Symmetrical Relation Between Mentor and Learner”. 3rd International Conference on Education and Psychological Sciences. Florence, Italy.

- Hans Götzsche and Farshad Badie (2016): "Towards Logical Analysis of Occurrence Values in Truth-Functional Independent Occurrence Logic", Logic Colloquium 2016 (by Association for Symbolic Logic). Session for Philosophical Logic. Leeds, UK.
- Farshad Badie (2016): "Conceptual Analysis of Inductive Meaning Constructing in Constructivist Interactions between Mentor and Learner". Creative University Conference (in Knowledge Cultures, Critical Creative Thinking and Innovative Learning Processes). Session for Learning Processes and Knowledge Building. Aalborg, Denmark.
- Farshad Badie (2016): "On Logical Characterisation of Human Concept Learning within Terminological Systems", 2nd Conference on Logic & Cognition, Poznań Reasoning Week, Poznań, Poland.
- Farshad Badie (2016): "A Conceptual Framework over Contextual Analysis of Concept Learning within Human-Machine Interplays". International Symposium on Emerging Technologies for Education. Track for 'Emerging Technologies of Design, Model and Framework of Learning Systems. Sapienza University, Rome, Italy.
- Farshad Badie (2016): "Towards Semantic Analysis of Mentoring-Learning Relationships within Constructivist Interactions". International Symposium on Emerging Technologies for Education. Track for 'Emerging Technologies of Design, Model and Framework of Learning Systems', Sapienza University, Rome, Italy.
- Farshad Badie (2017): "A Formal Semantics for Concept Understanding relying on Description Logics". 9th International Conference on Agents and Artificial Intelligence. Session for Artificial Intelligence. Porto, Portugal.

SEVEN

I have been a member of (or invited by) the following associations, societies, and conference committees:

- Executive Committee Member (Secretary) of Nordic Association of Linguists (since January of 2016)
- Member of Scandinavian Logic Society
- Member of Copenhagen Association for Dynamics Interaction Logic Language and Computation
- Member of Association for Symbolic Logic
- Member of Nordic Society for Phenomenology
- Member of Association for Neural-Symbolic Learning and Reasoning (NeSy)
- Artificial Intelligence's Session Chair, 9th International Conference on Agents and Artificial Intelligence, Porto, Portugal, 2017.
- Invited Chair and Speaker at Educational Leadership Forums, World Education Day–2017 (Inheritance, Innovation, Development and Philanthropy), Dalian, China, 2017.

- Program Committee member of the 2017 IEEE International Conference on INnovations in Intelligent SysTems and Applications, Special Session on Semantic Intelligence, Gdynia, Poland, 2017.
- Invited Editorial Board Member and Reviewer at the journal of 'Philosophy Study', David Publishing Company, New York, USA, 2017.
- Invited Committee Member and Reviewer at the 10th International Conference on Advances in Computer-Human Interactions, Nice, France, 2017.
- Invited Technical Committee Member and Reviewer at the 4th International Conference on Education and Psychological Sciences, Barcelona, Spain, 2017.
- Anonymous Reviewer of the 1st International Symposium on Emerging Technologies for Education. Roma, Italy, 2016.
- Invited Chair for 'Interactive Systems' session at the 9th International Conference on Advances in Computer-Human Interactions, Venice, Italy, 2016.
- Technical Committee Member and Reviewer of the 3rd International Conference on Education and Psychological Sciences, Florence, Italy, 2016.

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SECTION I

THE OUTLOOKS

PART I. CONSTRUCTIVISM

Constructivism is the philosophical and scientific position that knowledge arises through a process of active construction.

(Mascolol & Fischer, 2005)

Constructivism can be defined as a mapping from the phenomenon of ‘learning’ into individualistic knowledge structures.

(Badie, 2017)

1. THE PHENOMENON OF 'LEARNING'

The point of departure is my special focus on the phenomenon of 'learning'. I shall begin with reviewing and summarising some significant definitions and notions of 'learning'. Subsequently, I will focus on conceptualising the summarisations in order to use them in my research.

According to (Oxford Online Dictionaries, 2017), the term 'learning' is defined as "the acquisition of knowledge or skills through study, experience, or being taught". Also, (Cambridge Online Dictionary, 2017) defines 'learning' as "the activity of obtaining knowledge". According to (Wallace, 2015), learning, rather than teaching, is the central purpose of [any] education. It is usually defined as a change in someone's behaviour, knowledge, level of skill, or understanding which is long-lasting or permanent and is acquired through experiences rather than through the process of growth or ageing. Regarding (Colman, 2015), learning is any lasting change in behaviour resulting from experience, especially conditioning. Furthermore, (Colman, 2015) interprets learning as the act (or process) of acquiring knowledge or skill, or knowledge gained by study. According to (Honderich, 2005), 'learning' can be interpreted as the acquisition of a form of knowledge or ability through the use of experience. In addition, (Honderich, 2005) states that not all modifications of behaviour as a result of experience involve learning, although behaviourist theories of learning tend to assume otherwise. Furthermore, (Honderich, 2005) believes that it's far from clear that changes of behaviour brought about by conditioning should be thought of as involving learning; the same applies to the biological phenomenon of 'imprinting', whereby something that happens at a certain point of an animal's life determines a subsequent form of behaviour. Considering (McFarland, 2014), the phenomenon of 'learning' can be interpreted as an irreversible change in response to particular stimuli, as opposed to the reversible changes that result from changes in motivation. (McFarland, 2014) also expresses that learning is characterised by changes in memory of the contiguity of stimuli and of the consequences of responding to stimuli. Finally, regarding (Hine and Martin, 2015), leaning is a process by which an animal's experience may permanently alter their future behaviour, usually in a beneficial way. Also, (Hine and Martin, 2015) emphasises that the phenomena of 'learning' and 'memory' are intimately interconnected, because new information can be linked to past experience recorded in memory and new associations formed.

Taking into account the afore-mentioned descriptions, specifications, and characteristics of the phenomenon of 'learning', it shall be concluded that *learning, as a process of acquiring knowledge through the use of experience, causes long-lasting or permanent changes in an individual's level of knowledge and depth of understanding.* Note that it's extremely important to interpret the phenomenon of 'learning' as a 'process' and, in fact, as a 'series of actions and changes'. In fact, we shall take into consideration that any process is a series of changes with some sort of unity, or unifying principle, to it. Hence 'process' is to 'change', or 'event', rather like 'syndrome' is to 'symptom', see (Whitehead, 1979; Honderich, 2005). Consequently, the process of ‘learning’ could produce different changes in any organism, and the produced changes could be relatively permanent, see (Schacter et al., 2009).

It’s worth mentioning that ‘learning’ can be also interpreted as *the involvement of the self, by any individual, in increasing knowledge about a phenomenon.* This phenomenon could be a subject, object, process, or event. Accordingly, human beings could become concerned with [and be involved in] a learning process within their, e.g., education, schooling, storytelling, game playing, adventuring, consulting, training, and instructing. Actually, any learner can be characterised as *an individual who attempts to be concerned with [and be involved in] the process of learning and, in fact, she/he tackles to do learning.* In addition, any mentor can be described as *an individual who opens the world to the learner and opens the learner to the world in order to support and guide the learner through the learning process.*

From a general point of view, human beings can be connected with the phenomenon of ‘learning’ based on either their specific goals or their own motivations and, in fact, learning could occur either as a goal-oriented or as a motivation-based process. There are many approaches in biology, genetics, neuropsychology, cognitive psychology, educational psychology, learning sciences, and pedagogical sciences that focus on analysing of the HowNess of occurrences of learning processes.

A Country School. Painting by Edward Lamson Henry (1890), Yale University Art Gallery

2. LEARNING AND CONSTRUCTING

As mentioned, learning can be interpreted as a process of acquiring knowledge (over experiences) that could cause long-lasting or permanent changes in an individual's level of knowledge and depth of understanding. I shall, therefore, claim that the phenomenon of 'learning' could be interpreted as the process of constructing [something] and, in fact, as an 'activity of construction'. It is worth mentioning that some approaches (like (Selvi, 2013)) define 'learning' as "the process of creating and constructing knowledge". Correspondingly, the creation and construction of knowledge could be considered to be the main issues in the learning (and, of course, mentoring) processes. I take into consideration (Oxford Online Dictionaries, 2017) and (Cambridge Online Dictionary, 2017) and become more specific on the term 'construction':

- Construction is the action of building something or some structure.
- The term 'construction' can express "the creation of an abstract entity". For example, relying on this conception, we could say that languages play fundamental parts in human beings' constructions of reality. It's worth mentioning that there are ontological problems implied by saying that there exists 'abstract entities', see 'abstract objects' in (The Stanford Encyclopedia of Philosophy, 2017).
- The term 'construction' is, indirectly, interrelated with the phenomenon of 'interpretation' and, respectively, with the phenomenon of 'explanation'. For example, we can say that "a passionate teacher could put a helpful construction upon her/his students' questions".
- The term 'construction' has a specific usage in many languages and, in fact, it expresses the way in which the words (as parts and building blocks of sentences and phrases) are arranged and serialised. So, we can see that 'construction' could be strongly related to (and dependent on) arrangements and serialisations.

I shall, therefore, conclude that *construction is an action and process of producing and making. It has strong interrelationships with interpretation, explanation, arrangement, and serialisation.*

Note that any expressed interpretation, as well as an explanation of the concept of 'learning', could, subsequently, link the interpreter (and explainer) to some [more-]specific denotations of 'learning', like, e.g., (i) interpreting 'learning to learn' (Nisbet and Shucksmith, 1984), (ii) interpreting 'learning about learning' (Säljö, 1979; Watkins et al., 2000), (iii) interpreting 'learning how to learn' (Novak and Gowin, 1984), (iv) interpreting 'learning to think' (Ritchhart and Perkins, 2005), and (v) interpreting 'how learning supports making and producing' (Papert, 1993; Ackermann, 2002). Therefore, I shall conclude that understanding the phenomenon of 'learning' as 'the activity of construction', leads us to a number of categories, like, e.g.,

a) constructing something for learning something,
b) producing mental, logical, conceptual, and physical constructions of ‘learning’ something,
c) producing mental, logical, conceptual, and physical constructions for describing how to learn something,
d) producing mental, logical, conceptual, and physical constructions as the models of learning something,
e) constructing models of thinking for supporting learning something, and
f) sharing the concept of ‘construction’ through progressive internalisation of learners’ activities.

Construction is the process of producing and making. It has strong interrelationships with interpretation, explanation, and arrangement.

3. CONSTRUCTIVISM

Constructivism has become the central backbone of my Ph.D. research. From a general perspective, according to (Cambridge Online Dictionary, 2017) and (Oxford Online Dictionaries, 2017), constructivism could be seen as a theory that (i) interprets learning as an active process. Also, according to this theory, (ii) people gain knowledge and understanding through the combination of their experiences and ideas. Obviously, the most significant presupposition is that constructivism is correlated with and corresponds to the process of building, making, and constructing. Therefore, it’s possible to relate ‘learning as an activity of construction’ to ‘constructivism’.

It shall be stressed that constructivism as a style, movement, and HowNess can have various meanings and outcomes in different areas. Thus, I need to be very specific.

For this purpose, I shall focus on the relevant and underpinning conceptions and theories of a number of scientists, psychologists, and philosophers who influenced the concept of ‘constructivism’. Subsequently, I will uncover the key points, fundamental characteristics, and essences of constructivism, and thus, I will clarify what this dissertation means by constructivism.

The Most Influential Constructivists

3.1. Giambattista Vico (1668–1744)

Giambattista Vico was an Italian philosopher. It could be claimed that he was the first philosopher who proposed an explicit formulation of a constructivist theory of knowledge in his little-known Latin treatise, see (Vico, 1710). Vico coined the phrase "verum est ipsum factum" and explained that "to know something means to know what parts it is made of and how they have been put together", see (Glasersfeld, 1995).

Ernst von Glasersfeld in (Glasersfeld, 1995) has written that "Vico's notions that we can rationally know only what we ourselves have made, and that the knowledge of poets and myth-makers is of a different kind, fitted well between some of the disconnected ideas [about constructivism] in my head. Only very much later did I come to read Vico's treatise on epistemology (1710), which, as far as I know, is a first explicit formulation of constructivism". Regarding Vico's ideas, the objective and ontological reality, therefore, may be known to a creator, who constructed it, but not to a person who has access only to subjective experiences. According to (Vico, 1710, Ch. l, par.III, 2), "This is the norm to which all human truths should be compared; this is to say, among human cognitions those are true, whose elements are within ourselves and co-ordinated by ourselves and which, by means of postulates we continue to produce ad infinitum; and as we put together these elements, we become the makers of the truths that we know by composing them".

3.2. John Dewey (1859–1952)

John Dewey was an American philosopher, psychologist, and educational reformer. Dewey is known as one of the main founders of functional psychology. He is especially well-known for his philosophical school of pragmatism. Dewey can be seen as the philosophical founder and the organiser of the philosophical foundations of constructivism, see (Jennings et al., 2016). In fact, he rejected the notion that learning environments should focus on repetitive and rote memorisation. Dewey suggested a method of 'directed living', according to which, students would engage in real-world, practical workshops in which they would demonstrate their knowledge through their creativities and collaborations. He believed that students should be provided with opportunities to think by themselves. Dewey believed that learning environments must be filled with real experiences. More specifically, he believed in 'Experience and Education' and, more deeply, he put 'progressive education' against 'traditional education'. He wrote: "If you have doubts about how learning happens, engage in sustained inquiry: study, ponder, consider alternative possibilities and arrive at your belief grounded in evidence", see (Ültanır, 2012). One may conclude that his main purpose of focusing on 'progressive education' was facilitating the naturally developing tendencies and potential of the children, see (Matthews, 2003).

According to (Dewey, 1938; Dewey, 1998), and taking into consideration the conclusions of (Ültanır, 2012), the history of the theory of education has been shaped by two opposing ideas: the first idea expresses that education is an internal development based on the student's natural talent. On the other hand, the second idea argues that education is a process of external building, independent from talent or abilities. This process is one in which tendencies are replaced by the process of habits gained with the help of external interventions. Taking Dewey's school of pragmatism into account, I shall conclude that his pragmatist constructivism holds that the phenomena of 'meaning' and 'truth', with regard to multiple conceptions of learners and mentors, are some functions of those conceptions' practical outcomes. I shall, therefore, conclude that there can be strong conceptual connections between 'semantics' and 'pragmatics' in the framework of constructivism.

3.3. Maria Montessori (1870–1952)

Maria Montessori was an Italian physician and educator. She is acclaimed for her educational method that builds on the way children naturally learn. The field of Philosophy of Education bears her name as well, see (American Montessori Society, 2016). She developed her educational method based on a few premises (see 'Montessori method' in (Learning Theories, 2005–2016)) as follows:

1. Montessori had a special respect for children's choices and options. She believed that children must explore and discover the world in order to be prepared to become independent learners. This means that information shall not be presented to them from above.
2. Montessori believed that children are, constantly, learning in an inherent process of their everyday life. What any child absorbs is highly dependent on what types of information and experiences cross her/his paths, see (Montessori, 1949).
3. Montessori believed that children become ripe to learn different types of skills at specific points in their developments, see (Montessori, 1936). She believed that the age at which each sensitive period occurs varies from one child to another. In her opinion, teachers must be strongly aware of when the right time is to introduce concepts to each individual learner.
4. Montessori believed that classrooms as well as any other learning environment should be filled with readily available and well-organised learning materials, should be aesthetically pleasing, and only include things that the teacher wants the child to experience and to learn about. Such an environment should contain materials that children of different ages, different personal characteristics and backgrounds, and different interests could all engage in.
5. Montessori believed in educating children by themselves. It is also known as 'auto-education'. In her opinion, an appropriate learning environment gives multiple choices to learners. Additionally, different learners of different ages can assist and support, as well as collaborate with, each other through their learning processes.

3.4. Jean Piaget (1896–1980)

Jean Piaget was a Swiss biologist and clinical psychologist. He is well-known for his theory of cognitive development and, especially, for his theory of children's cognitive development. Piaget was highly interested in expressing the HowNess of 'meaning making' by human beings with regard to their own experiences and ideas of the world. According to that, he became concerned with the concept of 'constructivism'. Regarding (Piaget, 1971), the fundamental principle of constructivism states that "the mind organises the world by organising itself". Jean Piaget is known as a genetic epistemologist and cognitive constructivist. Genetic epistemology is a study of the origins (genesis) of knowledge and it is strongly related to the developmental theory of knowledge, see (Berly, 1977; Vuyk, 1981; Driscoll, 1994). In fact, the central focus of genetic epistemology has been on relating ‘knowledge and its existence’ with ‘the individual human’s model of knowledge construction’.

According to (Guenther, 1998; Merriam and Caffarella, 1991; Campbell, R. L., 2002), Piaget’s theory of constructivism stresses that the development of knowledge representation and manipulation is not genetically programmed into the brain of human beings. Jean Piaget used to observe children as young scientists who are driven to understand their world, and to change their understanding in the face of mistaken predictions about the world. Changes in knowledge structures drive changes in fundamental cognitive capabilities. According to Piaget this seemingly natural progression of cognitive capabilities emerges in an orderly way. This means that certain ways of thinking support subsequent ones. It shall be emphasised that the latter ones cannot emerge until the early ones have been mastered and specified.

3.5. Lev Vygotsky (1896–1934)

Lev Vygotsky was a Soviet psychologist and his main focus area was developmental psychology. He was the founder of a theory of human cultural and bio-social development; commonly referred to as cultural-historical psychology, see (Vygotsky, 1978a; Vygotsky, 1980; Wertsch, 1991; Crawford, 1996). In my opinion, Vygotsky’s theory has strongly developed constructivism with regard to the importance of humans’ social interactions.

According to Vygotsky’s constructivist theory of learning described in (Learning Theories, 2005–2016), his social learning theory asserts three major themes regarding:

a. social interactions,
b. the more knowledgeable other, and
c. the zone of proximal development.

Vygotsky believed that the first theme, ‘social interaction’, plays a fundamental role in the process of humans’ cognitive development. In (Vygotsky, 1978a) he states: “Every function in the child’s cultural development appears twice: first, on the social level, and, later, on the individual level; first, between people (inter-psychological) and then inside the child (intra-psychological)”.

Regarding the second theme, ‘the more knowledgable other’, he believed that the more knowledgable other (= MKO) refers to any individual who has a better understanding or a higher ability level than the learner with respect to a particular task, process or concept. According to the second theme, it could be assumed that MKO expresses ‘being a mentor as well as teacher, instructor, or tutor’.

Finally, regarding the third theme, the zone of proximal development (= ZPD) is, in fact, the distance between a learner’s ability to perform a task under adult guidance and/or with peer collaboration, and the learner's abilities in solving the problems independently. It shall be emphasised that Vygotsky believed that the phenomenon of 'learning' occurred in this zone. This means that the learners do learning in this zone, see (Vygotsky and Cole, 1978b; Stanlaw, 2005).

3.6. Heinz von Foerster (1911–2002)

Heinz von Foerster was an Austrian-American physicist and philosopher. Together with Warren McCulloch (American neurophysiologist: 1898–1969), Norbert Wiener (American mathematician and philosopher: 1894–1964), John von Neumann (Hungarian-American mathematician, physicist and computer scientist: 1903–1957), and others, he was the architect of cybernetics. In particular, he was the developer of new (see (Pask, 1996)) or second-order cybernetics that focuses on (i) self-referential systems and (ii) the importance of eigen-behaviours for the explanation of complex phenomena. Foerster tried to attach the origin of second-order cybernetics to classical cybernetics in order to construct a model of the mind, see "the Heinz von Foerster's page on (Univie, 1996–2010)".

Foerster had essential influences on many cognitive scientists and radical constructivists. The 'radical' version of constructivism was developed, independently, by Heinz von Foerster and Ernst von Glasersfeld. Knowledge, from the perspective of radical constructivism, is not a representation of 'objective' facts, see (Foerster, 1981; Segal, 2001). As a radical constructivist, Foerster believed that "Objectivity is a subject's delusion that observing can be done without him. Involving objectivity is abrogating responsibility – hence its popularity", cited in (Poerksen, 2004, p.3). See more about Foerster's influences in (Foerster, 1973; Foerster, 1984; Stewart, 1994; Varela, 1995).

3.7. Ernst von Glasersfeld (1917–2010)

Ernst von Glasersfeld was a German philosopher and psychologist. As mentioned earlier, the radical version of constructivism was developed independently by Foerster and Glasersfeld. Glasersfeld stated that "constructivism was introduced in the modern era by Jean Piaget as a way of thinking about cognition and knowledge, not as a metaphysical theory about what might exist".

Note that knowledge, through the lens of radical constructivism, is not a representation of objective facts and, respectively, of objective procedures. Knowledge, though, is a compendium of concepts, conceptual relationships, and rules that have proven useful in domesticating humans' experiential world, see (Glasersfeld, 1989; Glasersfeld, 1990; Glasersfeld, 1992; Glasersfeld, 2001). Glasersfeld believed that "...learners construct understanding. They do not simply mirror and reflect what they are told or what they read. Learners look for meaning and will try to find regularity and order in the events of the world even in the absence of full or complete information", see (Watzlawick, 1984) that is the English translation of (Glasersfeld, 1981; Glasersfeld, 1984).

The learners' subjective interpretation of texts and mentors' discourse and, thus, the subjective view of linguistically presented problems is increasingly being taken into account in educational practice and research (Glasersfeld, 1983). Such a constructivist perspective has noteworthy consequences. Glasersfeld in (Glasersfeld, 1995) wrote: "Twenty years ago, when Charles Smock and I put together our research report on epistemology and education, we chose as subtitle: 'The implications of radical constructivism for knowledge acquisition' (1974). It was the first time that the word 'radical' was associated with Piaget's genetic epistemology. Charles, who had worked with Piaget at Geneva, sent a copy of the report to the master. who had introduced the constructivist approach to cognition in the 1930s. A few weeks later, Charles received a most encouraging acknowledgment: 'I always appreciate what you write,' Piaget said, 'you are one of the few Americans who have understood me' (April, 1975) …".

3.8. George Kelly (1905–1967)

George Kelly was an American psychologist. He developed a theory of personality known as personal construct psychology whose focus has been on the distinctive ways in which individuals construct and reconstruct the meanings of their lives, see (Scheer, 2016). Because of the breadth of Kelly's approach, personal construct concepts and methods have been used to study such topics as cognitive complexity, psychological disturbance, the development and breakdown of close relationships, vocational decision making, education, and organisational behaviour. However, in keeping with Kelly's original focus on psychotherapy, his thinking has had its greatest impact in the areas of clinical and counselling psychology.

Regarding Kelly's 'personal construct theory (constructivism)', human beings interpret and understand their world and, correspondingly, construct their own versions (conceptions) of reality. Kelly believed that any individual tries to understand the world in different ways, see (Kelly, 1955; Kelly, 1963).

3.9. Jerome Bruner (1915–2016)

Jerome Bruner was an American psychologist and educationalist. According to Bruner's theoretical framework, learning is an active process and learners construct their new ideas or concepts based upon their existing knowledge. He believed that 'knowing' is how human beings get "beyond the information given" and, that knowing, getting to know the world, is not just perceiving something; it's constructing it. This means that Bruner believed that learning, as an active process of construction, includes information selection and information transformation, decision making, generating hypotheses, and making meaning from information and experiences. Bruner's theories emphasise the significance of categorisation (classification) in learning and, in fact, he believed that all cognitive activities of human beings involve categories. He stated that "To perceive is to categorize, to conceptualize is to categorize, to learn is to form categories, to make decisions is to categorize". More specifically, Bruner believed that categorisation is the process of constructing and it is dependent on representations and humans, by categorisation, make sense of their world. Furthermore, in his opinion, incoming information is organised in terms of pre-existing categories, or humans create new ones. Where humans cannot perceive things, they go beyond the information given and make inferences based on what they do know, see (Bruner et al., 1956; Bruner, 1974; Bruner, 1986; Bruner, 1990; Lefrancois, 1995; Bruner and Kalmar, 1998).

For Bruner, interpreting information and experiences by similarities and differences was a key concept. It could be assumed that his central focus was on 'Constructivism and Discovery Learning'. There are four features of Bruner's theory of instruction:

a) 'Predisposition to learn' that, as a feature, specifically states the experiences which move the learner towards a love of learning in general, or of learning something in particular.
b) 'Structure of knowledge' that expresses that it's possible to structure knowledge in a way that enables the learner to most readily grasp the information.
c) 'Modes of representation' (e.g., visual representations, words, and symbols), and
d) 'Effective sequencing' that expresses no one sequencing will fit every learner, but in general, increasing difficulty. Sequencing, or lack of it, can make learning easier or more difficult, see (Bruner, 1960).

Jerome Bruner proposed, based on his theory of constructivism, a discovery-oriented approach in schools. According to that, learning is developed as a process of constructing new ideas based on current/past knowledge. Students are encouraged to discover the facts and relationships for themselves and continually build on what they already know.

3.10. Paul Watzlawick (1921–2007)

Paul Watzlawick was an Austrian theoretician in communication theory and radical constructivism. He supervised the publication of (Watzlawick, 1984) as one of the most important contributions to constructivism. According to Watzlawick, "Radical constructivism, thus, is radical because it breaks with convention and develops a theory of knowledge in which knowledge does not reflect an objective ontological reality".

3.11. Edgar Morin (1921– …)

Edgar Morin is a French sociologist and epistemologist. He is especially well-known for his researches and theories in "complexity [of thoughts]". His research areas have been politics, sociology, education, systems biology, and visual anthropology, see (Morin, 2001; Morin and Montuori, 2008).

3.12. Humberto Maturana (1928– …)

Humberto Maturana is a Chilean biologist. He was one of the designers of the second-order cybernetics. He is well-known for creating the term ‘autopoiesis’. This term expresses a living system which re-organises itself. Such a system is self-generating and has a self-maintaining structure, see (Maturana and Varela, 1973/1980). Maturana’s work has influenced the concept of ‘constructivism’ in systems’ thinking and cybernetics.

4. WHAT DOES THIS RESEARCH MEAN BY CONSTRUCTIVISM?

As presented above, there are a number of scholars in different areas who have influenced the concept of ‘constructivism’. Anyway, it must be accepted that Jean Piaget was the introducer of the theory of constructivism. As mentioned, Glasersfled believed that “constructivism was introduced in the modern era by Jean Piaget as a way of thinking about cognition and knowledge”. Accordingly, the relevant chapters of this dissertation have cited Piaget as the originator of the theory of constructivism as well as the constructivist theory of learning. In the following I shall extract the most significant characteristics of constructivism in this research. The following itemised characteristics (under A and B) are conceptualised based on (i) my articles’ outcomes regarding the concept of ‘constructivism’, and (ii) the theories of Dewey, Montessori, Piaget, Vygotsky, Glasersfeld, Kelly, and Bruner (I am personally in favour of the theories of Piaget, Vygotsky and Bruner). The following items represent the most significant characteristics of ‘the constructivist model of knowing’ and ‘the constructivist model of learning’ in my dissertation.

A. The Constructivist Model of Knowing

1. Constructivism is an epistemology and is strongly supported by the study of the origins (genesis) of knowledge. Accordingly, it is a style of thinking about the phenomenon of ‘knowledge’. More specifically, the philosophy of constructivism focuses on epistemological junctions between the phenomena of ‘knowledge’ and ‘cognition’.

2. Constructivist epistemology is strongly concerned with the questions of

    a. how to know and to produce ‘knowing’?
    b. how knowledge may reasonably be constructed?

3. The central focus of constructivist epistemology is on the origin of any individual’s constructed knowledge. Accordingly, any constructed knowledge is recognised to be idiosyncratic.

4. Constructivism deals with ‘the developmental theory of knowledge’ and with ‘how knowledge may reasonably be assumed to be built up in an individual’s cognitive systems’.

5. The most significant attribute of the constructivist model of knowing is that there are strong interrelationships between the phenomenon of ‘knowledge’ and ‘the individualistic constructed and to-be-constructed models of knowledge’.

6. Constructivism expresses that any mind organises the world by organising itself. It is a philosophical viewpoint on how humans’ minds form and modify their personal understandings of reality. Consequently, it shall be taken into account that the phenomenon of ‘understanding’ cannot be imposed on a human being from outside (e.g., from other agents), because it only comes from the human’s within.

7. In the framework of constructivism, human beings interpret and, subsequently, understand their world. This means they can construct their own versions of the world based on their own conceptions.

8. In the framework of constructivism, there are very strong correlations

    a. between ‘constructing’ and ‘interpreting’, and
    b. between ‘constructing’ and ‘explaining’.

9. In the framework of constructivism, human beings, relying on their own conceptualisations of the world, provide supportive backgrounds for producing their own understandings.

10. Social Constructivism is a sociological theory of knowledge. According to that, the constructivist epistemology could be applied in social forms. Any human’s construction(s) in the framework of ‘social constructivism’ doesn’t negate her/his self-development and self-regulation, but, on the other hand, this kind of construction can aid the human beings in promoting the quality as well as supporting the HowNess of their self development processes.

11. In the framework of constructivism, human beings construct their own meanings with regard to their own conceptions of the world.

12. In the framework of constructivism, the semantic phenomenon of ‘meaning’ can be recognised as a function from any individual’s conceptions into her/his own updated and developed conceptions.

13. In the framework of constructivism, the constructed meanings by any individual become, directly, reflected in her/his personal understandings of the world.

B. The Constructivist Model of Learning

14. In the framework of constructivism, the phenomenon of ‘learning’ is interpreted as an active and dynamic process of knowledge construction.

15. In the framework of constructivism, the process of ‘learning’, as a role of the individual ‘learner’, corresponds strongly with her/his own constructed ‘model of knowing’. Therefore, it shall be assumed that knowledge is not passively received, but, it can be—dynamically—built up.

16. Constructivist learning is learner-centered and progressive. It states that the phenomenon of ‘learning’ is not an output of a process and it’s not a product of a development. Constructivist learning does, however, recognise the phenomenon of ‘learning’ as a development.

17. According to constructivist learning, any learner must be driven to understand the world and to change her/his understanding in the face of mistaken predictions about the world. It shall be stressed that constructivist learning highly respects any individual learner’s choices, admirations, and backgrounds.

18. Constructivist learning is strongly concerned with any individual human’s

    a. self-organisation and re-organisation and, respectively, self development,
    b. self-regulation, and
    c. auto-education.

19. Constructivist learning motivates any individual learner to explore and discover the world by her(him)self. We shall take into consideration that these explorations are how learners can make sense of the world and of themselves.

20. In the framework of constructivism, learners must be given open-ended problems.

21. In the framework of constructivism, knowledge could be interpreted to be constructed over humans’

    a. constructed concepts and their produced conceptions,
    b. constructed conceptual relationships between concepts and, subsequently, between their own conceptions, and
    c. produced rules based on their inferences and reasoning processes.

    Note that all these items become constructed and produced though humans’ [experience-based] constructed world.

22. In the framework of constructivism, learners must be allowed to express their own conceptions of the world. Their existing conceptions must be taken seriously. If not, learners will revert back to their own conceptions of the world outside of the learning environment.

23. In the framework of constructivism, learners

    a. relate their new conceptions to their pre-conceptions and experiences,

b. integrate their knowledge into interrelated conceptual structures,
c. design personal patterns for themselves, and
d. reflect on their own understanding of the world.

24. In the framework of constructivism, learners construct their own meanings [of what they learn] with regard to their own experiences and conceptions of the world.

25. In the framework of constructivism, a mentor is an individual who has higher abilities in particular domains. She/he must be interpreted as an advanced learner. Mentors are required to act as facilitators whose main functions are to help learners become active participants and constructors.

26. In the framework of constructivism, the process of mentoring proceeds towards developing constructed knowledge structures.

27. Any constructivist interaction and conversational exchange between a learner and her/his mentor as well as other learner(s) share multiple conceptions, cognitive contents, and understandings with varying degrees of consciousness and intentionality between them.

28. Any constructivist interaction and conversational exchange can be interpreted as agents' co-activations consisting of their shared actions and transactions. More specifically, both agents and, in fact, constructors, are concerned with their co-activations (i.e., their collaborations, co-operations, and co-ordinations), see (Allwood, 2013). In such a framework, mentor and learner exchange questions, answers, actions, and transactions concerning their multiple descriptions, specifications, explanations, and justifications.

29. Any learner, through her/his constructivist interactions and conversational exchanges, can

    a. construct her/his personal model (i.e., mental representation as well as mental construction) of knowledge, and
    b. explore the key elements that could lead her/his progress to the creation and shared understanding of various phenomena.

4.1. CONSTRUCTIONISM

As mentioned, interpreting the phenomenon of 'learning', could also link us to some [more-]specific denotations of 'learning'. For example, we can have various interpretations of (i) learning to learn, (ii) learning about learning, (iii) learning how to learn, and (iv) how learning supports making and producing. Such concepts are mainly interpretable by 'constructionism'. The central focus of constructionism, as a learning theory, is on the constructivist presupposition that 'learners create, as well as develop, their own constructions of knowledge by constructing their personal conceptualisations and conceptual (as well as mental) representations'. It could be assumed that the central focus of constructionism is on:

a. conceptualising the phenomenon of 'learning', and
b. learning how a human being can learn.

Papert's theory of constructionism is concerned more with the 'art of learning'. Papert's constructionism is strongly related with 'making' and 'productivity' through learning processes, see (Papert, 1993; Ackermann, 2002). Papert was interested in interpreting and expressing how learners could relate themselves with either their own or other's knowledge construction(s). Consequently, he was interested in analysing how these relations ultimately facilitate the construction of more-organised knowledge structures.

In my opinion, constructivism and constructionism are not two distinct and separated concepts, but they are strongly tied together and support each other. Actually, constructionism is a constructivist learning theory as well. The most significant mutual goals of constructivism and constructionism is creating one's own knowledge by constructing conceptual (and mental) representations. Conceptual representations might be recognised to be of the most important cognitive functions in human beings, see (Hampton and Moss, 2003). Furthermore, it shall be taken into account that constructionism is a complement for constructivism. This means that constructionism shares constructivism's view of learning as 'building knowledge structures' through progressive internalisation of processes. It's plausible to conclude that:

- the main idea of constructionism is that human beings learn effectively through creating, constructing, and developing [by themselves].

At this point, I shall stress that what I have analysed, and will analyse, under the label of 'constructivism', could be expressed by constructionism (and to be shared with constructionism) as well. In fact,

a. the most central framework of this PhD research has been 'constructivist model of knowing and constructivist theory of learning', and
b. the constructivist model of knowing has analysed 'personal knowledge building by constructing personal conceptual representations'.

It shall be taken into consideration that the afore-mentioned items could be believed to cover constructionism as well.

5. CONSTRUCTIVISM AND MEANING CONSTRUCTION

The meaning of something is what it expresses or represents, see (Cambridge Online Dictionary, 2017). Meanings are related to the importance, value, worthiness, authentication, authenticity, and precision of what they express. A process of meaning making by a human being, or so-called ‘meaning construction’, is strongly related to her/his own ‘interpretation’ and ‘construing’, see (Ignelzi, 2000).

Human beings, by constructing meanings for themselves, move towards making sense of their world, their lives, their goals, and themselves. It can be assumed that there is a logical bi-directional relationship between ‘meaning construction’ and ‘constructivism’. According to (Hein, 1999), ‘meaning construction’ and ‘constructivism’ are related to each other in important ways. These two concepts can be realised to be the conclusions of each other in different contexts.

The modern educational and pedagogical theories have stressed the learners’ active interconnections with the phenomenon of ‘learning’ and with their active participations in ‘learning environments’. It’s undeniable that significant contributions of modern researches in learning and knowledge acquisition sciences have been achieved based on special focuses on the processes that human beings use and/or become concerned with, more than based on the structure and context of the material to be learned. It shall be taken into consideration that all humans, constantly, organise and select the information their senses take in from the natural world as well as from the symbolic and cultural worlds of words, signs, and symbols. This is how any individual human being makes sense of her/his world.

Meaning construction and making sense of the world is beyond ‘learning’, but it is all about being ‘human’, being intelligent, and having interaction with self and with the environment. As mentioned, Piaget believed that human beings, with regard to their own experiences and ideas of the world, become concerned with meaning making. Piaget strongly believed in construction of meanings by human beings with regard to their own made senses of the reality and conceptions of the world. I assume that Piaget’s belief in the concept of ‘meaning construction’ motivated him to focus on the development of his theory of constructivism. Piaget’s theory of constructivism argues that humans produce knowledge and form meanings based upon their experiences.

In this dissertation, ‘meaning construction’ is considered the most significant task of constructivist learning. This means that ‘meaning construction’ is located at the center of constructivist learning/mentoring processes. In my theoretical model, learning is

- an active and dynamic process of knowledge construction,
- a role of learners that corresponds with their own constructed models of knowing, and
- strongly correlated with learners’ own built-up constructions of meanings.

More specifically, human beings construct meanings in order to make proper linkages between ‘their conceptions of the world’ and ‘their understandings of the world’. More clearly, in the framework of constructivism, there are strong connections amongst

a. Models of Knowings,

b. Knowledge Construction,
c. Meaning Construction, and
d. Meaningful Understanding Production

According to Figure 1, the produced meaningful understandings are supported by constructed meanings. Also, the constructed meanings are created and developed by the constructed knowledge as well as the models of knowings. It shall be emphasised that 'any constructed knowledge by an individual is an outcome of her/his modelled knowings'. In other words, the constructed knowledge has strong roots in how humans, mentally, shape and sketch what they know. In addition, as the most significant conclusion of Figure 1, it must be stated that the constructed meanings are conceptual as well as semantic linkages between the constructed knowledge and the produced understandings.

It shall be claimed that the constructed meanings by any individual human being are the basic materials of her/his produced meaningful understandings. This conception is highly in line with (Selvi, 2013). For example, Selvi has stated that (i) 'knowing' refers to making meanings and (ii) 'knowing' is the creative process of becoming 'self-being' in life-world. In addition, he has expressed that (iii) learning is the way of knowing that occurs during the search for meaning, which is an individualistic process.

At this point, I also feel the need to pick up a quote by Bruner that states: "In reference to right answers – Knowing is a process, not a product". In my opinion, "knowing, as an active process of 'being a human', formulates and constructs meanings in order to become valuable and influential and, consequently, to support meaningful understandings".

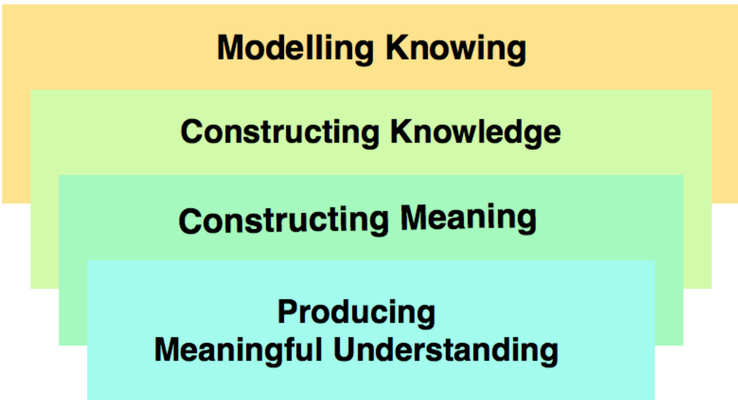


Figure 1. *From Knowing to Understanding in the Framework of Constructivism*

5.1. MEANINGS IN THE FRAMEWORK OF CONSTRUCTIVISM

In this research, meanings are 'conceptual structures'. As such, in the framework of constructivism, meanings, to a large extent, influence the individuals' constructions and developments of their personal experiential reality and even their fictions. Hence, in the framework of constructivism, meanings, as conceptual structures, are constructed based on conceptual entities. Additionally, meanings, as open-ended conceptual structures, are shaped based on undetermined numbers of developments and updatings of the linked collection(s) of the conceptual entities. By a metaphor, any conceptual entity could be described as a building block of a conceptual structure. Furthermore, in my opinion, conceptual entities are made of mental entities. More specifically, regarding my theoretical model, human beings' mental images of conceptual entities are visualised based on their mental structures. Also, the mental structures are constructed based on the mental entities.

At this point, I shall stress that we can have different conceptions of conceptual entities and, of course, there is no absolute schema for conceptual entities. In my theoretical model, any conceptual entity is known as what some approaches, like, e.g., (Bartlett, 1932; Peacocke, 1992; Zalta, 2001), have called 'concepts'. Therefore, in my model, any conceptual entity, as a representation of a part of reality/fiction in individual's mind, is a building block of a meaning. In parallel, we cannot have an absolute conception of mental entities. As Piaget argued, all learning was mediated by the construction of mental entities (schemata). I also believe that these mental entities are equivalent to what I have expressed under the label 'mental entities', see Figure 2. I will propose a more specific description of Figure 2 in Part II (Constructivist Concept Learning).

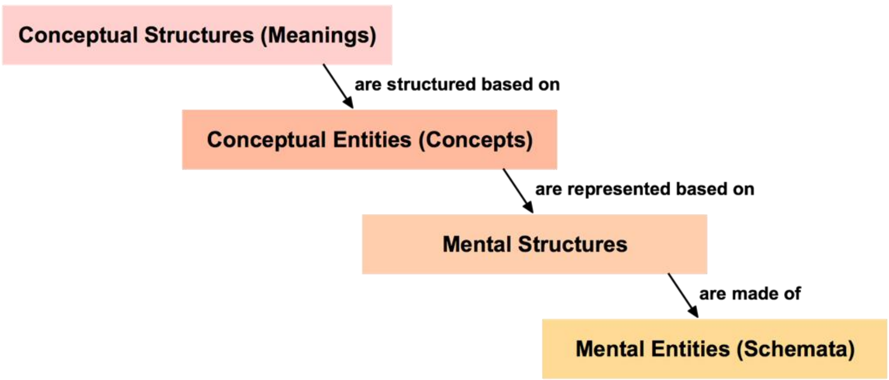


Figure 2. *From Meanings to Schemata in the Framework of Constructivism*

5.2. MEANING CONSTRUCTION IN THE CONTEXT OF INTERACTIONS

Constructivism has a stable root in the analysis of communication between language users, see (Postlethwaite and Husén, 1994). According to (Shannon, 1948), Shannon's mathematical theory of communication confirmed that only directives of choice and combination could travel and be transformed between communicators, but not the meanings that have to be ‘selected’ and ‘combined’ to interpret a message. Therefore, it shall be taken into consideration that any human being, as a language user, builds up her/his own meanings on the basis of her/his individual experiences. Hence, the meanings remain subjective and internal; no matter how much any constructed meaning becomes modified, developed, and homogenised through the subject’s interactions with other humans.

In the context of social interactions, learners internalise their constructed meanings. Respectively, they construct their personal meanings with regard to their interactions with other agents and, thus, they internalise those meanings. This seems to be in line with Vygotsky’s analysis of meaning making processes, see (Holbrook, 2012).

In the context of human-human communications and interactions, every individual constructs her/his own experiential reality and, respectively, produces her/his own realisation and meaningful understanding. For example, one can find some applications of this approach in psychotherapy, e.g., (Elkaïm, 1983; Keeney, 1983), or some approaches in literary studies, e.g., (Schmidt, 1983). According to (Schmidt, 1983), meanings are not materially inherent in words as well as texts which are becoming transformed between agents, but they must be supplied by any human from her/his individual stores of experiential abstractions.

Meaning influences one’s constructions of her/his personal experiential reality/fiction (Painting: Sun in Hand by Ian Lee Oliver)

PART II. CONSTRUCTIVIST CONCEPT LEARNING

1. CONCEPTS

As you will see in section II, there has always been a general problem concerning the sense, notion, and concept of ‘concepts’, in linguistics, psychology, philosophy, and computer science. Over the years, the concept of ‘concepts’ has not been used consistently and it is not always transparent

- if concepts are some mental representations (images) of various phenomena, or
- whether they always have to be bound up with some linguistic expression (in order to be expressed).

There is no doubt that the concept of ‘concepts’ is rather vague and imprecise. Note that this research doesn’t deal with the historical and basic epistemological treatment of concepts. This is taken up by, e.g., (Gauker, 2011) including the modern relevance of the historical notions. This dissertation, regarding the heuristic question of how knowledge may reasonably and logically be assumed to be constructed by a human being in order to be meaningful and be understood, focuses on logical conceptualisation of concepts. In order to conceptualise a concept from the perspective of logics, we need to consider it as a logical-assessable phenomenon (e.g., logical mathematical phenomenon). Consequently, such a logical-assessable phenomenon can be applied to different contexts, e.g., learning and knowledge acquisition systems.

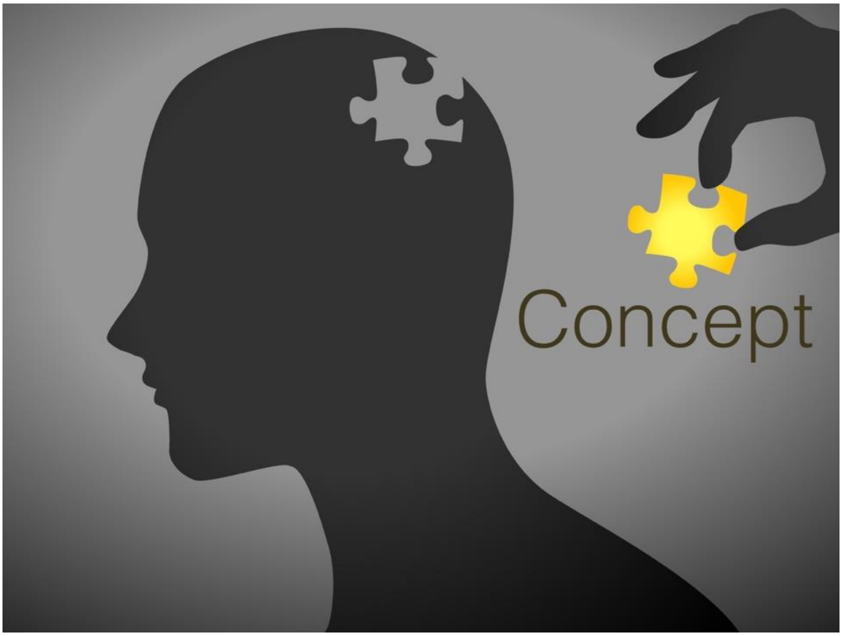
My research has aimed at providing a conceptual, logical, terminological, and semantic analysis of the application of concepts and thus, I have needed to assume concepts’ usages in order to be comprehensible in descriptions, theorisations, and logical formalism. In this research, ‘concepts’ are the main conceptual constituents. They have been considered as the primary and fundamental units of humans’ [background and to-be-constructed] knowledge, and as the basic materials of humans’ [pre-constructed and to-be constructed] meanings. I have decided to focus on the concept of ‘concepts’ and assess them through the lenses of Predicate Logic and Set Theory. This means I have perceived any concept as a ‘class’. In fact, a concept has—mainly—been utilised, analysed, and applied as a ‘set’ or as a ‘collection of multiple sets’. Accordingly, I have assumed that any individual phenomenon, as a member of a class, holds its own characteristics and attributes. Respectively, all members of a class share, at least, one significant characteristic with each other. The common characteristic(s) which is/are sharable by:

i. all members of a class (as elements of a set), and
ii. all sub-classes of a class (as subsets of a set)

could be seen as the conceptual label(s)/index(es) of that class. In addition, different classes are relatable to each other by means of logical symbols and operations. My included articles in Section II, either directly or indirectly, focus on denotational (extensional) and connotational (intensional) aspects in learning contexts. Specifically, the articles (M, N, O) focus on logical analysis of these aspects.

1.1. MANAGING CONCEPTS

My theoretical model has focused on concepts through the lenses of Predicate Logic and Set Theory. This means that my model is strongly dependent on the concept of ‘classification’. I have also had a special attention to the concept of ‘induction’. Correspondingly, the model expresses that human beings’ reasoning processes are mainly structured over their mental abilities of classification and induction. Consequently, (a) Classification-based Learning and (b) Induction-based Learning (Inductive Learning) are the most central reasoning as well as learning theories within my approach. You will see that Concept Learning has been specified and analysed based upon these two theories and relying on a constructivist model of learning.



There has always been a general problem concerning the sense, notion and concept of ‘concepts’.

2. CLASSIFICATION-BASED LEARNING

This section focuses on the question of how we can find a conceptual and epistemological linkage between the concepts of ‘classification’ and ‘constructivism’. According to Bruner, “To perceive is to categorise, to conceptualise is to categorise, to learn is to form categories, to make decisions is to categorise”. In his opinion, there are bi-conditional and supportive relationships between the phenomenon of ‘classification’ and the phenomena of ‘perceivedness’, ‘conceptualisation’, ‘learning’, and ‘decision making’. More particularly,

a. there is a strong correlation between ‘perceiving something’ and ‘classifying that thing’, and
b. conceptualising something is strongly related with classifying that thing and considering it as a member of a class.

These two items are highly in line with my ideas. I shall draw your attention to the most important points regarding the classification-based learning theory in the framework of constructivism,

- In the framework of constructivist epistemology, the classification-based learning, as a theory of reasoning and learning, assesses concepts as ‘classes’. According to ontological conceptions, ‘classes’ could be seen as ‘entities’. In my opinion, constructivist epistemology interprets these entities as ‘conceptual entities’, see Figure 2 (that was presented in Part I).

- There are, of course, ontological problems implied by saying that ‘concepts’ are ‘materials’ and, subsequently, they are the makers of the phenomenon of ‘semantics’. Taking the phenomenon of ‘class’ into consideration, I shall specify this conception and say that, in the framework of constructivist epistemology, the theory of classification-based learning can interpret ‘concepts’ as:

  a. basic materials of meanings, and
  b. fundamental constructors of semantics.

- In the framework of constructivist epistemology, the theory of classification-based learning can provide a supportive background for concepts to become ‘manifested’ and, accordingly, to become ‘expressed’.

- In the framework of constructivism, the phenomenon of ‘classification’ can be interpreted as a ‘process of constructing’.

- In the framework of constructivism, the concept of ‘constructing’ is highly interrelated with the concept of ‘representation’.

- In the framework of constructivism, any ‘classification’ corresponds to an ‘assignment’. In other words, ‘classifying’, as a task as well as role of human beings, deals with their ‘assigning’.

- In the framework of constructivism, classifications are actualised with regard to one’s determined and specified labels on the basis of her/his own ‘conceptualisations’.

- In the framework of constructivism, humans create multiple either related or non-related classes in their minds in order to construct as well as develop knowledge based on those classes. Respectively, they can construct and develop knowledge based on the members of those classes.

- The previous process might be activated inversely. In fact, humans may move from the elements of classes (‘children’) to the main classes (‘parents’) in order to focus on developing their constructed knowledge.

- In the framework of constructivism, classes are representable in the form of conceptual entities in individuals’ minds and within their own conceptual structures. More specifically, classifications are strongly dependent on mental representations of one’s constructed as well as to-be-constructed conceptual constructions, see Figure 2.

- In the framework of constructivism, the mental structures become built up based on mental entities (schemata), see Figure 2.

- In the framework of constructivism, the concept of ‘construction’ can express ‘the creation of an abstract entity’. Accordingly, there is a logical equivalence between

    a. the creation of an abstract entity, and
    b. the creation of a class as a conceptual entity.

- In the framework of constructivism, there is a strong bi-conditional relationship between

    a. classification (classifying a phenomenon as belonging to a class) regarding hierarchical viewpoints, and
    b. construction (providing a mental construction and representation of a phenomenon).

- In the framework of constructivism and through the process of classification-based learning, the notions of ‘classification’, ‘representation’, ‘construction’, and ‘abstraction’ are linked with each other.

- In the framework of constructivism, human beings, with regard to their own conceptions of the world, make their own sense of the world by their own classification and, subsequently, their own construction processes based on their own conceptual structures, conceptual entities, mental representations, mental structures, and mental entities, see Figure 2.

3. INDUCTION-BASED LEARNING

Through induction-based (= inductive) reasoning, human beings extrapolate and analogise their own body of evidences, which are achieved based on their own experiments, to more extended situations in the form of logical conclusions. Inductive reasoning as well as inductive learning processes based on the concept of ‘induction’ provide the humans with a mental system of evidential support that extends the concept of ‘deduction’, as well as the process of deductive reasoning, to less-than-certain inferences. Note that deductive reasoning is the process of reasoning from a set of established, certain, and meaningful logical premises.

Human beings construct logical (and meaningful) premises for their own inductive inferences/arguments in order to become prepared to be capable of providing some degree of support for coming up with their own logical (and meaningful) conclusions. Therefore, it’s assumed that the obtained meaningful conclusions work as conceptual as well as logical linkages between their own body of evidences and their own meaningful understandings, see Figure 3.

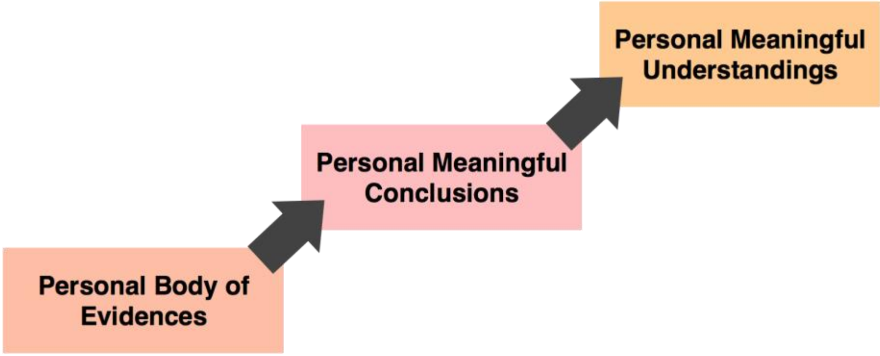


Figure 3. *Meanings within Inductive Reasoning (and Learning)*

4. CONCEPT LEARNING

In this research, the expression ‘Concept Learning’ will be analysed based upon the concepts of ‘classification’ and ‘induction’. The mechanism is assumed to be supported by the supposition that the generated logical premises presumed any individual human being are collectable, classifiable, and archivable in her/his mind. Therefore, these are some important assumptions:

- Through concept learning processes, the quality and HowNess of mental representations of logical premises are collectable, classifiable, and serialisable in the form of ‘hypotheses’ in order to be induced.

- Any human being deals with specification, with various degrees of complexities, of her/his own conceptualisations through her/his concept learning processes.

- Concept learning is meaningful and activable

    a. based on humans’ preconceptions,

b. over humans’ background knowledge with regard to their own conceptualisations of significant characteristics and properties of concepts, and
c. through experiencing various groups of examples [that share the central characteristics] of those concepts.

5. CONSTRUCTIVIST CONCEPT LEARNING

Certainly, there is no compelling reason to claim that concept learning must necessarily be structured based on the processes supported by constructivism. I do, though, strongly believe that there is a considerable epistemological junction between constructivism and concept learning that could support me in assessing and interpreting ‘concept learning’ with regard to a constructivist model of knowing as well as learning. Concerning this epistemological interconnection, the following conclusions can be offered:

- Constructivism as an epistemological framework is capable of generating Constructivist Concept Learning, see Figure 4.

- Constructivist Concept Learning might be seen as any individual’s conditional reasoning with regard to her/his own pre-constructed, constructed, and to-be-constructed concepts.

- Relying on the previous conclusion, Concept Learning is theorisable as one’s conditional reasoning with regard to her/his own pre-conceptions, conceptions, and to-be-expressed conceptions of the world.

- In the framework of constructivist concept learning, the conceptual and logical interrelationships between ‘meanings (as conceptual structures)’ and ‘concepts (as basic materials)’ establish a kind of semantics based upon humans’ personal conceptions of the world.

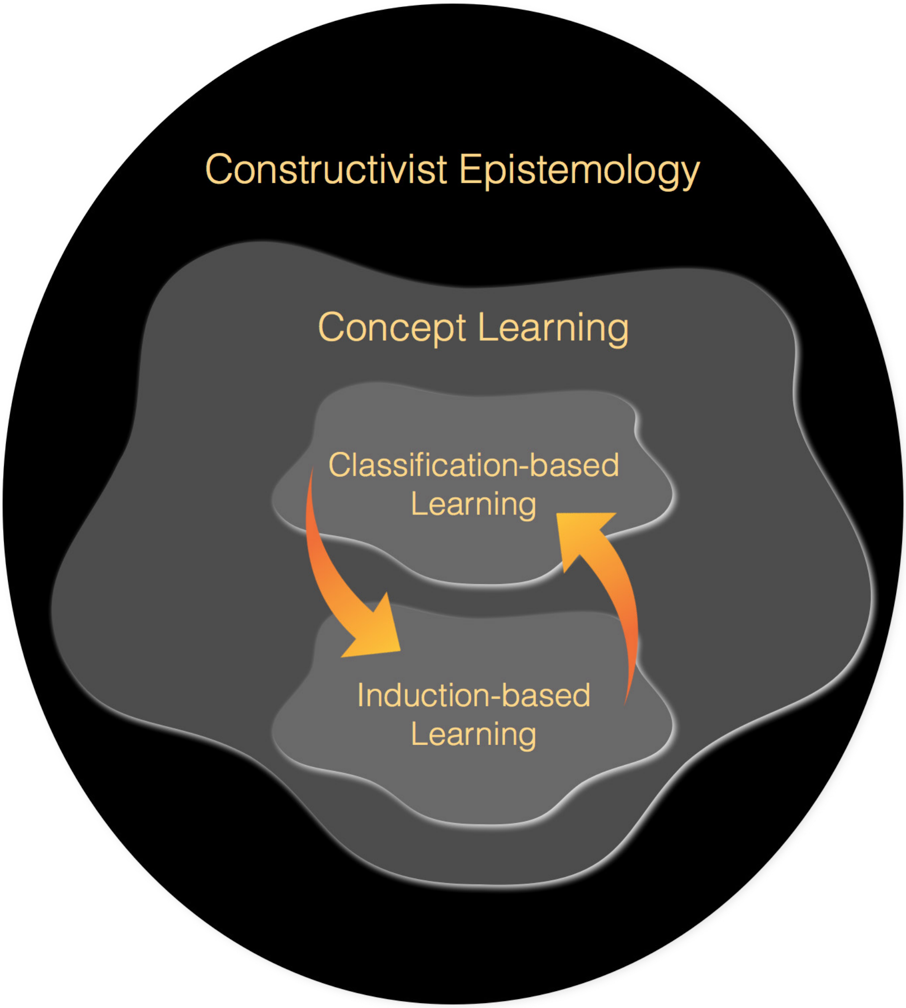


Figure 4. *The Structure of Constructivist Concept Learning*

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SECTION II

ARTICLES

PAPER A. A SEMANTIC BASIS FOR MEANING CONSTRUCTION IN CONSTRUCTIVIST INTERACTIONS

Farshad Badie

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The layout has been revised.

ABSTRACT

Regarding constructivism as a learning philosophy and/or a model of knowing, a person (learner or mentor) based on her/his preconceptions and on personal knowings could actively participate in an interaction with another person (learner or mentor) in order to construct her/his personal knowledge. In this research, I will analyse 'meaning construction' in the framework of constructivism. I will focus on a semantic loop that the learner and mentor as intentional participants move through and organise their personal constructed conceptions in order to construct meanings and produce their individual meaningful comprehensions. Subsequently, I will provide a semantic framework for analysing the process of meaning construction based on personal knowings and personal conceptions within constructivist interactions. This research could propose a new scheme for interpretation based on semantics and on interaction.

1. INTRODUCTION

An interaction between learners and mentors as intentional participants could exchange questions and answers concerning, e.g., description, specification, explanation, argumentation, analysing, justification, formulation, theorising etc. The multilevel agreement-oriented interactions among learners and mentors could be considered, be interpreted and be analysed based on the models of the underlying processes involved in complex human learning. As such they could be seen as a radical constructivist account of human cognition and comprehension. They are actually shaping a kind of ontology of human beings. They enable learners (and mentors) to develop their own understandings of the complex real underlying systematic processes, and also of themselves, see (Scott, 2001). Learning based on constructivism with reference to Conversation Theory, which is designed by Gordon Pask, the enterprise begins with the negotiation of an agreement between learners and mentors to converse about a given domain and learn about some particular topics and skills in that domain. It could work as an explanatory, heuristic and developmental framework. For more detailed information see (Pask, 1975; Pask, 1980).

In fact, learning based on constructivism could heuristically be concerned with the questions focusing on 'What is/does ...?', 'Why is/does ...?' and 'How is/does …?'. A person whose insights are based on her/his pre-structured knowledge, personal knowings and preconceptions may ask these heuristic questions and ask the interlocutor to produce some heuristic answers or some modified heuristic questions. What could be offered by learning based on constructivist interaction is a framework for thought and a semantic model to account for the emergence of the domain of human conceptual knowledge. As an abstract model, it is able to explain how the interactions lead humans to construct personal knowledge. In this framework, the learner (mentor) manages to construct her/his personal knowledge within interaction. Consequently, she/he gains an opportunity to attain a deeper personal understanding and greater motivation. According to constructivism, a learner is highly concerned with active 'creation' of personal mental structures. Constructivism requires negotiation of 'meaning' and 'reflection' of prior and new knowledge. Jean Piaget, the originator of constructivism, argued that all learning was mediated by the construction of ‘mental objects’ that he called ‘schemata’. For Piaget, schemata first emerge as concrete actions and then gradually develop into more abstract and conceptual mental entities, see (Husén and Postlethwaite, 1989; Spiro et al., 1992; McGaw and Peterson, 2007; Sawyer, 2014).

In this research, I will, from a new perspective, analyse meaning construction as the most significant production of learning based on constructivism. I will analyse the semantic loop that the learner (and mentor) move through in order to construct their personal meanings. I will deal with how the learner (mentor)

i. organises the personally constructed concepts,
ii. produces meaningful meanings, and
iii. attains a deeper comprehension.

I will finally provide a framework in order to demonstrate different steps of meaning construction based on personal constructed concepts within constructivist interactions.

2. CONCEPTS

I shall emphasise that there is a general problem concerning the notion of ‘concept’, in linguistics, in psychology, in philosophy, in metaphysics, in computer and information sciences, but, for now, I assume the use of the expression concept to be comprehensible in the context. Walter Parker writes: “concepts are the furniture of our minds. A well-furnished mind is a source of success and lifelong learning. When a student forms a concept from its examples, he or she knows more than the definition of a term. This is deep conceptual learning rather than superficial knowledge of a vocabulary word” (Link a). He also says that “a concept is defined by critical characteristics shared by all examples of the concept. For something to be an example of a concept, it must contain all these critical characteristics”, see (Parker, 2008). Generally, a concept is a unifying theme for something. In ontology, a concept is a fundamental category of existence. Following (Margolis and Lawrence, 2011), concepts could be understood as the mental representations, where concepts are entities that exist in the brain. They could also be understood as abstract objects, where objects are the constituents of propositions that mediate between thought, language, and referents. In my research, a concept is an idea which corresponds to some ‘distinct entity’ or ’class of entities’ or to its ‘essential features and attributes’. It can determine the application of a term (especially a predicate), and thus plays a part in the use of reason or language (cf. Rudolph, 2011; Baader et al., 2010). Analytically, a concept is a linkage between linguistic expressions and the mental images (representations of the world, of inner experiences etc.) that humans have in their minds, see (Götzsche, 2013).

I focus on concepts (classes) because concepts and the relationships between them are used to establish the basic terminology adopted in my modelled pedagogical domain regarding the hierarchical structure. For example, relying on Description Logics (see (Baader et al., 2010)), the concept 'Mentor' can be analysed as a 'concept description' (descriptions mainly follow the inductive rules) that demonstrates the mentor as "a person who has a learner and the learner is a person". Formally: $\mathsf{Mentor} \equiv \mathsf{Person} \sqcap \exists \mathsf{hasLearner.Person}$. This concept construction is able to support the formal, explicit specification of a shared conceptualisation of Mentor based on the constructor's conceptions.

3. DEFINITION AND DEFINITENESS

A definition could semantically be seen as a kind of equation whose left-hand side is a concept and whose right-hand side is a description. They are used to introduce symbolic names for complex descriptions. In a pedagogical system learners and mentors could define something based upon multiple concept descriptions. Actually, these definitions could be constructed based on their own conceptions and background knowings. Logically, a set of definitions is (and must be) 'explicit' and 'unequivocal', ie. not vague and not ambiguous. In fact, 'explicitness' and 'unequivocality' are the prerequisites and preconditions for definiteness. So, a set of someone's definitions in a category of her/his constructed concepts could provide a backbone for any construction. Then the provided backbone supports the person in defining more complex concepts and descriptions over abstract concepts. Subsequently, the learner (mentor) could employ inductive rules on her/his personal definitions of abstract concepts in order to produce more complex definitions for more complicated described concepts. For example, considering the description '$\mathsf{Person} \sqcap \mathsf{Male} \sqcap \exists \mathsf{hasLearner.Person}$' for '$\mathsf{MaleMentor}$', the '$\mathsf{MaleMentor}$' has been defined by being associated to a description.

4. SEMANTIC INTERPRETATION

Generally, the act of elucidation, explication and explaining the meaning of something is called interpretation, see (Simpson and Weiner, 1989). Humans need to attempt to provide a way to determine the truth values of sentences. Linguistically, interpretation is the continually adjusted relation between the conventional meanings of sentences/statements and the actual mental universe of the individual (based on accumulated experience of that individual). Logically, an interpretation is an assignment of meanings to non-logical symbols. For instance, it can not assign meaning to logical symbols $\{$Not ($\neg$), And ($\sqcap$), Or ($\sqcup$), Equality ($=$), Equivalency ($\equiv$)$\}$. Actually, we cannot assign any meaning to a description until the non-logical symbols are given interpretations. Considering C and D as two concepts and $R(C,D)$ as any possible binary relationship between them, one could have different types of interpretations based on them. More specifically, the interpretation I assigns to C a set that contains the interpretation(s) of C, and it also assigns to R a binary relation between the elements of two sets (interpretation(s) of C and interpretation(s) of D). Actually, in translating from an informal (commonly English) language into a formal language, we need to provide symbolisation keys, which are the interpretations of all the non-logical symbols we use in the translation, see (Prior, 1955). In case a given interpretation could assign the value True to a sentence (or theory), that interpretation is called a model of that sentence (or theory). In fact, designing a proper model can make the definitions adequate.

5. INTERACTION AND MEANING CONSTRUCTION

A person who undertakes to learn (train) something within a constructivist interaction primarily focuses on 'concept formation'. She/He initiates to ask some questions and answer other questions asked by the interlocutor. In my research, the most significant necessity of modelling meaning construction in a constructivist interaction is to consider the personal mental structures (schemata). They have been created regarding what the participants (both learner and mentor) have been affected by. The person inductively develops her/his mental entities. The learner (mentor) finds new concepts for herself/himself. Hearing different words from the interlocutor could be conducive to new conceptions. A person may have formed a concept before participating in the constructivist interaction and then, regarding the feedbacks produced by interlocutor, tackles to reform them. So, in fact, 'forming', 'transforming' and 'reforming' concepts are three significant matters in constructivist interactions. The individual has to 'generalise' from different examples and this may lead her/him to discovering new concept(s). She/He searches for and lists attributes and properties that can be used to distinguish exemplars from non-exemplars of various concepts (classes). But what she/he really does is more than generalising from different examples that she/he hears or produces. More specifically, she/he identifies, specifies and relates the generalised examples and 'compares' different examples. In fact, a very efficient way to form a new concept and induce new categorisation rules in constructivism is to compare a few individuals when their categorical relation(s) is known. On the other hand, she/he could be able to make her/his personal labels for categorising the concepts in order to direct and employ different classes of concepts.

5.1. THE SEMANTIC PROCESS

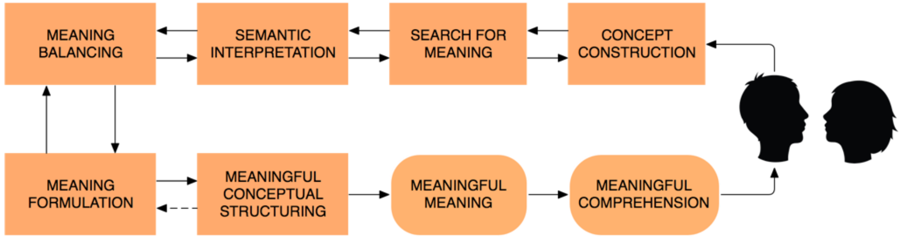

Figure 1. *Meaning Construction within Constructivist Interaction*

At the beginning, the person, mentally, designs some 'schemata'. Then, she/he gradually develops them and divides them into more abstract concepts (conceptual entities). A proposed schema describes a pattern of the person's thought. I have already said that she/he could either categorise different concepts or follow the categorised concepts. So, in fact, her/his designed schema could support her/him in managing those concepts. I label the first semantic phase ‘Concept Construction’ (presented in Figure 1), where the individual constructs her/his personal concepts and conceptions based on her/his personal schemata.

Subsequently, she/he needs to focus on the ‘reflection’ of prior knowledge (what have been acquired/created before interaction) and new knowledge (what is being acquired/created during the interaction) and the initial meanings. So, in fact, she/he ‘searches for the (initiative) meanings’ of the class/classes of constructed concepts and their significant relationships, see ‘Search for Meaning’ in Figure 1. Accordingly, she/he defines her/his constructed concepts and searches for the initial definitions of the constructed concepts.

Consequently, she/he focuses on the ‘interpretation’ of the initial meanings and definitions. From the logical point of view, the interpretation of a constructed concept is a ‘function’. Generally, this function assigns a ‘meaning’ to a ‘symbol’. Formally: *Interpretation*: *Meaning → Symbol*. The Interpretation functions operate the person’s definitions based on her/his constructed concepts. Therefore, they ‘activate’ the meanings. This phase is presented as ‘Semantic Interpretation’ in Figure 1.

Accordingly, concerning ‘*Interpretation*: *Meaning → Symbol*’, meaning is the product of the inverse of interpretation function (*Symbol → Meaning*). I label this phase ‘Meaning Balancing’, see Figure 1. There is a strong relationship between Semantic Interpretation and Meaning Balancing. The person could be able to balance and adjust the initial meanings based on the interrelationships between ‘the interpretation’ and ‘the inverse of the interpretation’.

The conclusions make an appropriate background for verifying the [personally] found meanings based on personal constructed concepts. Meaning Balancing is quite supportive in balancing the personal definitions and vice versa, see ‘Meaning Formulation’ in Figure 1. The person formulates the balanced meanings based on the balanced definitions of her/his personal constructed concepts. There is an appropriate relationship between formulated meanings and balanced definitions. In fact, a meaning would be given a better shape after checking the balanced definitions.

The formulated meanings organise (as well as re-organise) and reinforce the mental structures that the learner (mentor) uses them (as the pattern of her/his thought) in order to develop the individual conceptual knowledge. Subsequently, the formulated meanings are some applicable prerequisites for Meaningful Conceptual Structuring (see Figure 1) upon personally formulated meanings and based on personal constructed concepts. On the other hand, the meaningful conceptual structures could induce new formulated meanings on higher conceptual levels (presented by dashed arrow in Figure 1). And, furthermore, the new formulated meanings are considered as new schemata in constructing higher levels of conceptions.

Finally, the meaningful conceptual structures support her/him in providing meaningful meanings. The meaningful meanings highly reflect on the constructor and support her/him in proposing the modified schemata on higher conceptual levels. So, the person moves through this semantic loop in order to organise her/his personal constructed concepts, to construct her/his personal meanings, and to produce meaningful meanings.

6. CONCLUSIONS AND FUTURE WORK

Constructivism has been known as a learning philosophy and/or a model of knowing. In the framework of constructivism, two persons whose insights are based on their preconceptions and on personal knowings can actively participate in an interaction. The most important objective is to construct their personal knowledge, to learn from each other and to train each other. Therefore, they have an opportunity to attain a deeper personal understanding, comprehension and greater motivation. In this research, I have focused on 'meaning construction' as the most significant production of constructivist learning. I have worked on a semantic loop that the intentional participant in a constructivist interaction moves through. She/He constructs her/his personal concepts based on concept formation, defines them (produces individual definitions based on constructed concepts), and organises concepts and definitions in order to construct meanings and produce meaningful meanings. Meaningful meanings support her/him in constructing knowledge, producing meaningful comprehension and reacting more appropriately in front of the interlocutor's acts. I have provided a framework for analysing meaning construction based on individual comprehension and personal concept constructions within constructivist interactions.

In fact, the proposed loop, semantically, transforms multiple constructed concepts into meaningful meanings (and meaningful comprehensions). It could be observed as a new scheme for interpretation based on 'semantics' and on 'interpretation'. Obviously, the proposed semantic loop is self-organised. Equivalently, it promotes itself on higher conceptual levels. In future research, I will focus on the logical analysis of meaning construction within constructivist interactions and work on its formal semantics. I will employ some fundamental descriptions in Concept Language (Description Logics: DLs) in order to analyse multiple semantic concepts within my progress and provide a DLs-based formal semantics for analysing meaning construction in constructivist interactions. Subsequently, I will be concerned with semantic analysis of meaning construction based on personal knowings and personal concept constructions within constructivist interactions. I will check the validity of the logical descriptions in conceptualising constructivist learning concerning the 'Structure of Observed Learning Outcomes (SOLO)'. The consequences will make a backbone for better conceptualisation of human's understanding. And, the results will be employed in the analysis of formal semantics in terminological knowledge for pedagogical knowledge representation systems. They can conceptually analyse pedagogical developments in the framework of constructivism and in the context of interactions for promoting human's understanding.

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Links

Link a: http://teachinghistory.org/teaching-materials/teaching-guides/25184

PAPER B. TOWARDS A SEMANTICS-BASED FRAMEWORK FOR MEANING CONSTRUCTION IN CONSTRUCTIVIST INTERACTIONS

Farshad Badie

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ABSTRACT

Constructivism is known as either a learning philosophy or a model of knowing. It is possible to say that a successful learning science may be constructed and developed based on the proper foundation that is provided by constructivism. Jean Piaget, the originator of constructivism, argued that all learning was mediated by the construction of mental objects that he called schemas. From his point of view, schemas gradually develop into more conceptual mental entities. According to constructivism, a learner and a mentor, can—based on their preconceptions (concept pre-formations), based on their pre-structured knowledge, and based on their personal knowings—actively participate in an interaction with each other in order to construct their personal knowledge. The most significant objective of constructivism is construction of personal knowledge and its development, and producing the own understanding of a world (universe of discourse) within the interaction. Therefore, the learner and mentor gain an opportunity to attain deeper personal understandings and greater motivations. The main contribution of the present research is the conceptual and logical analysis of meaning construction within constructivist interactions based on my conceptualisation of Definition and Definiteness, Meaning and Meaningfully, and Semantic Interpretation. In this research, I will analyse meaning construction and meaningful comprehension production within constructivist interactions. I will employ some fundamentals from 'functions and functionality' and 'Description Logics (DLs)' in order to analyse the semantics of concepts and conceptions in the course of work. I will provide a DLs-based formal semantics for analysing meaning construction in the framework of constructivism. The central focus will be on my proposed semantic loop that the learner and mentor as intentional participants move through, which organises their personal constructed conceptions in order to construct meanings and produce their meaningful comprehensions. This research will be concerned with meanings and definitions and proposes a new scheme for interpretation based on semantics and on interactions.

1. INTRODUCTION

An interaction between two (or more) agents exchanges a number of questions, answers and actions concerning their personal conceptions and conceptualisations. First, I shall emphasise that 'concept' is a term that should be used with caution and I need to explain what I mean by this term. The specification of concepts makes a proper background for describing and understanding of conceptions and conceptualisations. There has always been a general problem with concepts. For example, in linguistics, in psychology, in philosophy, in metaphysics and in information sciences we may have different notions and visions of what a concept is. So, what I will use and will express under the label of 'concept' aims at providing a comprehensible characteristic of conceptions and conceptualisations. In my semantic approach, a concept is a linkage between linguistic expressions and the mental images that a human may have in her/his mind, see (Götzsche, 2013). For instance, these mental images could be interpreted and seen as the representation of aspects of the world (and the universe of discourse). And, in fact, what is being identified as a conception of an agent within an interaction is her/his act of visualising different concepts and linking her/his expressions with regard to individual mental images and schemas.

The context of this research is the interaction between the learner and her/his mentor. Accordingly, I need to focus on multilevel agreement-oriented interactions among learners and mentors. Such an interaction could be viewed as a radical constructivist account of the learner's (and mentor's) cognition, comprehension and understanding, see (Scott, 2001) for more details. The constructivist account of an agent's understanding is capable of enabling her/him in developing the individual understandings of the more specified (and complex) concepts with regard to her/his understanding of the general concepts. Producing one's own understanding of a world and developing it during the interactions with the interlocutor could be said to be the most valuable product of the constructivist interactions. Understanding more specified concepts enable learners (and mentors) to develop their own understandings of the underlying systematic processes in reality, and also their understandings of themselves through the universe of discourse.

I also need to be careful with the usage of the term 'Constructivism'. Constructivism can be used in many disciplines, see (Baker et al., 2007). Generally, it can be classified into two main parts:

i. Constructivism as a learning philosophy and an educational theory of learning,
ii. Constructivism as a model of knowing.

By all means there seems to be a very strong interrelationship between (i) and (ii). (Husén and Postlethwaite, 1989) has specified constructivism as a theory of knowledge with separated roots in philosophy, psychology and cybernetics. Also, I have observed constructivism as a 'theory of knowledge and knowing' in my semantic approach. In my opinion, knowledge is actively constructed based upon the agent's comprehension of meanings with regard to the definitions of concepts. We will see that the knowledge of a part of the universe of discourse, will be more understandable and comprehensible by both agents after their interactions. The important common background of (i) and (ii) is that 'a learner (and even a mentor) based on her/his personal knowings and on her/his pre-structured knowledge gains an opportunity to attain a deeper personal comprehension and to develop her/his understanding'. Accordingly, she/he will succeed in constructing knowledge. I acknowledge this as the most central assumption of the constructivist interactions.

Jean Piaget, the originator of constructivism, argued that all learning was mediated by the construction of mental objects that he called 'schemas'. For Piaget, schemas first emerge as concrete actions and gradually develop into more abstract and conceptual mental entities, see (KeithSawyer, 2014; Spiro et al., 1992; McGawand Peterson, 2007; Husén and Postlethwaite, 1989; Moallem, 2001). Considering learning in the framework of constructivism with reference to Conversation Theory, which is designed by Gordon Pask, the enterprise begins with the negotiation of an agreement between the learner and the mentor to converse about a given domain and learn (and train) about some particular topics and skills in that domain. It could work as an explanatory, heuristic and developmental framework. For more detailed information see (Pask, 1975; Pask, 1980; McIntyre Boyd, 2004).

In this research, I will focus on the conceptual and the logical analysis of meaning construction within constructivist interactions. As mentioned, knowledge is actively constructed based upon the agent's comprehension of meanings with regard to the definitions of concepts. So, it seems quite important to conceptualise 'concept', 'meaning' and 'definition' and their interrelationships. I will need to analyse 'definition and definiteness', 'meaning and meaningfully', and 'semantic interpretation' within my approach. In order to analyse concepts and conceptions in the process, I will provide a formal semantics based on Description Logics (DLs), see (Baader et al., 2010). DLs is the most well-known knowledge representation formalism. The concepts (and their interrelationships) are used to establish the basic terminology adopted in my modelled pedagogical domain regarding the hierarchical structure.

I will analyse 'meaning construction' and 'meaningful comprehension production' within constructivist interactions. My main focus is on my own semantics-based framework, see (Badie, 2015). According to that, learner and mentor as intentional participants move through a semantic loop and organise their personal constructed conceptions in order to construct meanings, to improve the constructed meanings, and to produce their individual meaningful comprehensions. In this research, considering the analysis of definitions and meanings, I will develop my framework. What could be offered by learning constructively in this framework and in the context of interactions, is 'a body of thought' and 'a semantic model to account for the emergence of the domain of the learner's (mentor's) conceptual knowledge'.

2. TERMINOLOGY

Description Logics (*DLs*) is a well-known knowledge representation formalism and represents knowledge in terms of:

- individuals or objects,
- concepts or classes, and
- roles or relationships.

Individuals correspond to constants, concepts to unary predicates, and roles to binary predicates in First-Order (Predicate) Logic (FOL). In DLs individuals are the instances (or members) of a concept. For instance, John as an individual can also be recognised as an instance of the concept Person. Focusing on predicate P, which is one of the most important constructors of FOL-based expressions, we could say that P is capable of covering something, let me name that thing 'variable X'. Then, we have a $P(X)$. $P(X)$ expresses that P is describing X, see (Borgida, 1994). Reconsidering the predicate P in DLs and naming it a role, we have achieved something that can establish a relationship between various individuals.

In DLs the concepts could be divided into two types:

a. The Atomic Concepts (Concept Names), e.g., Person, Thing, Computer, Logic, Management. An atomic concept is identified by *A* in DLs. They are very general concepts and could be identified as the labels of the classes of ontologies within ontology representation systems.

b. The Complex Concepts (the specified atomic concepts), e.g., FemalePerson, WhiteWing, ComputerScience, InductiveLogicProgramming, ManagementInformationSystems.

Obviously, an agent (learner or mentor) could understand an atomic concept easier than its specified case. From another point of view, there are two kinds of atomic symbols.

a. Atomic Concepts,
b. Atomic Roles.

From this perspective, atomic symbols are defined as the elementary descriptions from which humans inductively build and construct complex (specified) descriptions by means of 'concept constructors' and 'role constructors'. The set of the main connectors in the base Description Logic is:

{Conjunction ($\sqcap$: and), Disjunction ($\sqcup$: or), Negation ($\neg$: not), Existential Restriction ($\exists$: there exists …), Universal Restriction ($\forall$: for all ...), Top Concept ($\top$: everything), Bottom Concept ($\bot$: nothing)}.

A knowledge in DLs usually consists of the 'terminological axioms' and 'assertions'. Considering *C* and *D* as concepts, *R* and *S* as roles, and *a* and *b* as individuals, we have the following terminological axioms:

i. the Concept Inclusion: $C \sqsubseteq D$,
ii. the Role Inclusion: $R \sqsubseteq S$,
iii. the Concept Equality: $C \equiv D$, and
iv. the Role Equality: $R \equiv S$.

Additionally, $C(a)$ and $R(a,b)$ denote the concept and the role assertions, see (Schmidt-Schaulss and Smolka, 1991; Baader et al., 2010).

Example. Anna has been asked to describe the concept 'Teacher'. She conceptualises 'Teacher' by "Teacher is a person who has [at least] one student". This description has been built up following the inductive rules. Translated into DLs we have: Teacher $\equiv$ Person $\sqcap$ $\exists$hasStudent.Person. This concept description supports the formal, explicit specification of the conceptualisation of the concept 'Teacher' based on

Anna's conception. Obviously, her constructed description for 'Teacher' influences her other conceptions based on 'Teacher'. On the other hand, it's also obvious that Anna's description of the concept 'Teacher' is dependent on the concept 'Student'. Therefore, for a stronger and more specified conceptualisation of 'Teacher', she must describe 'Student' on one higher conceptual level. This conceptual dependency expresses that she has been concerned with the taxonomy of various concepts and with the explicit specification of the conceptualisation of the concept 'Teacher'. Actually, she is indirectly getting concerned with an ontology. Suppose that she is an agent in an interaction, and that she will utter her conception of 'Teacher' (and, Thus, of 'Student') to her interlocutor. So, she has provided a prescription for building a block of a constructivist learning support system in the context of her interaction.

3. EXPLANATION

In this research, I assume that an explanation is the actual explaining of definitions and meanings. The main objective of explanations is to shed light on the produced personal comprehension. In this section I will focus on analysis of definitions and meanings. They will be applied in section 5.

3.1. MEANING

Linguistically, meaning can be realised as a context-update function, see (Larsson, 2012). So, the input of the meaning function is a context and the output is its updated form. Considering X as a context and X' as the updated form of X we have '*Meaning*: $X \rightarrow X'$'. Any context comprises different types (and different numbers) of concepts, i.e. atomic concepts and specified concepts. Then, we terminologically have $C \sqsubseteq X$. Therefore, I describe a 'meaning' as a 'concept-update function' like:

$$Meaning: C \rightarrow C'.$$

I will need to reconsider the meanings while I will be concerned with the interpretation functions in section 4.

3.2. DEFINITION AND DEFINITENESS

In a semantics based system, a definition can be figured out as a kind of equation whose left-hand side is a concept (the concept that is going to be defined) and whose right-hand side is a description (generally built up using the inductive rules). In the current approach, a definition is a type of introduction. Actually, a description is expressed in order to introduce atomic (and complex) concepts and roles, and their possible combinations for constructing more specified descriptions. In a constructivist interaction, the learner and the mentor introduce and define different concepts, which are produced by their individual conceptions and based on multiple concept descriptions. All these definitions have been constructed over their pre-structured knowledge. Logically, a set of definitions must be 'explicit' and 'unequivocal', i.e., not vague and not ambiguous.

Example. Though the lenses of Bob's conception, the concept 'Spring' can be defined as "the season of the moderate weather; when all the trees are green". His description can be translated into a DLs-based formalism. Then,

$$\text{Spring} \equiv \text{Season} \sqcap \exists \text{hasWeather.Moderate} \sqcap \forall \text{hasTree.Green}.$$

In fact, he has defined the concept 'Spring' by associating 'Spring' with a description, which has been built up based on his own conception.

In a constructivist interaction, any of the agents may define a concept based on her/his individual conception. Accordingly, regarding the feedbacks of the interlocutor, she/he modifies and updates her/his definition. I have called it 'definition updating'. Here are two important facts:

i. A defined definition by the learner (mentor) provides a supportive backbone for performing a more developed (and organised) concept description(s).

ii. A more developed (and more well-organised) concept description supports the learner (mentor) in providing a more understandable meaning on higher levels of interaction.

In this example, the interlocutor can update the spring's definition. For instance, she/he can make the definition more specified and can define Spring by "Spring is the season of the moderate weather; when all the trees are green. And April is one of its months". Equivalently,

$$\text{Spring} \equiv \text{Season} \sqcap \exists \text{hasWeather.Moderate} \sqcap \forall \text{hasTree.Green} \sqcap \exists \text{hasMonth. April}.$$

4. THE SEMANTIC INTERPRETATION

Generally, to elucidate and to explicate what we mean by something, and to explain our meanings, is noticed as our interpretation of that thing, see (Simpson and Weiner,1989). An agent who constructively focuses on the concept description C in an interaction, needs to provide a way to determine the truth values of her/his sentences concerning C. Actually, these sentences have been explained based on her/his mental images of C. Subsequently, she/he transforms her/his mental images into some linguistic expressions. It's possible to translate the final linguistic expressions into a description language like DLs. Linguistically, an interpretation is known as the continually adjusted relationship between two salient items:

i. The conventional meanings of the agent's sentences/statements. In other words, the agent's intention behind her/his expression of those sentences/statements.

ii. The actual mental universe of C, which is based on the accumulated experience of that individual.

From the logical point of view, an interpretation is an assignment of meanings to the non-logical symbols and to the non-logical parts of the sentences/statements. The interpretation cannot assign meanings to the logical symbols, like, e.g., not, and, or, equal, equivalent. An agent is not able to assign any meaning to a description until she/he interprets all the non-logical symbols of that description in her/his mind. Considering C and D as two concepts and $R(C,D)$ as any possible binary relationship between them, an agent could have different interpretations over them. More specifically, the interpretation I assigns to the concept description C and D two sets that contain their interpretation(s). It also assigns to the role R a binary relationship between the elements of those two sets (between the interpretation(s) of C and the interpretation(s) of D), see (Prior, 1955). In case a given interpretation could assign the value True to a sentence (built up over a concept description), that interpretation is called a 'model' of that sentence. And if an agent's sentence is assigned the value True for all possible interpretations, it would be a 'tautology model'.

Designing a proper model can make the definitions adequate. A compassionate mentor and a dutiful learner attempt to provide adequate models for their utterances. They also wish to perform a tautology model in order to satisfy all possible interlocutor's interpretations and to satisfy the interlocutor's mentality.

From the formal point of view, I observe an interpretation as an assignment of meanings to the non-logical building blocks of the agent's language and her/his linguistic expressions. In order to bring forth the formal semantics of the approach, I employ the function I from a non-empty set D into the set D^I. Then:

$$I : D \rightarrow D^I.$$

D may contain multiple atomic concepts and concept descriptions that the agent has in mind. Therefore, the function I

- assigns to every atomic concept A, a set like A^I (formally, $I : A \rightarrow A^I$), and
- assigns to every concept C, a set like C^I (formally, $I : C \rightarrow C^I$),

which are both the subsets of the set D^I. This procedure is also definable on every atomic (and described) role. So, considering R as a binary relation between two concepts C and D, there is a I, such that:

$$I : D \rightarrow D^I \times D^I.$$

As mentioned in the analysis of meanings, I need to reconsider the meanings concerning the interpretation functions. Here I figure out a 'function' as a 'functional role' between two concepts. Suppose that C is a concept. The interpretation I interprets *Meaning* as a functional role if and only if:

$$\{(C,C_1), (C,C_2)\} \subseteq Meaning^I \quad \Rightarrow \quad C_1 = C_2.$$

Therefore:

$$Meaning(C) = C_1 = C_2.$$

Suppose that C is a concept. C has been produced based on the agent's conception. She/he—according to her/his interpretations—produces the Meaning function by meaning C (finding an adequate meaning for it) for her(him)self.

5. MEANING CONSTRUCTION: THE SEMANTIC PROCESS

The primary step towards learning constructively within an interaction can be built up over an asked question or a committed action. The schemata could be seen as different types of concrete actions and questions. The learner (mentor) gradually develops and divides them into more general concepts. A person who undertakes to learn something and to understand it within a constructivist interaction primarily becomes concerned with various general concepts related to that thing. More specifically, she/he becomes concerned with forming (see Link a) or reforming concepts.

5.1. DESIGNING SCHEMATA

Obviously, at the beginning the learner has a shallow understanding of what she/he directs to learn, and of its situation within the universe of discourse, see (Parker, 2008). The preliminary understanding supports the learner in building various patterns in her/his mind, and any of these patterns describes the learner's thought over a conception related to a matter. These patterns could be seen as the mental structures. The learner uses these mental structures to organise her/his conceptual knowledge, and to provide strong backbones for her/his interpretations. The learner gradually elaborates the network of her/his general mental structures. Any of these patterns could be named a 'schema', see (Bartlett, 1932) and (Link b, Link c). In my semantic approach schemata are employed to

i. provide a background for the learner's concepts,
ii. specify the learner's inferences and argumentations,
iii. describe different theories based on terminological axioms and assertions, and
iv. give sufficient and satisfying conditions for definitions of truth.

The most primary phase is 'Designing Schema', see Figure 1. I shall emphasise that schemata are the most significant necessities of an adequate model for meaning construction and meaning production in my semantic approach. Note that all mentioned matters relevant for a schema are semantically valid in the case of the mentor as well.

5.2. CONCEPT CONSTRUCTION & MEANING FORMULATION

From the methodological point of view, the learner is the developer of her/his personal conceptions over the individual designed schemata. She/He needs to employ inductive rules in order to expand her/his general concepts into more specified ones. Additionally, during the interaction she/he investigates new concepts and applies them in her/his concept constructions. Hearing any word from the interlocutor, e.g., why, tell, think, compare, mean, is conducive to new conceptions and new concepts. As mentioned, a learner may have formed a concept before participating in the interaction. For example, Mary initially has conceptualised and defined the concept 'Information System' by "the Information system is a system whose components are information". This definition is translatable into '$\text{InformationSystem} \equiv \text{System} \sqcap \exists \text{hasComponent.Information}$'. Suppose that she utters her definition. Later on, regarding the feedbacks produced by the mentor, she will be conduced to performing either a more specified (and accurate) definition or a modified (reformed) definition for the concept 'Information System'.

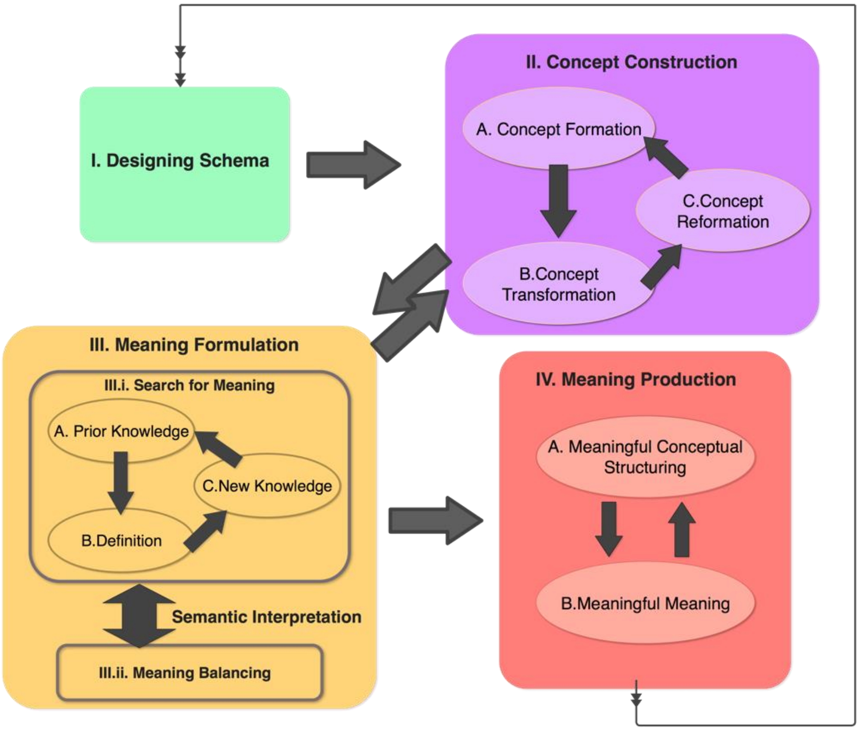

Figure 1. *Meaning Construction in Constructivist Interactions*

Notice that the collection of 'formation', 'transformation' and 'reformation' of the concepts is a significant matter in the development of the concept constructions within constructivist interactions. I identify the process

‘forming the concept → transforming the concept → reforming the concept’

as the main basis of ‘Concept Construction’ in my framework, see Figure 1. The generalisation of various specified concepts supports the learner (and even the mentor) in discovering new concept(s). She/He searches for and lists attributes and properties that can be used to distinguish exemplars (of various concepts) from non-exemplars. But what she/he really does is more than just specifying and generalising from different examples that she/he hears or produces. She/He is also concerned with identifying and relating the induced examples. Then, she/he needs to ‘compare’ different achievements. The efficient way to induce a new categorisation method in the framework of constructivism is to compare a few individuals when their categorical relation(s) is known. Then, she/he can make her/his personal labels of the concepts categorisation in order to manage and direct different classes of concepts.

According to the constructed concepts, the reflection of the prior knowledge (what has been acquired or created before interaction) and the new knowledge (what is being acquired or created during the interaction) and the initial definitions must be negotiated. So, the sequence

‘Prior Knowledge → Definition → New Knowledge’

supports the learner in searching for the initiative meanings of the class/classes of constructed concepts and their relationships, see ‘Search for Meaning’ in Figure 1. This phase is highly affected by ‘Semantic Interpretation’. In fact, initiative meanings are needed to be interpreted in order to be balanced, see Figure 1.

From the logical point of view, the interpretation of a defined constructed concept is a function. This function has turned a ‘definition’ into a ‘meaning’. Formally:

Interpretation: *Definition* → *Meaning*.

The Interpretation functions operate the person’s definitions based on her/his constructed concepts. Therefore, they activate the meanings. This transformation has been presented as ‘Semantic Interpretation’ in Figure 1. Accordingly, concerning ‘Interpretation: Definition → Meaning’, the definition could be seen as the product of the inverse of interpretation function (*Meaning* → *Definition*). Logically, we have iterative loops between ‘definition’ and ‘meaning’. The agent could be able to balance and adjust the initial meanings based on the interrelationships between ‘interpretation’ and ‘the inverse of the interpretation’. The conclusions make an appropriate background for verifying the [personally] found meanings based on personal defined constructed concepts. Therefore, there are many transformations from ‘Search for Meaning’ into ‘Meaning Balancing’ and vice versa. Let me say that “a meaning would be given a better shape after checking the balanced definitions based on personal constructed concepts”. The conclusions will formulate meanings, see ‘Meaning

Formulation' in Figure 1. In this phase the agent formulates the balanced meanings based on the balanced definitions for her/his personal constructed concepts.

5.3. MEANING PRODUCTION

The formulated meanings will be organised in order to be produced by the agent. Methodologically, any formulated meaning is a basis for providing a meaningful conceptual structure; let me name it 'Meaningful Conceptual Structuring'. The meaningful conceptual structures are all personally formulated based on personal constructed concepts and definitions. On the other hand, the meaningful conceptual structures could induce new formulated meanings on higher conceptual levels and on higher levels of interaction.

Furthermore, the new produced formulated meanings are considered as the developed schemata in constructing higher levels of conceptions. Finally, the meaningful conceptual structures support the agent in providing meaningful meanings. The meaningful meanings highly reflect on the constructor and support her/him in proposing the modified schemata on higher conceptual levels and on higher levels of interaction. So, the person has moved through this semantic loop in order to organise her/his personal constructed concepts, to construct her/his personal meanings, and to produce the meaningful comprehensions.

6. CONCLUSIONS

Constructivism is a learning philosophy and an educational theory of learning that could be known as a proper foundation for modern learning sciences. A learner and her/his mentor, whose insights are based on their preconceptions, on personal knowings and on pre-structured knowledge can actively participate in a constructivist interaction. The most salient aim could be recognised as 'constructing the personal knowledge in the context of interaction'. Then the learner and the mentor may have the opportunity to attain a deeper personal comprehension and greater motivation within the universe of discourse.

In this research, I have focused on the conceptual and the logical analysis of meaning construction in the framework of constructivism and in the context of mentor-learner interactions. I have been concerned with my own semantics-based framework. My framework considers the agents (learner and mentor) as two intentional participants. It represents a loop that the learner and mentor move through, which organises their personal constructed conceptions in order to construct meanings and produce their individual meaningful comprehensions. I have analysed 'Definition and Definiteness', 'Meaning and Meaningfully', and 'Semantic Interpretation' using Description Logics (DLs), and have employed the results in development of my framework. This research has proposed a new scheme for interpretation based on semantics and on interpretation. Obviously, the proposed semantic loop is self-organised and can promote itself on higher conceptual levels and on higher levels of interaction.

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Links

Link a: http://teachinghistory.org/teaching-materials/teaching-guides/25184

Link b: http://plato.stanford.edu/entries/schema

Link c: http://www.britannica.com/topic/schema-cognitive

PAPER C. A CONCEPTUAL MIRROR: TOWARDS A REFLECTIONAL SYMMETRICAL RELATION BETWEEN MENTOR AND LEARNER

Farshad Badie

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ABSTRACT

The multilevel interactions between a mentor and her/his learner could exchange various conceptions (that are supported by their own conceptualisations) between them. Producing the own realisation of a world and developing it in the context of interactions could be said to be the most valuable product of the constructivist interactions. The most significant matter in meaning construction is producing the own meaningful understanding. Here the learner gets to know how to develop her/his thinking. In this research, I will focus on relating (i) meaning construction through the lenses of the learner's conceptions and (ii) meaning construction through the lenses of constructivism. Constructivism is an educational theory of learning and a model of knowing. The main contribution of this research is analysing the symmetrical relationship between learner and mentor. I will analyse the logical dependencies between learner and mentor and will check their reflectional symmetrical relationship in a conceptual mirror. The conceptual mirror is a phenomenon that represents the meeting point of the mentor's and the learner's conceptual knowledge.

1. INTRODUCTION AND MOTIVATION

In an interaction between a mentor and her/his learner a number of questions, answers, actions and reactions concerning their personal conceptions are exchanged. First, I shall emphasise that what I use and express under the label of a 'concept' aims at providing a comprehensible characterisation of conceptions and conceptualisations. In my approach, a concept is a linkage between linguistic expressions and the mental images (in a broad sense) that the learner (mentor) may have in her/his mind, see (Götzsche, 2013). For instance, these mental images could be interpreted and seen as the learner's representation of aspects of the world (of the universe of discourse). Also, the mentor's construction of the universe of discourse is another instance of mental images. According to the features of concepts just mentioned, a learner's (mentor's) conception within an interaction is equivalent to her/his act of imaging various concepts and linking her/his expressions with regard to the own mental images and schemata, see (Link a and Link b). In my approach, a learner's (mentor's) schemata (*i*) provide backgrounds for her/his concepts, (*ii*) specify her/his inferences and reasonings, (*iii*) describe various theories based on terminologies and world descriptions and, finally, (*iv*) give sufficient and satisfying conditions for definitions of truth.

The multilevel, and commonly agreement-oriented, interactions between a mentor and her/his learner could be viewed as the radical constructivist account of the learner's (and the mentor's) realisation and comprehension, see (Spiro et al., 1992) for more details. The constructivist account of an agent's realisation is capable of enabling her/him in developing the individual realisations of the concepts. Producing the own realisation (and thus, understanding) of a world and developing it during the interaction with the interlocutor could be said to be the most valuable product of the constructivist interactions. In a constructivist interaction learner and mentor develop their own realisations of the underlying systematic processes in reality, and also their realisations of themselves through the universe of discourse.

Constructivism is a model of knowing and an educational theory of learning. It conceives of learning as the process of construction. In the framework of constructivism, a learner attempts to construct knowledge based upon her/his preconceptions (pre-concept formations: see (Link c)) and pre-structured knowledge. The main focus of the mentor could be said to be on the learner's knowledge construction. Consequently, the learner will have the opportunity to attain deeper personal realisations and greater motivations, see (Spiro et al., 1992; Piaget, 1967; Husén and Postlethwaite, 1989; McGaw and Peterson, 2007; Keith Sawyer, 2014; Jaworski, 1995). A learner, either by acquiring new concepts or by modifying existing concepts, decides to construct knowledge. And the mentor constructs parts of the learner's mind by performing the constructive mentoring methods and theories. Actually, what a learner constructs could be analysed as the reflection of what the mentor has provided for her/him (e.g., asked her/him a question). Also, what a mentor constructs in the learner's mind could be seen to be the reflection of what the learner has done (e.g., answered a question to the mentor).

In this research, I see 'learning' from the functional point of view and think of causation in the process of 'construction'. In my opinion, knowledge can actively be constructed based upon the learner's realisation of the meanings of various concepts with regard to their descriptions and definitions. I have focused on this area in (Badie, 2015a; Badie, 2015b). Kindly observe that the definition of a concept is an equation whose left-hand side is a concept and whose right-hand side is a description for that concept, see (Baader, et al., 2010). Also, I have defined a meaning as a concept-update function in my approach, see (Badie, 2015b).

Focusing on the learners' conceptions of successful learning and effective mentoring, a learner can describe the steps of learning from two distinct points of view in order to provide a satisfactory description of knowledge construction development:

- the first one is her/his own point of view, and
- the second one is her/his mentor's point of view.

Learners usually observe learning through the lenses of their mentors in order to see themselves and their own requirements. Additionally, a learner needs to realise and to figure out how her/his own conceptions of learning about an object may be reflected in the mentor's conceptions of that object and vice versa.

Now I take a model of students' developing conceptions of learning into consideration. The model sketches on Säljö's seminal studies on learning conceptions, see (Säljö, 1979). Säljö focused especially on describing learning from the learner's point of view and identified five categories and levels for a learner's developing conceptions of 'learning'. Also, (Rossum and Schenk, 1984) suggested a new category and added it as the sixth level to the Säljö's model. Here I summarise the model as the following items:

- Knowing More: The learner observes ‘learning’ as knowing new things. She/He wants her/his mentor to impart the well-structured information into separated and isolated facts.

- Memorising: The learner reproduces what she/he has acquired and known. So, it’s all about memorising. She/He still tries to know more in order to reproduce more.

- Selection: The learner selects and memorises the facts and might be able to apply her/his knowledge in practical approaches. She/He expects the mentor to shape and to motivate her/him during the interaction.

- Meaning Construction: The learner is realising that constructing knowledge is very important and it will be plausible in the shadow of meaning construction. The mentor would guide the learner to find out how to think logically, analytically and productive.

- Reality Interpretation: Learning as an interpretative process should support the learner in interpreting and understanding the reality. Then the learner characterises ‘learning’ as ‘the process of self development’.

- Self Awareness: This category is about self realisation. This process is always going to be continued. The learner is always going to expand her(him)self. Obviously, this is the most excellent and the most transcendental conception

In this article my main focus is on ‘Meaning Construction’ (level four). I have focused on meaning construction in the context of interactions and have written some of my research products in (Badie, 2015a; Badie, 2015b). In my opinion, this level is the most definitive level. Let me make a conceptual linkage between my own approach and Säljö’s model. Focusing on meaning construction, the learner deals with her/his individual concept constructions for developing her/his conceptualisations. So, I shall bring your attention to the fact that my approach recognises the collection {Concept Formation, Concept Transformation, Concept Reformation} as the most significant matter in the development of concept constructions within constructivist interactions. I have identified the process ‘Concept Formation → Concept Transformation → Concept Reformation’ as the process of Concept Construction (CC) in the context of interaction. The most significant expression at this level is ‘meaningful comprehension’. Here the learner gets to know (and gets to identify) how to relate different ideas. In fact, she/he is about to develop her/his thinking. In my opinion, knowledge can actively be constructed based upon the learner’s realisation of the meanings of various concepts with regard to their descriptions and definitions. So, meaning construction [in the framework of constructivism and in the context of interactions (and dialogues) between the learner and the mentor] finds its real significance here. Subsequently, the learner describes her/his individual concepts and attempts to produce meanings, to formulate them and to develop their constructions. At this level, the compassionate mentor is the developer of the learner’s thinking. This development will support the learner in finding how to think logically, analytically and productive.

The main focus of this research is on a reflectional symmetrical relation between learner and mentor. I will logically analyse it while I will focus on conceptual knowledge. Thus, I need to analyse the logical dependencies between learner and mentor, and see the reflections in a conceptual mirror. The conceptual mirror represents the meeting point of the mentor's and the learner's conceptual knowledge. In the following sections I will present the followings: The Learner's Conceptual Knowledge, The Relationships between Learner and Mentor, Conceptual Mirror: A Reflection-Symmetry and Conclusions.

2. THE LEARNER'S CONCEPTUAL KNOWLEDGE

Bloom's taxonomy is a framework for classifying pedagogical objectives, which could be interpreted as the statements of what educators and mentors expect their learners to have learned, see (Furst et al., 1956; Krathwohl, 2002). According to Bloom's researches, knowledge has a strong relationship with recognition of various materials, ideas, methods, processes, structures and settings. Bloom's taxonomy divides a knowledge class into multiple classes (e.g., knowledge of terminologies, knowledge of ways and means, knowledge of trends and sequences, knowledge of classifications and categorisations, knowledge of criteria, knowledge of methodologies, knowledge of quantifications, knowledge of principles – generalisations and specifications, knowledge of theories and structures). Since then, (Krathwohl, 2002) has proposed a knowledge dimension in the revised version of Bloom's taxonomy. The revised taxonomy consists of:

- Factual Knowledge (e.g., terminological knowledge),
- Conceptual Knowledge (e.g., knowledge of theories, models and structures),
- Procedural Knowledge (e.g., knowledge of methods and algorithms), and
- Metacognitive Knowledge (e.g., contextual knowledge, conditional knowledge).

In fact, learning consists of a sort of Transformation functions from knowledge (that is going to be known) into the sets of 'facts', 'procedures', and 'concepts' in different 'contexts'. And subsequently, the learners transform facts, procedures and concepts into their minds. I formally describe learning as the conjunction of the following transformations:

i. $Knowledge \rightarrow \{Fact, Procedure, Concept\}$

ii. $\{Fact, Procedure, Concept\} \rightarrow Mind.$

In this research my main concern is 'concepts'. I am focusing on conceptual knowledge acquisition.

- In my opinion, there is a concept behind every fact. Then any factual knowledge can be supported by a conceptual knowledge. For instance, according to a fundamental characteristic of terminological knowledge (as a type of factual knowledge), we can represent terminologies within taxonomical structures. A taxonomy could be constructed based upon concepts. Then, a terminological knowledge has been supported by a conceptual knowledge. Also, as another instance, we can define a body of the related elements and interpret it as a set of constructors for denoting various concepts and their interrelationships. That's how the concept languages and descriptive languages appear. Then, we could be able to represent knowledge over concepts, their instances and their relationships.

- Additionally, in my opinion, any procedure could be observed as the conclusion of the sequence of a number of facts. Therefore:

$$Fact \rightarrow Fact \rightarrow Fact \ ... \ \Leftrightarrow \ Procedure.$$

And actually:

1. A procedure could be viewed as a body of a few number of facts.
2. A fact is supported by a concept.
3. A procedure is supported by a concept.

According to the afore-mentioned items, a learner acquires facts and procedures and they all become supported by concepts in her/his mind. These concepts are considered as the building blocks of her/his conceptual knowledge and can be considered as the elements and ingredients of a conceptual system and, thus, they support the learners' developing conceptualisation of 'learning'. Then, she/he can think of learning (mentoring), successful learning and satisfactory mentoring.

Here I describe learning as the conjunction of the processes (*i*) and (*ii*):

i. $Knowledge \rightarrow \{Fact, Procedure, Concept\} \rightarrow \{Concept, Concept, Concept\}.$

ii. $\{Concept, Concept, Concept\} \rightarrow \{Fact, Procedure, Concept\} \rightarrow Mind.$

3. THE LEARNER'S CONCEPTUAL KNOWLEDGE

In logic, the reflexive relation R is a binary relation between an object and itself. Let R be a relationship on set A. If and only if R relates every element of A with itself, then R is identified as a reflexive relation on A. Formally:

$$\forall a_i \in A : \quad a_i \, R \, a_i.$$

A binary relation R between two elements of set A is identified as a symmetrical relation if and only if for any a_i and a_j belonging to A, in the case R relates a_i with a_j, then R relates a_j with a_i as well. Formally:

$$\forall a_i \in A \quad \exists a_j \in A :$$
$$a_i \, R \, a_j \;\Rightarrow\; a_j \, R \, a_i.$$

Let set E be a learning environment. A lot of elements could exist in a learning environment. The main focus of this research is on elements L and M, where L and M denote *Learner* and *Mentor* respectively. Formally:

$$L \in E \;,\; M \in E.$$

M can, metaphorically, be seen as a mirror that shows the L's self, see (Grow, 1991). In my opinion, the multi-level agreement oriented interactions between a mentor and her/his learner constructs a symmetrical relationship between them. Let me conclude that a real constructive and productive relationship between L and M is inherently a symmetrical relationship. It may not be symmetric (and may be asymmetric) in some existing relationships over E, but it is potentially a symmetry and may represent a willingness to achieve more symmetrical properties and preserve them in the context of interactions.

A responsible learner (in parallel with her/his constructive and compassionate mentor) attempts to survive this symmetrical relationship. I see this characteristic as the most excellent and valuable realisation of the phenomenon of 'learning' in the context of relationships between mentor and learner. Moreover, any person is able to observe the reflection of her/his own conceptions of the phenomenon of 'learning' with regard to perceived facts, procedures and concepts, in her/his self (e.g., individuality, personality). In fact, this is also the most excellent and transcendental realisation of the phenomenon of 'learning'. Similarly, 'growing self awareness' is the most valuable product of the complement model of Säljö's model and has been manifested in the learner's self. Thus, there exists a reflexive relationship between 'growing self awareness' and 'self'. For another instance, the 'reality interpretation' is the product of the last level of Säljö's model. I assume that reality interpretation is also reflected in L's (and M's) interpretations and these interpretations could be made in the shadow of the learner's (and mentor's) self. Further, 'meaning construction' as the product of layer four of Säljö's model has been reflected in personal understanding based on individual interpretations in the shadow of the learner's (mentor's) self.

4. THE LEARNER'S CONCEPTUAL KNOWLEDGE

I will define the conceptual mirror in order to clarify the interrelationships between the learner's and the mentor's conceptual knowledge. A conceptual mirror can be a supportive point for mentor and learner. In fact, a mentor could have a better understanding of her/his learner's knowledge by looking in the mirror and the learner can have a better realisation of mentoring by looking in the mirror.

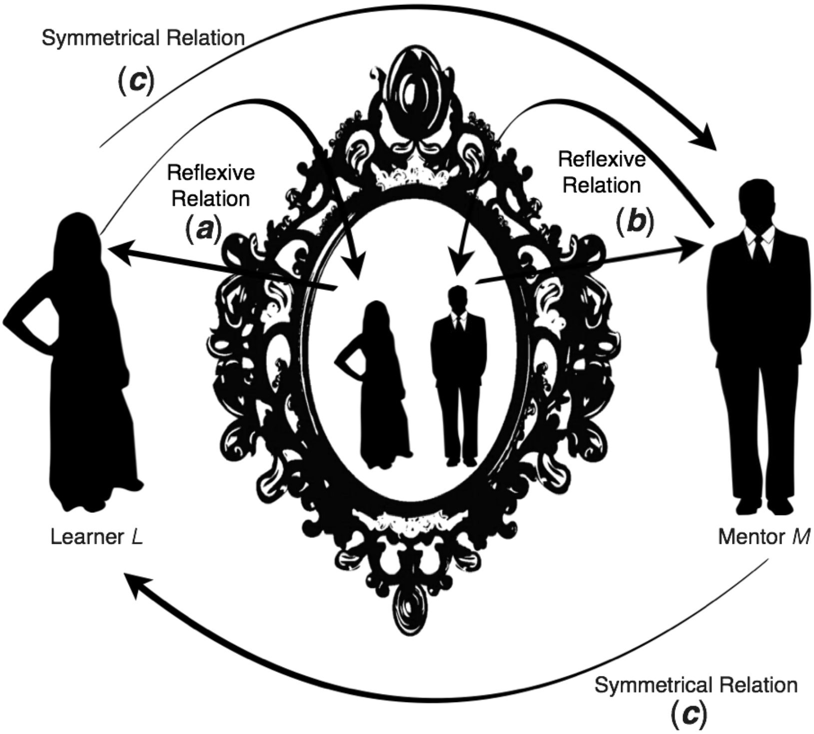

Figure 1. *A Conceptual Mirror*

I define the conceptual mirror on the meeting point of:

A. The learner's learning.
B. The reflections of the learner's conceptions in her(him)self.

Similarly, the conceptual mirror is located on the meeting point of:

C. The mentor's mentoring.
D. The reflections of the mentor's conceptions in her(him)self.

I shall emphasise that the learner's trust is reflected in the learner's self. Thus, the learner's trust can be seen in the conceptual mirror at the meeting point of (B) and (D).

4.1. LOGICAL ANALYSIS OF THE CONCEPTUAL MIRROR

There is a symmetrical relationship between Learner L and Mentor M that is represented by ***c*** in Figure 1. The symmetrical relationship ***c*** could be described as the product of the concatenation of two reflexive relationships. These reflexive relationships are represented by ***a*** and ***b*** in Figure 1. Let me clarify what this concatenation is. There are two important results:

1. In this system, the learner sees the reflection of her/his individual conceptions of the phenomenon of 'learning' in the conceptual mirror. She/He also observes the reflection of the phenomenon of 'mentoring' (of the mentor) in the conceptual mirror and, subsequently, in her(him)self.

2. On the other hand, the mentor has seen the reflection of the phenomenon of 'mentoring' in the conceptual mirror. Moreover, the mentor observes the reflection of the learner's conceptions of the phenomenon of 'learning' in the conceptual mirror and, thus, she/he sees the reflection of the phenomenon of 'mentoring' in the learner's self.

As for the results 1 and 2, the learner observes the reflection of the phenomenon of 'mentoring' in her(him)self and the mentor observes the reflection of the phenomenon of 'mentoring' in the learner's self. This demonstrates a symmetrical relationship between a mentor and a learner. So, we have seen that the concatenation of the reflexive relation ***a*** and reflexive relation ***b*** could produce the symmetrical relationship ***c***.

Let R_r and R_s denote the reflexive and symmetrical relationships respectively. Therefore, we have the following system:

i. $L \, R_r \, L$
ii. $M \, R_r \, M$
iii. $(\ldots R_r \ldots) \;\; R_s \;\; (\ldots R_r \ldots)$

The first premise represents the reflexive relation between learner and her(him)self. Also, the second premise represents the reflexive relation between the mentor and her(him)self. The third premise represents that there is a symmetrical relation between two reflexive relations. Therefore, I can conclude that there is a symmetrical relation between 'the reflexive relation between learner and her(him)self' and 'the reflexive relation between mentor and her(him)self'. Formally:

$$(L \, R_r \, L) \;\; R_s \;\; (M \, R_r \, M) \qquad (I)$$

As described, the learner and mentor observe the relationship of their interlocutors (mentor and learner) with themselves in the conceptual mirror. In fact, the symmetrical relationship '$(L \, R_r \, L) \;\; R_s \;\; (M \, R_r \, M)$' enters the conceptual mirror.

Reconsidering section 2, a learner, by learning, transforms knowledge into multiple concepts. Regarding the reflexive relationship between a learner and her(him)self in a learning process, I can conclude that her/his own conceptual knowledge (and produced concepts) reflects in her(him)self.

Considering L_C as a learned concept (a produced concept based on learning), and taking the result (I) into account, I can propose the following system:

i . $L_C \, R_r \, L_C$,
ii. $M_C \, R_r \, M_C$,
iii. $(\ldots R_r \ldots) \;\; R_s \;\; (\ldots R_r \ldots)$.

The first premise represents the reflexive relation between a learned concept and itself. The second one represents the reflexive relation between a mentored concept and itself. Also, the third premise represents that there is a symmetrical relation between two reflexive relations. So, there is a symmetrical relation between a 'reflexive relation between a learned concept and itself' and a 'reflexive relation between a mentored concept and itself'. Then, formally:

$$(L_C \, R_r \, L_C) \quad R_s \quad (M_C \, R_r \, M_C) \; .$$

Moreover, this conclusion denotes that the learner observes the reflection of the mentor's conceptual knowledge in the conceptual mirror, where she/he has observed the reflection of her/his conceptual knowledge. Therefore:

$$L_C \, R_r \, L_C \;\; \Leftrightarrow \;\; M_C \, R_r \, M_C \; .$$

It can be divided into two conclusions:

i. $L_C \, R_r \, L_C \; \Rightarrow \; M_C \, R_r \, M_C$,
ii. $M_C \, R_r \, M_C \; \Rightarrow \; L_C \, R_r \, L_C$.

According to (*i*), the learner observes the learned concept in her(him)self. This concludes that the mentor observes the mentored concept in her(him)self. According to (*ii*), the mentor observes the mentored concept in her(him)self and, therefore, the learner observes the learned concept in her(him)self. These conclusions demonstrate an equivalence and stability between learner's and mentor's conceptual knowledge.

5. SUMMARY AND CONCLUSIONS

An interaction between a mentor and her/his learner could exchange their personal conceptions. The multilevel interactions between them could be viewed as the radical constructivist accounts of their realisations and comprehensions. Producing one's own realisation of the world and developing it in the context of interaction could be said to be the most valuable product of the constructivist interactions. In this research, I have employed Säljö's model of students' developing conceptions of learning and have focused on one of its levels, so-called 'Meaning Construction'. The main reasons for this consideration have been my special interest in meaning construction in the context of interactions and my own research in the analysis of meaning construction through the lenses of the theory and philosophy of constructivism. I believe that this level is the most definitive and determinative level in Säljö's model of students' developing conceptions of learning. In my opinion, this level can appropriately describe the interrelationship between the learners' and the mentors' observations. And, in fact, at this level, the learner initiates the developing of her/his conceptualisations. The most significant matter at this level is the production of the own meaningful comprehension. Here the learner gets to know (and to identify) how to relate different ideas and how to develop her/his thinking. In this article, I have made a conceptual linkage between my own approaches and Säljö's model. I have focused on the conceptual knowledge in the revised version of Bloom's taxonomy with regard to my goals. The main contribution of this research has been logical representations and the analysis of the reflectional symmetrical relation between learner and mentor. Thus, I have analysed the logical dependencies between learner and mentor, and have checked their reflections in a conceptual mirror. A conceptual mirror is a phenomenon that represents the meeting point of (i) the learner's learning and (ii) the reflections of the learner's conceptions in her(him)self. It also represents the meeting point of (i) the mentor's mentoring and (ii) the reflections of the mentor's conceptions in her(him)self. Accordingly, a conceptual mirror represents the junction (and dependency) of the mentor's and the learner's conceptual knowledge in the context of their interactions.

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Links

Link a: http://plato.stanford.edu/entries/schema

Link b: http://www.britannica.com/topic/schema-cognitive

Link c: http://teachinghistory.org/teaching-materials/teaching-guides/25184

PAPER D. A SEMANTIC REPRESENTATION OF ADULT LEARNERS' DEVELOPING CONCEPTIONS OF SELF REALISATION THROUGH LEARNING PROCESS

Farshad Badie

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ABSTRACT

Learning is a reflective activity that enables the learner to draw upon her/his previous experiences and background knowledge to conceptualise, realise, understand and evaluate the present, so as to shape her/his future actions and to construct and develop new knowledge for her(him)self. Learning strongly depends on meetings of awarenesses (or self realisations) which we see as achieved through the experiences that mentors and learners undertake jointly. Self realisation is a type of self organisation process and always organises itself. In fact, self realisation is always going to be continued. This research will conceptually focus on multiple categories through the adult learners' developing conceptions of learning. The focus will be on different categories from the basic conceptions to excellent ones. I will take an appropriate model of students' developing conceptions of 'learning' into my consideration. The model sketches on Säljö's seminal studies on learning in the learner's perspective. I will employ the most significant characterised learning conceptions that are seen through the lenses of the adult learners. This characterisation will be applied over the most significant categories of learners' developing conceptions of learning. Some categories like 'Meaning Construction', 'Reality Interpretation' and 'Self Awareness (Self Realisation)' could be known as the most excellent and the most transcendental conceptions used by learners. Consequently, I will employ the highlighted concepts in order to design a semantic representation of learners' developing conceptions of learning. This semantic representation will be a graph whose nodes represent the main identified concepts and whose arcs represent the relations between the concepts. According to Description Logics and their descriptive features, a concept can be observed as an idea and that idea can be transformed into a hypothesis in order to be corresponded to a distinct entity [and, respectively, to a group of entities] or to its/their essential features. The ideas determine the applications of terms and phrases. An idea is a significant part in the use of reasons and languages. My desired semantic structure will provide a structural representation of the learners' developing conceptions of learning that could support a kind of top-level ontology, i.e., an ontology representing the learners' developing conceptions of learning based on educational informatics. I shall draw your attention to the fact that in information sciences an ontology is described as an explicit (and formal) specification of a shared conceptualisation on the domain of interest. Ontologies of a thing/phenomenon support different researchers in providing appropriate specified descriptions with regard to various concepts relevant for that thing/phenomenon. I shall emphasise that I will need to employ some descriptive features of Description Logics (DLs) for representing fundamental terminologies and world descriptions as the backbones of the desired semantic representation. Description Logics are the most well-known family of knowledge representation formalism. They are supported by First-Order Predicate Logic.

1. INTRODUCTION

The state of knowledge representation art in information sciences has experienced significant improvements during last decades, see (Randall et al., 1993). Knowledge representation systems could assist us in constructing ontologies, see (Staab and Studer, 2004; Grimm et al., 2007). From the philosophical point of view, an ontology is described as the science of 'being' and 'existence'. The ontologies demonstrate the structure of the reality of a thing/phenomenon within the world. An ontology checks the attributes that belong to a thing because of its nature and its existence. On the other hand, in information sciences an ontology is described as an explicit and formal specification of a shared conceptualisation on a domain of interest. However, I shall claim that there is a very strong interrelationship between philosophical and informatical descriptions of the ontologies. In fact, any ontology within information sciences describes the world through the lenses of structuralism and existentialism and attempts to specify its viewpoint on different levels of its conceptualisation. In this article my notion of 'ontology' is beyond the use in the information sciences. A formalism based on formal and schematic ontologies can be represented over

a. concepts (as the classes of individuals and objects), and
b. their relations (i.e., roles of concepts).

According to Predicate Logic and its descriptive features within information sciences, a 'unary predicate' has been supposed to be equivalent to a 'concept'. Also, a concept has been realised and has been seen as an 'idea' which can be transformed into a 'hypothesis' in order to be corresponded to a distinct entity [and, respectively, to a group of entities] or to its/their essential attributes, characteristics and properties. The hypotheses can describe theories relying on terminologies and world descriptions. Also, they support inferential and reasoning processes and satisfy the conditions for definitions of truth. I shall contemplate that they focus on both WhyNess and HowNess. Actually, considering the provided world descriptions and supported inferential processes, the hypotheses can determine the applications of predicates, statements and terms. Therefore, a hypothesis is a significant part in the use of reasons and languages.

Let me go back to ontologies. The ontologies form the backbone of a huge number of semantic applications (in information sciences). The underlying Description Logics (Concept Languages) are now one of the most widely used knowledge representation formalism. They have emerged from semantic networks (Quillian, 1968) and frame-based systems (Minsky, 1974) and help us to represent the inferential processes within knowledge representation systems. Reasoning over represented knowledge can be identified to be the most remarkable objective of knowledge representation systems, e.g., (Bader et al., 2010; Buchheit et al., 1993; Rudolph, 2011).

In this article, I will focus on conceptualising learning within the upper ontology of adult learners' developing conceptions of the phenomenon of 'learning'. An upper (= top-level, foundation) ontology describes the general concepts that are the same across all knowledge domains. In fact, splitting a general concept into more specific ones is conducive to get from top-level ontology to the deeper ontologies with more specific information. This research focuses on the learners' conceptions of successful and effective learning. Obviously, any learner can describe the steps and levels of learning in order to provide a satisfactory description of the development of knowledge construction for her(him)self. I will take into consideration a model of students' developing conceptions of learning that is based on Säljö's seminal studies on learning conceptions and on learning in the learners' perspectives, see (Säljö, 1979). Those studies have focused on description of learning from the learner's point of view and identified five categories and levels for a learner's developing conceptions of learning. Additionally, (Rossum and Schenk, 1984) suggested a new category and added it as the sixth level to the Säljö's model. According to the final model, 'self awareness' and 'self realisation' is recognised as the most excellent conception of learners. So, conceptually, it can include the other conceptions within lower categories.

In this article, I will employ DLs to specify the adult learners' conceptions of learning relying on Säljö's model. Thus, I will highlight the most effective and advantageous concepts and roles within that model. The desired logical description can provide a proper foundation for formalising knowledge in order to denote the learners' developing conceptions of the phenomenon of 'learning'. The formalism will provide a supportive backbone for representing the main constructive concepts and their interrelationships. It also supports the specification of the conceptualisation of the learners' grasping of the phenomenon of 'learning'. In the following sections of this article you will be offered the followings: Knowledge Representation Formalism, Learners' Developing Conceptions of Learning and Logical Representation, Semantic Representation, and Summary and Conclusions.

2. KNOWLEDGE REPRESENTATION FORMALISM

According to (Baader et al., 2010) and (Schmidt-Schaulss and Smolka, 1991), Description Logics (DLs) are well-known knowledge representation formalism and represent knowledge in terms of concepts (classes), roles (relations) and individuals (objects). Concepts correspond to unary predicates, roles to binary predicates and individuals to constants in First-Order Predicate Logic (FOL). In DLs individuals are the instances (or members) of a concept. For instance, blue as an individual can also be recognised as an instance of the concept Colour. From certain point of view, atomic concepts and atomic roles are two kinds of atomic symbols. The atomic concepts and roles are defined as the elementary descriptions from which humans inductively build and construct complex (specified) descriptions by means of concept constructors and role constructors. The set of the main connectors in the basic Description Logic is:

{Conjunction ($\sqcap$: and), Disjunction ($\sqcup$: or), Negation ($\neg$: not), Existential Restriction ($\exists$: there exists …), Universal Restriction ($\forall$: for all ...), Top Concept ($\top$: everything), Bottom Concept ($\perp$: nothing)}.

A knowledge base in DLs usually consists of the terminological axioms (for terminological and grammatical descriptions) and assertions (for world descriptions). Let *C* and *D* be concepts, *R* and *S* be roles, and *a* and *b* be individuals. Then we have the following terminological axioms:

- Concept inclusion $C \sqsubseteq D$ (e.g., Bird $\sqsubseteq$ Animal),
- Role inclusion $R \sqsubseteq S$ (e.g., walking $\sqsubseteq$ moving),
- Concept equality $C \equiv D$ (e.g., Person $\equiv$ Human), and
- Role equality $R \equiv S$ (e.g., designing $\equiv$ sketching).

Additionally, $C(a)$ and $R(a,b)$ denote concept assertion and role assertion respectively. For example, Colour(blue) denotes a concept assertion and eating(Person,Apple) represents a role assertion.

3. LEARNERS' DEVELOPING CONCEPTIONS OF LEARNING

According to (Rossum and Hame, 2010) and (Pratt, 1992), learning conceptions play an important role in learners' study behaviour in higher (tertiary) education. In fact, the human being views the world through the lenses of her/his conceptions, interpretations and actualisations in accordance with her/his realisation of the world. In this section I generally and succinctly describe different categories of the learners' developing conceptions of the phenomenon of 'learning' with regard to Säljö's [and Rossum's] model.

1. Knowing More. At this point, the learner identifies learning as knowing new things. Here the learner wants and needs her/his mentor to impart the well-structured information into separated, isolated and realisable facts for her/him.

2. Memorisation. Here the learner reproduces what she/he has acquired and what she/he has known. So, this category is all about memorising. The learner still tries to know more [and more] in order to reproduce more [and more] knowledge.

3. Selection. The learner selects (chooses) and memorises the facts and might be able to apply her/his knowledge in the practical approaches. She/he expects the mentor to shape and to organise the information for her/him and to motivate her/him during their interactions.

4. Meaning Construction. The learner is realising that constructing knowledge is very important. She/he understands that knowledge construction could be plausible with regard to 'meaning construction'. The mentor would guide the learner to find out how to think logically, analytically and productive. Let me conclude that she/he has initiated to become concerned with developmental aspects of learning[1].

[1] Jean Piaget (1896 - 1980) is the originator of constructivism. Constructivism is a learning philosophy and a pedagogical theory of learning that can be recognised as a model and a theory of knowing. Piaget was the first psychologist to make a systematic study of cognitive development and developmental theory of learning. However, a learner may not know about the developmental theory of learning, but she/he, in levels 4 and 5 of Säljö's model has highly become concerned with developmental aspects of learning.

5. Reality Interpretation. Here the learner recognises learning as an interpretative process that could support her/him in realising and understanding the ‘reality’ of the world. According to this insight, the learner characterises learning as the process of ‘self development’. I shall claim that she/he has highly become concerned with developmental aspects of learning.

6. Self Realisation. This category is about self awareness. It’s all about the appreciation of the self. The process of self realisation is always going to be continued. The learner will be going to expand her(him)self. Obviously, this is the most excellent and the most transcendental learning conception of a learner.

Focusing on the afore-mentioned levels of conceptions, we can realise that there is a strong dependency between them. Any lower conception provides a conceptual [and logical] presupposition for its upper conception. And any upper conception is the consequence of its lower one. This is extremely advantageous in logical modelling of the learning conceptions using DLs. I have figured out the learners’ conceptions of their significant roles within learning processes in Figure 1. Figure 1 is represented with regard to Säljö’s and Rossum’s model.

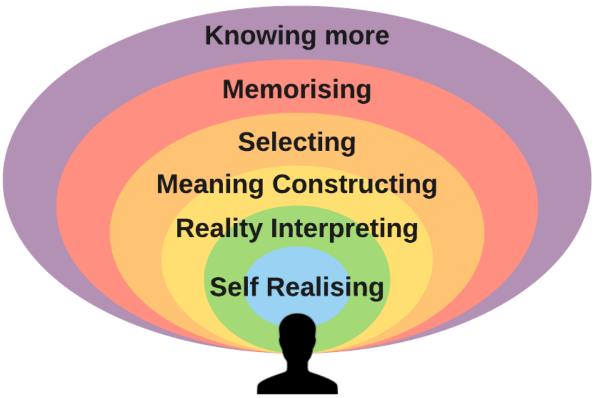

Figure 1. *The Learners’ Conceptions of their Roles within Learning Processes*

3.1. LOGICAL DESCRIPTION

In this section I employ DLs to provide a formal description of the levels of the model and their specifications. I also employ concept inclusion and role inclusion axioms and role assertion to, formally, describe the significant roles of the learner.

See the following formalism:

1. Here I focus on formal description of ‘learning’ as a role of the concept ‘Learner’. The description is represented in the form of the conjunction of a number of role

inclusions. We have the following formal description:

$(\text{learning} \sqsubseteq \text{transforming}) \sqcap$
$(\text{learning} \sqsubseteq \text{memorising}) \sqcap$
$(\text{learning} \sqsubseteq \text{reproducing}) \sqcap$
$(\text{learning} \sqsubseteq \text{selecting}) \sqcap$
$(\text{learning} \sqsubseteq \text{abstracting}) \sqcap$
$(\text{learning} \sqsubseteq \text{developing}) \sqcap$
$(\text{learning} \sqsubseteq \text{expecting}) \sqcap$
$(\text{learning} \sqsubseteq \text{beingAware})$.

2. As mentioned, any lower conception provides a logical premise for its upper conception and any upper conception is the logical consequence of its lower one. Thus, any lower conception supports its upper conception. For instance, Memorisation is a concept relevant for the role memorising. Memorisation supports Transformation as the concept relevant for the role transforming. Therefore, we have a number of conjunctions between various relevant role assertions. Additionally, we have one concept inclusion. This concept inclusion is "$\text{Transformation} \sqsubseteq \text{Process}$" and denotes the fact that the most primary conception of 'learning' sees learning as transforming. Also, Transformation is the concept of the role transforming and is originally a process. We have following formal description:

$(\text{Transformation} \sqsubseteq \text{Process}) \sqcap$
$\text{support(Transformation,Memorisation)} \sqcap$
$\text{support(Memorisation,Reproduction)} \sqcap$
$\text{support(Reproduction,Selection)} \sqcap$
$\text{support(Selection,Comprehension)} \sqcap$
$\text{hasInput(Transformation,Fact)} \sqcap$
$\text{hasOutput(Transformation,Knowing)} \sqcap$
$\text{support (Knowing,Learner)} \sqcap$
$\text{hasInput(Reproduction,Production)} \sqcap$
$\text{hasOutput(Reproduction,Production)} \sqcap$
$(\text{Production} \sqsubseteq \text{Process}) \sqcap$
$\text{hasInput(Production,Fact)} \sqcap$
$\text{hasOutput(Production,Fact)} \sqcap$
$\text{support(Expectation,Interpretation)} \sqcap$
$\text{support(Interpretation,Comprehension)} \sqcap$
$\text{hasInput(Comprehension,Concept)} \sqcap$
$\text{hasOutput(Comprehension,Meaning)} \sqcap$
$\text{support(Meaning,Abstraction)} \sqcap$
$\text{support(Abstraction,Development)} \sqcap$
$\text{hasInput(Development,Thought)} \sqcap$
$\text{hasOutput(Development,Thought)} \sqcap$
$\text{support(Development,Expectation)} \sqcap$
$\text{support(Expectation,Realisation)} \sqcap$

hasInput(Realisation,Self) ⊓
hasOutput(Realisation,Self) ⊓
support(Self,Awareness) ⊓
support(Awareness,Learner).

4. SEMANTIC REPRESENTATION

As mentioned, in information sciences we tackle to provide an appropriate description of a thing/phenomenon and its structure concerning multiple concepts relevant for that thing/phenomenon. Considering ontologies as the specifications of the shared conceptualisations, they are definitely more descriptive in the shadow of the semantic networks and semantic representations. A semantic network is a graph (and is a representation) whose nodes represent concepts and whose arcs represent relations between the concepts (e.g., isA, produce, support, hasInput). Semantic representations can provide applicable structural representations of the statements about a thing/phenomenon within the domain of interest. In Figure 2, you see a semantic network that represents the main concepts that an adult learner is concerned with through her/his conceptions of the phenomenon of 'learning' (and, the concept Learning). I have designed this semantic network with regard to the provided logical description in section 3. In fact, the described world descriptions in DLs have been represented in Figure 2. Thus, you can see that the conceptual and logical relationships between the most significant hypotheses within the learner's developing conceptions of learning are represented. This network is capable of supporting a proper background for building the top-ontology of the adult learners' developing conceptions of learning. It can also support the educators', curriculum designers', educational psychologists' and mentors' reasoning processes for discovering the key points within their learners' conceptions from 'knowing, only, an isolated fact' to 'self realisation and self awareness'.

5. SUMMARY AND CONCLUSIONS

In this article, I have focused on specification and conceptualisation of the concept 'learning' within the top-ontology of adult learners' developing conceptions of successful learning. I have taken Säljö's [and Rossum's] model of students' developing conceptions of learning into consideration. According to this epistemological model, self awareness and self realisation is the most excellent conception of learners. So, it can conceptually include all the other conceptions within lower categories. According to (Rossum and Hame, 2010), the model has proven its worth as an instrument for curriculum design, measurement of epistemological development and as a tool for staff development. I have employed Description Logics to describe and specify various categories of the explained model, and have highlighted the most effective concepts and roles. The proposed logical description has provided a proper foundation for a DLs-based formalism that describes the learners' developing conceptions of learning. Also, the formalism provides a strong backbone for representing the main constructive concepts and their interrelationships.

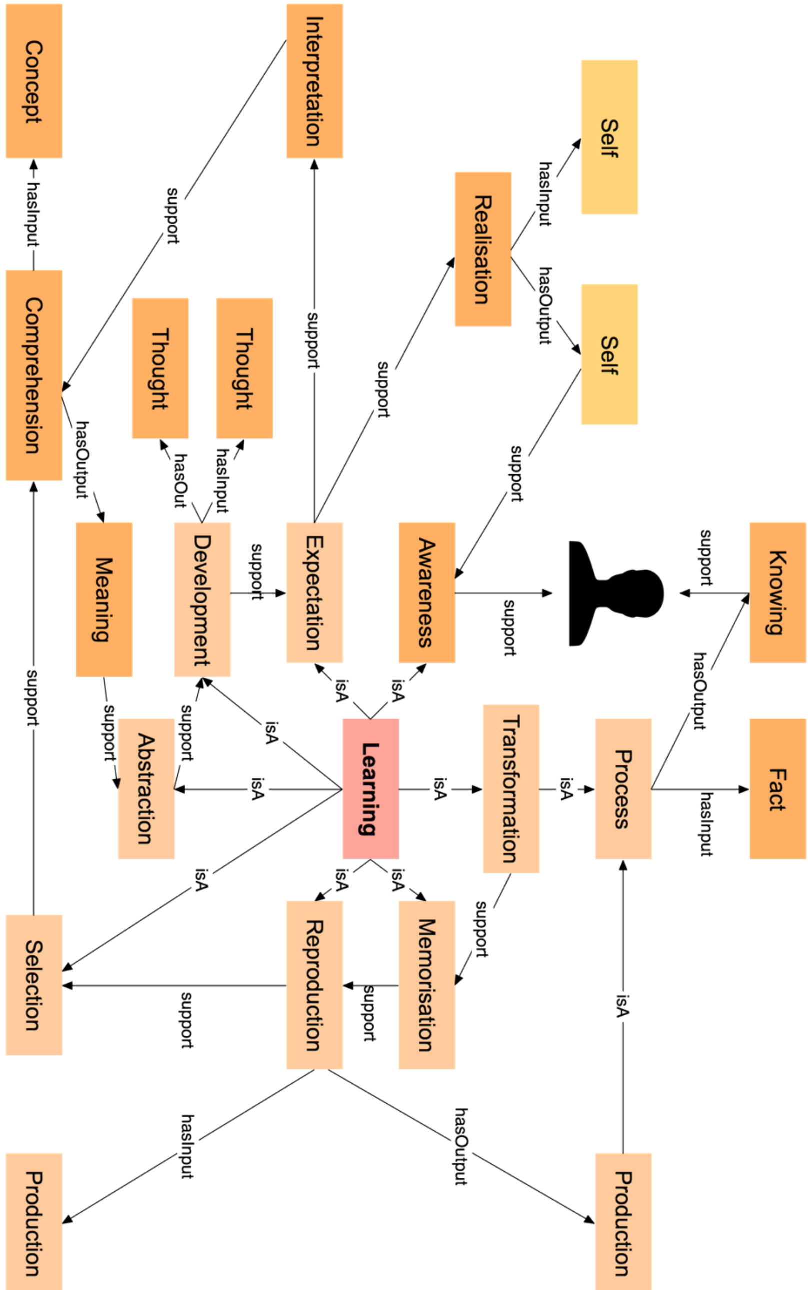

Figure 2. *A Semantic Representation of Learners' Developing Conceptions of Learning*

It has also supported the specification of the conceptualisation of learners' grasping of the phenomenon of 'learning'. Accordingly, regarding the proposed logical description I have sketched out a semantic representation. It can support an appropriate scheme for building the top-ontology of the learners' conceptions of learning. Additionally, it can support educators, curriculum designers, educational psychologists and mentors in elaborating their realisation of their learners' needs within educational systems.

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PAPER E. CONCEPT REPRESENTATION ANALYSIS IN THE CONTEXT OF HUMAN-MACHINE INTERACTIONS

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ABSTRACT

This article attempts to make a conceptual and epistemological junction between human learning and machine learning. I will be concerned with specifying and analysing the structure of concepts in the common ground between a concept-based human learning theory and a concept-based machine learning paradigm. I will focus on (i) humans' conceptual representations in the framework of 'constructivism' (as an educational theory of learning and a proper model of knowing) and 'constructionism' (as a theory for conceptualising learning) and (ii) concept representations in the framework of 'inductive concept learning' (as an inductive machine learning paradigm). The results will support figuring out the most significant key points for constructing a conceptual linkage between a human learning theory and a machine learning paradigm. Accordingly, I will construct a conceptual ground for expressing and analysing concepts in the common ground of educational and informatics sciences and in the context of human-machine interplays.

1. MOTIVATION

Regarding a very general definition, the act [and the role] of learning can be identified as related to acquiring new or modifying existing knowledge. Often, the ability to acquire knowledge is seen as a sign of, or even a prerequisite for, intelligent behaviour. I shall stress that 'knowledge' is a very complicated and sensitive term that must be used with caution. Considering the structures of human and information sciences and their interrelationships, I need to focus on specifying knowledge and on analysing the phenomena that we can use under the label of 'knowledge'. It seems quite important to investigate what the term 'knowledge' stands for (and can stand for) to be assumed and to be comprehensible in various frameworks of learning within different systems. This article attempts to construct a conceptual and epistemological linkage between human learning and machine learning and to analyse the structure and description of concepts in the common ground between a theory (and a philosophy) in the framework of human learning and a paradigm in the framework of machine learning. Before getting into the details, I contemplate the term 'Machine Learning'. Later on, I focus on knowledge to provide a proper background for my desired contributions.

Machine Learning has been recognised as a subfield of Artificial Intelligence and Computer Science. According to (Mitchell, 1997), "a machine learning approach attempts to develop strong algorithms that allow machines to improve [the productivity of] their performances on a given goal [and on an objective function]". In machine learning, the word 'learning' has been utilised as a binary predicate for machine. Learning as a binary predicate describes a role that is being performed by a machine. It is important to focus on the term 'learning' within the context of the analysis of knowledge. My main goal is figuring out the most significant key points for building a conceptual link between humans and machines.

In order to analyse knowledge I take Bloom's taxonomy into consideration. This taxonomy is a framework for classifying pedagogical objectives, which could be interpreted as the statements of what teachers [, tutors and mentors] expect their learners to have learned, see (Furst et al., 1956; Krathwohl, 2002). Consequently, knowledge has a strong relationship with recognition of materials, ideas, methods, processes, structures and settings. Bloom's taxonomy divides a body of knowledge into multiple classes like, e.g., knowledge of terminologies, knowledge of ways and means, knowledge of trends and sequences, knowledge of classifications and categorisations, knowledge of methodologies, knowledge of universals and abstractions, knowledge of principles and generalisations, knowledge of theories and structures. Later on, (Krathwohl, 2002) has proposed a knowledge dimension in the revised version of Bloom's taxonomy. The revised taxonomy consists of four categories: (1) Factual Knowledge (e.g., terminological knowledge), (2) Procedural Knowledge (e.g., knowledge of methods and algorithms), (3) Conceptual Knowledge (e.g., knowledge of theories, models and structures), and (4) Meta-cognitive Knowledge (e.g., contextual knowledge, conditional knowledge). According to this categorisation I can say that "knowledge acquisition consists of a sort of Transformation functions from reality into the sets and categories of facts, procedures, concepts and contexts". The human being has this ability to deal with multiple classes of facts, procedures, concepts and contexts and can transform them into her/his mind.

Transformations can be interpreted as the outcomes of self-involvement in increasing knowledge about a subject matter. In human sciences, a learner is someone who intentionally attempts to know more about something in order to construct her/his knowledge about that thing. Any human has a background knowledge and tackles to carry on constructing knowledge over her/his existing knowledge. This consideration conduces me to observe and to interpret human knowledge acquisition (and human learning) as the 'activity of construction'. Any person tackles to develop her/his constructed knowledge constructions and to gain an opportunity to attain deeper realisations and understandings. Also, human's deeper understandings support her/his greater motivations. Here I feel the need to concentrate on 'conceptualisations' in order to provide a supportive analysis of realisation and understanding.

In my opinion, an understanding expresses a local manifestation of a global conceptualisation. More specifically, any understanding (based on a concept) could be interpreted as a local manifestation of a global conceptualisation (of that concept). It shall be claimed that human beings' grasps of concepts could provide proper foundations for generating their own conceptualisations. Additionally, the personal conceptualisation could be identified as the action or the process of forming a concept with regard to the basis that has been provided by the individual realisation (see Appendix II: Concept Understanding and Conceptualisation).

In this research, I will mainly focus on concepts, conceptions and concept representations. I have believed that the main focus of process of knowledge acquisition (and learning) is on concepts and concept representations in the ground of conceptualisations. Knowledge acquisition based on concepts can be based on the following definition. This definition draws out the key elements, which have individual and social implications for intelligent learners, see (Watkins et al., 2002).

- Knowledge acquisition is a reflective activity that enables the learner to draw upon her/his previous experiences (and her/his background knowledge) to conceptualise (and, respectively, understand) and evaluate the present, so as to build up and shape future actions and to construct (and develop) new knowledge.

Let me go back to machines and machine learning. A machine program is said to learn from an experience if:

1. there is a set of tasks for machine,
2. there is a machine's performance measure, and also
3. the machine's performance at those tasks, as measured, improves with its experiences.

Here I present a problem in human learning to make a comparison between human learning and machine learning. This example can clarify what the afore-mentioned concepts in a machine learning problem are. Suppose we think of the problem that focuses on students' mathematical problem solving. Considering this problem,

- the most significant task of a student is 'to find proper solutions for mathematical problems',
- the set of tasks must consist of 'the student's tasks and obligations for solving mathematical problems',
- the performance measure could be known as 'the percentage of correctly solved problems', and
- the experience could consist of 'the existing transformations and alterations between observed problems and solved problems'.

Hence, a student can improve her/his ability in performing proper solutions for different mathematical problems after further experiments (experiencing more transformations). Subsequently, this student will have a better capability and more qualified competences in solving mathematical problems when more transformations (experiences) are provided for her/him. Providing more transformations for a student could be achievable by showing and providing her/him with more positive (sample) and negative (non-sample) examples of the solved mathematical problems.

Here I shall claim that the word 'learning' in 'machine learning' is metaphorical, and is a reflection of human knowledge acquisition and learning in machines and artificial agents. Let me express that 'machine learning' is a metaphor that describes what ingredients and concepts are concerned with effective knowledge acquisition and learning within reality. In my opinion, the most important concepts in a machine learning problem (e.g., problem, experience, task, performance, ability, learning) are 'conceptual reflections'. They are some mappings from reality into usable and applicable labels. In the following sections I will focus on:

i. humans' conceptual representations in the framework of constructivism (as an educational theory of learning and a proper model of knowing) and constructionism (as a theory for conceptualising learning that could be identified as a complement for constructivism), and
ii. hypothesis generation and concept representation in the framework of inductive concept learning (as a supervised machine learning paradigm).

Accordingly, the main contribution of this research is figuring out the most significant key points for 'constructing a conceptual and epistemological linkage between a [concept-based] human learning theory and a [concept-based] machine learning paradigm'. I will analyse the structural and logical specifications of concepts and conceptual representations and will analyse a common ground for expressing and analysing concepts in the context of human-machine interplays. I will also relate my specifications with Kantian account of schemata (and schemata-based concepts). Consequently, I will provide a list of the most significant transformations (from human into machine) and reflections (of human in machine) that make conjunctions between human learning and machine learning.

2. CONCEPTUAL REPRESENTATION

In this section I focus on (i) human conceptual representation in the framework of constructivism and constructionism and on (ii) hypothesis generation in the framework of inductive concept learning.

2.1. CONSTRUCTIVISM AND CONSTRUCTIONISM

Constructivism is a philosophy that appears in a variety of guises, some of them pedagogical, some epistemological and some in complex combinations, see (Phillips, 1995). In this research, I see 'constructivism' as a model of knowing with roots in philosophy, psychology and cybernetics that could support constructivist learning. In my opinion, the successful theories of learning are always getting supported by strong models of knowing, and thus, constructivism as a learning philosophy and as a theory of learning is highly dependent on constructivism as a model of knowing. According to these characteristics, it's possible to say that 'a successful theory of knowledge and an effective learning science may be constructed and developed based on the proper foundation that is provided by constructivism'. Jean Piaget, the originator of constructivism, argued that all learning was mediated by the construction of mental objects that he called schemata. Schemata gradually develop into more conceptual mental entities, see (Bartlett, 1932; Parker, 2008). Let me explain the schemata in more detail. In constructivist learning the human's mental structures manifest themselves in the form of schemata. The schemata demonstrate the human's realisation of the world. They conceptually represent the constituents of one's thoughts for knowledge acquisition with regard to her/his realisation of the world. Anyhow, in the framework of constructivism, a human being with respect to her/his pre-structured knowledge and her/his preconceptions attempts to develop the construction of knowledge. The most significant objective of constructivism is producing one's own understanding of the world, see (Husén and Postlethwaite, 1989; Keith Sawyer, 2014; McGawand Peterson, 2007) for more detailed information.

Constructionism is a framework central to the learning sciences, and it posits that 'learners create their own knowledge by the construction of conceptual representations'. Constructionism focuses on:

- conceptualising learning, and
- learning how a human can learn.

Papert's constructionism focuses more on 'the art of learning' and on 'the significance of making and producing things in learning'. Papert is interested in

- how learners engage in a relationship with [their own or other's] knowledge construction(s), and
- how these relations ultimately facilitate the construction of new knowledge.

Constructionism is a constructivist learning theory. It shares constructivism's view of learning as 'building knowledge structures' through progressive internalisation of action, see (Spiro et al., 1992; Ackermann, 2002; Papert, 1993). I may conclude that the main idea of constructionism is that human beings learn effectively through creating, constructing and developing things. Additionally, by adding experiences to the constructivism approach, constructionism attempts to conceptualise learning and to specify and analyse 'learning to learn'.

The most significant mutual objective of constructivism and constructionism is creating one's own knowledge by constructing conceptual representations. According to (Hampton and Moss, 2003), conceptual representations are arguably the most important cognitive functions in humans. They stand at the centre of the information processing flow, with input from perceptual modules of differing kinds. Also, the most important building block of constructivism and constructionism is schemata, see (Bartlett, 1932). Schemata provide proper backgrounds for the learner's concept [and conceptual] representations. They specify the learner's inferences and can satisfy various conditions for definitions of truth.

We saw that everything is about concepts and conceptual representations. Conceptual representations attempt to investigate the origins of human's thought and roots of the constructed knowledge. In section 3 I will elaborate the description of schemata and will focus on structural and logical specifications of concepts as the key elements of the conceptual domains representation.

2.2. INDUCTIVE CONCEPT LEARNING (ICL)

Machine learning problems can be seen and analysed from different points of view and be divided into several categories. One categorisation could divide them into supervised, unsupervised, and reinforcement learning methods. In supervised learning method, the pair (input,output) training examples are supplied by a trainer who is a human. So, the learner (that is a machine) searches for function mappings from the inputs into the outputs. In this research, I am dealing with ‘inductive learning from examples’, which is a subfield of supervised machine learning. To induce means to infer general principles and rules from specific facts as the instances. I shall emphasise that these facts are different from the facts presented in the revised Bloom’s taxonomy. All existing facts, procedures, concepts and contexts in Bloom’s taxonomy could be captured as some principles (i.e. actuality, objectivity and reality) in machines. In Inductive Learning, we describe the main terminologies, axioms and rules by descriptive logical languages, e.g., First Order Predicate Logic (FOL) and Description Logics (DLs).

Inductive Concept Learning (ICL) is a specified Inductive Learning. ICL attempts to logically describe concepts and their relationships. It employs the members (instances) and non-members of a concept that may be known as a class. A characteristic feature of most inductive learning approaches is the use of background knowledge. This feature supports more complicated and specific learning scenarios, because not only the factual description of the given examples can be used by the machine, but structurally rich knowledge representations can be taken into account as well, see (Mitchell, 1997; Lehmann, 2010).

In parallel with (Lavrac and Dzeroski, 1994), I focus on specification of concept learning with background knowledge. In concept learning with background knowledge, a machine with regard to the given set of training examples and background knowledge finds a ‘hypothesis’. A hypothesis can be expressible in concept description languages. Also, based on the background knowledge and given examples (to machine), a hypothesis can be complete and consistent, i.e. correct. So, one may assume that a hypothesis is generated based on ideas and can determine the applications of a term and a phrase. Furthermore, a hypothesis is a significant part in the use of reason and language. It has a very strong dependency to the background knowledge.

3. CONCEPTS: STRUCTURAL AND LOGICAL SPECIFICATIONS

There has always been a general problem concerning the notion of ‘concept’, in philosophy, in linguistics, in psychology and in computer and information sciences. This research is focusing on knowledge acquisition and learning relying on concepts and concept representations. Thus, I need to ascertain a realisable interrelationship between the description of concepts within human and information sciences. Actually, I am constructing a conceptual linkage between constructivism/constructionism and inductive concept learning. As mentioned, schemata provide proper backgrounds for the learner’s concept [and conceptual] representations. In a simplified version of Kantian philosophy, a non-empirical (pure) concept has been defined as a category. According to Kantian philosophy, schemata are the procedural rules by which a category is associated with a sense impression. Kant claimed that the schemata provide a reference to intuition in a way similar to the manner of empirical concepts.

According to the Kantian account of schemata there are three types of concepts that employ schemata:

1. Empirical concepts: For instance, the concept 'Spring' can describe a rule according to which human's imagination can visualise a general figure of 'a green season with beautiful trees and colourful flowers' without being restricted and closed to any particular and specific shape produced by experience, see (Link a).

2. Pure mathematical concepts: They are the construction or mental drawing of what is common to several geometrical figures. They can be concerned with numbers, algebras and arithmetics. I shall stress that these concepts are not based on objective visual images, see (Link b).

3. Pure concepts of the understanding: They focus on characteristics, predicates, attributes, qualities or properties of an object, that are, also objects in general or as such, see (Link c).

The third employs transcendental schemata, see (Kant, 1781; Kant, 1790; Kant, 1999). Here I focus on some specifications of concepts and then relate them to the Kantian philosophy.

Concepts are the furniture of human beings' minds. A well-furnished mind can be a source of successful knowledge acquisition and learning, see (Parker, 2008). Concepts are realised (by some philosophers and psychologists) as representations of reality in mind. Regarding this grasp of concepts, they could be understood as some general objects and labels, where objects are the constituents of propositions that mediate between thought, language, and referents, see (Bartlett, 1932). From these characteristics, I conclude that it's possible to say that concepts might be understood to be the representations of actualities and objectivities in humans' minds. The mental representations of actualities can affect the human's languages. More precisely, a concept can be said to be a linkage between linguistic expressions (descriptions) and the mental images (e.g., representations of the world, representations of inner experiences) that a human being has in her/his mind, see (Götzsche, 2013).

Relying on logics and their descriptive features, a concept can be seen as an 'idea' and the idea can be transformed into a hypothesis in order to correspond to a distinct entity (or even to a group of entities) or to its/their essential features. The ideas determine the application(s) of terms and phrases. It's really important to say that any idea is a significant part in the use of reason and language.

These characteristics and properties become applied in order to support the metaphorical usages of concepts in machine applications. In fact, the existing linkages between mental images and linguistic expressions can be mapped (be transformed) as multiple ideas into hypotheses in order to determine different applications in artificial systems. As mentioned, a concept can be expressible in some concept description languages and it's possible only in virtue of terminologies. In fact, various concepts and the relationships between them can be used to establish the fundamental terminologies adopted in a modelled conceptual domain regarding the hierarchical structures.

According to the characteristics of humans' ideas, when a human being forms (see Link d) an idea from its examples, she/he gets to know more than just some definitions. This demonstrates the deep learning rather than superficial knowledge, see (Parker, 2008). I shall emphasise that the human learner is the developer of her/his personal conceptions over her/his own designed schemata. In my opinion, the relationships between 'Kantian account of schemata' and the 'empirical concepts' supports the human's mental representations of the objects. It also sees a 'pure concept of the understanding' as a characteristic and predicate of an object (that can express what has been said about that object). The first one employs schemata and the second one employs transcendental schemata. In fact, this is how a learner deals with fundamental concepts through constructivist learning.

Accordingly, the leaner employs inductive rules to expand her/his general ideas into more specified ones. The generalisation of various specified hypotheses (based on ideas) supports the learner in discovering new hypotheses and generating new ideas. She/He searches for and lists attributes and properties that can be used to distinguish exemplars (of various hypotheses) from non-exemplars. But what she/he really does is more than just specifying and generalising from different examples; she/he is highly concerned with identifying and relating the induced examples. Let me be more specific.

As mentioned in 2.2, a machine with regard to the given set of examples and its background knowledge, finds hypotheses. The logical description of a concept, which arises during the knowledge acquisition and learning processes, is called a hypothesis, since it is an experimental explanation of why the objects are members (or non-members) of the hypotheses (concepts). Also, considering a concept as a hypothesis, if an example belongs to a hypothesis, we are able to conclude that the hypothesis covers the example. Then, the example has all features and characteristics of that concept, see (Baader et al., 2010).

3.1. CONCEPTS IN THE COMMON GROUND BETWEEN HUMAN CONSTRUCTIVIST LEARNING AND MACHINE ICL

Obviously, there is an important characteristic of concepts held in common ground. The concepts in the common ground are the images of the Idea transformations (the transformations from human beings into machines). The mappings epitomise humans' conceptual representations and generate hypotheses. In the common ground a concept is a specialised or generalised experience. The concepts could be recognised by their instances (that are concepts as well) and they all can be represented in different hierarchies.

In human scientific approaches an experimental explanation of why some objects are the members of a concept may support learners in representing their own ideas and in providing ideal (and conceptual) representations. In fact, the quality and the modality of the concept representation is affected by observing 'empirical concepts' and the 'pure concept of understanding' with regard to a Kantian account of schemata and transcendental schemata. On the other hand, in machine learning approaches, a machine generates the represented concept from its given instances. In the common ground, an experimental and empirical explanation of WhyNess of existence of some 'concepts, ideas and hypotheses' as the instances of other 'concepts, ideas and hypotheses' can provide a strong background for improving the quality of conceptualisations. Here are a number of transformations (from human into machine) and reflections (of human in machine) that make a conceptual and epistemological connection between human learning and machine learning:

- Transformation of a human being's knowledge and knowings into multiple principles (and axioms) in machines that are mainly object-oriented. Accordingly, the human being's knowings get classified into the specified classes (and under the determined labels) in machine's knowledge base.

- Transformation of a human being's experimental and empirical achievements into various categories of positive and negative examples in machines. Thus, the human being's experiments become divided into exemplars and non-exemplars of the specified classes with determined labels.

- Transformation of a human being's real 'problems', real 'tasks for solving problems' and real 'performances' into provided classes with the same labels (Problem, Task and Performance) in machines.

- The reflection of human learning and knowledge acquisition in machines and artificial agents. This reflection is equivalent to transforming a taken metaphorical image of learning and knowledge acquisition into machines and artificial agents.

- The reflection of human concepts in the hypotheses. The linkages between a human being's mental representations and linguistic expressions (and descriptions) are getting mapped as some ideas into hypotheses in machines. They correspond to multiple entities or to their essential features in order to express different significant parts in the use of reasons and languages.

- The reflection of humans' conceptual representations in hypothesis representations and representation of hierarchy of hypotheses in machines' knowledge bases.

4. SUMMARY AND CONCLUSIONS

In this article, I have focused on a conceptual and epistemological linkage between concept-based human learning and concept-based machine learning. Regarding the structures of human-oriented sciences and information sciences and according to the fact that human-oriented sciences and information sciences support distinct types of frameworks, I have had to specify and to analyse knowledge, knowledge acquisition and learning from two separated points of view. In human systems, knowledge acquisition is a reflective activity that enables a human being to draw upon her/his experiences and background knowledge to conceptualise, understand and evaluate the present, so as to build up and shape her/his future actions and to construct and develop new knowledge. On the other hand, a machine program is said to learn (and acquire knowledge) from an experience if there is a set of tasks and a performance measure for it and, also, if its performance at those tasks, as measured, improves with its given experiences.

In this article, according to (i) constructivism as a model of knowing and a theory of learning, and constructionism as a theory of conceptualising learning, and (ii) inductive concept learning as a supervised machine learning paradigm, I have focused on building a conceptual linkage between human learning and machine learning. The constructivist and constructionist theories of human learning and the [machine] inductive concept learning paradigm are shaped based upon concepts. The first two are focusing on concepts and conceptual representations and the third one focuses on representing concepts in informations sciences within electronic systems for hypothesis representation and hypothesis generation. My central focus has been on analysing concept representations in the mentioned frameworks and on their common ground. A concept can be seen as a linkage between linguistic expressions and the mental images that a human has in mind. It can be observed as an idea and be transformed into a hypothesis in order to be corresponded to entities or to their essential features. In fact, schemata provide proper backgrounds for the learner's concept (and conceptual) representations. A Kantian account of schemata sees the empirical concepts in the human's mental representation of the objects. It also sees a pure concept of the understanding as a characteristic and predicate of an object. It can express what has been said about a thing. The first one employs schemata and the second one employs transcendental schemata. In fact, this is how a learner deals with fundamental concepts within constructivist learning and, accordingly, she/he transforms her/his concepts into multiple hypotheses in order to apply them in inductive concept learning frameworks in machines.

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Links

Link a: http://kantwesley.com/Kant/EmpiricalConcepts.html

Link b: http://plato.stanford.edu/entries/kant-mathematics

Link c: http://userpages.bright.net/~jclarke/kant/concept1.html

Link d: http://teachinghistory.org/teaching-materials/teaching-guides/25184

PAPER F. LOGICAL CHARACTERISATION OF CONCEPT TRANSFORMATIONS FROM HUMAN INTO MACHINE RELYING ON PREDICATE LOGIC

Farshad Badie

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ABSTRACT

Providing more human-like concept learning in machines has always been one of the most significant goals of machine learning paradigms and of human-machine interaction techniques. This article attempts to provide a logical specification of conceptual mappings from humans' minds into machines' knowledge bases. We will focus on the representation of the mappings (transformations) relying on First-Order Predicate Logic. Additionally, the structure of concepts in the common ground between humans and machines will be analysed. It seems quite necessary to pay attention to the philosophy of constructivism and constructivist models of knowing. This research constructs a conceptual ground for expressing and analysing concepts in the common ground between humanistic and informatics sciences and in the context of human-machine interplays.

1. INTRODUCTION AND MOTIVATION

In an interaction between human beings (as intentional, aware and intelligent agents) and machines (as unaware and artificial agents), they exchange multiple actions and transactions concerning, e.g., identifications, descriptions, specifications and reasonings. According to (McIntyre Boyd, 2004) and based on our epistemological approach, the multilevel interactions between a trainer (a human being) and an artificial and a metaphorical learner (a machine), could be seen as a radical constructivist account of human cognition and comprehension. Also, these interactions could shape a kind of ontology. Obviously, the human-machine interactions are not agreement-oriented, because an aware agent cannot make an agreement with an unaware agent, but we suppose there is a type of agreement and convention between the human being and herself/himself to forward information about a given domain to the machine and to train the machine about some particular topics and concepts in that domain. In section two, we will focus on the expression 'concept'. In interactions between human beings and machines, humans can develop their non-evidential and non-axiomatical conceptions of the specified underlying systematic processes in the world.

Training machines based upon personal mental images of reality in the context of human-machine interactions, could provide a proper ground for constructivist machine training. At this point, we take the philosophy of constructivism into consideration. Constructivism appears in a variety of guises (e.g., pedagogical, epistemological and complex combinatorial). It has been known as a philosophical theory of learning and as a model of knowing, see (Husén and Postlethwaite, 1989; Spiro et al., 1992; Phillips, 1995; McGawand Peterson, 2007). According to constructivism, a human being is always concerned with the active creation of personal mental representations. As for learning in the framework of constructivism, any agent generates her/his own schemata, see (Link a). Relying on our approach, any schema is the product of the trainer's understanding of the world. It conceptually represents the constituents of the trainer's thought about training something. Schemata support the trainer in constructing and in developing her/his concepts (that have been constructed with regard to her/his own realisation of the world). Additionally, they provide strong backbones for the trainer's interpretations and provide proper backgrounds for describing terminologies and world descriptions. The constructivist machine training framework is heuristic, explanatory and developmental for human being's thoughts and reasoning. Actually, any constructivist machine training in the context of human-machine interaction is concerned with heuristic questions focusing on (i) 'What/Which is …?', (ii) 'How is …?', and (iii) 'Why is …?'. The first group of questions focus on the factual, structural, existential and ontological aspects of the world, the second group focus on procedural, methodological and technical aspects of the world and the third group focus on inferential aspects of the world.

This article attempts to construct a conceptual and logical linkage between human's knowledge and machine learning. So, before getting into the details we contemplate the term 'Machine Learning'. Machine Learning is a subfield of Artificial Intelligence and Computer Science. According to (Mitchell, 1997), a machine learning approach attempts to develop strong algorithms that allow machines to improve [the productivity of] their performances on a given goal [and on an objective function]. In machine learning, the word 'learning' has been utilised as a predicate for the expression 'machine'. 'Learning' as a binary predicate describes a role that is being performed by the machine. More specifically, machines' concept learning approaches try to provide appropriate logical descriptions and specifications for transformed concepts and their interrelationships after being transformed concerning their relationships with reality. A characteristic feature of most concept learning approaches is the use of background knowledge (e.g., internal knowledge base, ontological description). This feature supports more complicated and specific learning scenarios, because not only a factual (e.g., terminological) description of given examples can be used by the machine, but also, structurally, rich knowledge representations are taken into account, see (Mitchell, 1997; Lavrac and Dzeroski, 1994). In concept learning with background knowledge, with regard to the given set of training examples and background knowledge a machine focuses on hypothesis generation. In this article, we will provide a logical specification of mental mappings from humans into machines. We will focus on representations of transformations from humans' conceptions into machines' knowledge bases relying on First Order [Predicate] Logic (FOL). The results will support figuring out and analysing the most significant components of the logical characterisation of concept transformations. In the second section, we will focus on concepts and transforming concepts. The third section will deal with concept transformation process consisting of logic of transformation and the analysis of transformation. Section four will summarises the conclusions.

2. CONCEPTS AND TRANSFORMING CONCEPTS

First, we shall stress the fact that the notion of 'concept' is a very sensitive term that must be used with caution, but we assume the use of 'concept' to be comprehensible in this context and in the logical formalisms. In our opinion, a human being's specified realisation of the world finds its real significance with regard to her/his grasp of the various concepts. Concepts support thoughts. Thoughts are also highly dependent on a human's interpretations and realisations of whether a given thing/phenomenon is an instance of a [constructed] concept or not. According to (Götzsche, 2013) and based on our conceptual approach, a concept is a linkage (relationship) between humans' mental images of reality (for instance, "an image of the Spring") and her/his linguistic expressions and statements (for instance, "Spring is one of the four conventional temperate seasons, following winter and preceding summer").

Let us represent the described linkage by —*R*—. In descriptive logical approaches, these expressions support the definitions. A 'definition' is a kind of equivalence between a term referring to a thing (the thing that is going to be defined) and a description (generally built up using the inductive rules). Also, there is a strong relationship between the mental images and the mental representations of different aspects of the world. In fact, human beings need to, logically, apply —*R*— in their world descriptions, e.g., in assertions about real-world objects, in assertions about the empirical world, in assertional knowledge representations, in assertions about the ontologies, and in descriptions of terminologies and terminological knowledge. Therefore, human being transform —*R*— into discrete classes of things in order to see its applications. Thus, transformations play a very efficient part in the use of reasons and languages. Actually, transformations allow human to divide a continuously varying world into discrete classes of things, see (Lake, 2014).

At this point, we focus on the concept formation process (see (Parker: Concept Formation)) and acknowledge this process as the most fundamental step towards constructivist machine training. By forming concepts, a trainer (who is a human being) sorts her/his specific experiences and empirical studies into general classes [or even rules]. For instance, regarding the fact 'Drinking is a sign of thirst', she/he represents the classes Drinking and Thirst and the rule 'Drinking $\rightarrow$ Thirst' in the machine's knowledge base. Consequently, the machine expresses the proposed classes and generates the proposed rule over the background knowledge in machine's knowledge base and with regard to other experiences of the trainer. Moreover, the machine utilises the expressed classes and the generated rules in class-based and rule-based reasoning processes. We have introduced the term 'concept construction' process in (Badie, 2015a; Badie, 2015b) and have interpreted it as the super-category of concept formation processes. A concept construction process consists of 'forming concepts' and 'reforming constructed concepts'. The trainer is highly concerned with main characteristics and features of a thing/a phenomenon in order to consider it as an instance of a class. The trainer must employ the examples that can lead her/him to discovering new classes. She/he searches for [and itemises] the attributes and properties that can be used to distinguish exemplars from non-exemplars of various classes. Additionally, she/he identifies, specifies and relates the generalised examples and compares different examples. The following statements are derived from the above-mentioned characteristics of concepts.

- The descriptive logical languages and logical techniques transform the relationships between a human's mental images and her/his linguistic expressions into various ideas that are representable in the form of entities (discrete classes of things). The ideas specify the human's definitions (that are supported by linguistic expressions) by employing the logical rules that are (could be) existing between the same classes in the world. Accordingly, an idea is transformed into a hypothesis in order to correspond to a discrete class.

- As for the fundamental characteristics of concepts, a human being's conception within her/his interactions with a machine is equivalent to her/his act of representing various concepts and linking her/his explanations and, respectively, definitions, with regard to her/his own mental images.

3. CONCEPT TRANSFORMATION PROCESS

As accounted from above, "a concept is a relation and, in fact, a binary predicate between humans' mental images of the world and their linguistic expressions [and, thus, definitions]". Obviously, the definitions always attempt to provide appropriate descriptions for the mental images. Subsequently, the existing interrelationships and dependencies between mental images and the provided descriptions support idea generation. At this point, we focus on the analysis of idea transformation from humans' minds into machines' knowledge bases. Suppose that the trainer has considered n objects. For instance, the set of n objects is equal to {sofa_1, glass_2, plate_3, …, brown_n}. We shall draw your attention to the logical description of the transformation process.

3.1. LOGIC OF TRANSFORMATION

ONE. The trainer assigns her/his ideas to the objects and focuses on 'idea assertion'. For instance, she/he assigns her/his first idea to the first object. So, she/he constructs $Idea_1(object_1)$. For instance, she/he constructs Furniture(sofa) to express the fact that sofa is a furniture (or sofa is a member of the class Furniture). Similarly, she/he assigns the second [and, respectively, the third, fourth, …, and nth] ideas to the second [, third, fourth, …, and nth] objects. Therefore, there are totally n assignments like:

$$Idea_1(object_1),\ Idea_2(object_2),\ \ldots,\ Idea_n(object_n).$$

This conclusion represents a linear model. Considering $i \in [1,n]$ and relying on FOL, $Idea_i$ represents a unary predicate and $object_i$ represents a constant symbol (as an instance of the unary predicate $Idea_i$).

TWO. The trainer makes a relation between her/his achievements. Employing FOL, there exists a

$$Relation\ [Idea_1(object_1),\ Idea_2(object_2),\ \ldots,\ Idea_n(object_n)].$$

For instance, she/he can relate the assertions (the world descriptions) Furniture(sofa) and Colour(brown) to each other. Then, *Relation* [Furniture(sofa), Colour(brown)] is capable of representing different types of relationships between sofa and brown with regard to their labels in the trainer's mind. Actually, the proposed world descriptions can actively develop her/his knowledge. Also, the relationship between the world descriptions can establish various expressions in her/his mind. Let us conclude that these relationships construct more specified ideas based upon the proposed world descriptions. Relying on FOL and considering $p, q \in [1,n]$, *Relation* [$Idea_p(object_p)$, $Idea_q(object_q)$] represents a binary predicate between two unary predicates (between $Idea_p$ and $Idea_q$). This relation is also valid between $object_p$ and $object_q$ as the instances of $Idea_p$ and $Idea_q$ respectively. In this step, the trainer has produced a linear relational model, see Table 1.

$Idea_1(object_1)$	…	$Idea_n(object_n)$

Table 1. *Linear Relational Model*

THREE. The approached linear relational model is based on FOL. But it could also be represented in the form of a j-by-i matrix like I, where $i, j \in [1, n]$. This step represents the most significant assumption of the transformation. We shall stress the fact that we have represented the linear description '*Relation* [$Idea_1(object_1)$, $Idea_2(object_2)$, …, $Idea_n(object_n)$]' in the form of a j-by-i matrix in order to allow the required linear transformation (that reflects the ideas) to be represented in a well-structured format. Additionally, a matrix can appropriately be used in establishing a transformation. Here, we have a matrix (relational model), see Table 2.

$Idea_1(object_1)$	…	$Idea_i(object_i)$
…		
$Idea_j(object_j)$	…	$Idea_n(object_n)$

Table 2. *Relational Model*

FOUR. This step focuses on reflection. The idea assertion $Idea_1(object_1)$ (that is located in the first row and the first column of Table 2) becomes reflected in $Predicate_1(constant_1)$ (located in the first row and the first column of Table 3 that is the product of the transformation) and $Idea_n(object_n)$ (located in the jth row and the ith column of Table 2) gets reflected in $Predicate_n(constant_n)$ (located in the jth row and the ith column of Table 3). Thus, all cells in the relational model 2 are, collectively, reflected in an equivalent relational model (matrix), see Table 3.

$Predicate_1(constant_1)$	…	$Predicate_i(constant_i)$
…		
$Predicate_j(constant_j)$	…	$Predicate_n(constant_n)$

Table 3. *Relational Model*

FIVE. Table 3, as a relational model, represents a relationship between

$$Predicate_1(constant_1),\ Predicate_2(constant_2),\ \ldots,\ \text{and } Predicate_n(constant_n).$$

Therefore, we have a description like:

$$Relation\ [Predicate_1(constant_1),\ \ldots,\ Predicate_n(constant_n)].$$

Consequently, there are n assignments from the [unary] $Predicate_1$ into $constant_1$, from $Predicate_2$ into $constant_2$, …, and finally, from $Predicate_n$ into $constant_n$. These assignments have been related with each other by means of n-ary *Relation*. Based on FOL, the effect of n-ary *Relation* is equivalent to:

$$Predicate\ [Predicate_1(constant_1),\ Predicate_2(constant_2),\ \ldots,\ Predicate_n(constant_n)].$$

Note that the outer predicate is n-ary and works on n internal unary predicates. Then, the trainer has produced a linear relational model, see Table 4.

$Predicate_1(constant_1)$	…	$Predicate_n(constant_n)$

Table 4. *Linear Relational Model*

SIX. This step focuses on generating the 'relational hypothesis model'. Actually, the effect of the first unary predicate on the first constant symbol generates the first hypothesis (or $Hypothesis_1$), the effect of the second unary predicate on the second constant symbol generates the second hypothesis (or $Hypothesis_2$), …, and the effect of the nth unary predicate on the nth constant symbol generates the nth hypothesis (or $Hypothesis_n$). Subsequently, the outer n-ary predicate relates $Hypothesis_1$, $Hypothesis_2$, …, and $Hypothesis_n$. Therefore, there is a relationship between all generated hypotheses. Thus, we have:

$$Predicate\ [Hypothesis_1,\ Hypothesis_2,\ \ldots,\ Hypothesis_n].$$

Therefore, we have a relational hypothesis model, see Table 5.

$Hypothesis_1$	…	$Hypothesis_n$

Table 5. *Linear Relational Model*

SEVEN. Finally, there is a set like:

$$\{Hypothesis_1, Hypothesis_2, \ldots, Hypothesis_n\}$$

that represents the generated hypotheses for the machine.

3.2. ANALYSIS OF TRANSFORMATION

Suppose that:

i. I_n denotes the n-component linear relational model $[Idea_1(object_1), Idea_2(object_2), \ldots, Idea_n(object_n)]$,

ii. P_n denotes the n-component linear relational model $[Predicate_1(constant_1), Predicate_2(constant_2), \ldots, Predicate_n(constant_n)]$, and

iii. H_n denotes the n-component linear relational model $[Hypothesis_1, Hypothesis_2, \ldots, Hypothesis_n]$.

First, we focus on the forward direction from human to machine. There are *reflection* functions like $\boldsymbol{R}_i$ from human being's ideas into predicates. Let us represent the set of $\boldsymbol{R}_i$ by $\boldsymbol{R}$. So:

$$\boldsymbol{R}: I_n \rightarrow P_n .$$

Then, $\boldsymbol{R}$ represents the transformed ideas into predicates. Semantically, the reflection functions $\boldsymbol{R}$ satisfy the n-component model $[Hypothesis_1, Hypothesis_2, \ldots, Hypothesis_n]$ (i.e., provide proper models that attempt to satisfy the hypotheses). Then, there is a model like:

$$\boldsymbol{R} \vDash H_n .$$

Therefore, the reflection functions $\boldsymbol{R}$, semantically, satisfy the set of hypotheses in the machine (Result 1).

At this point, we focus on the backward direction from machine to human. There are various *conformation* functions like $\boldsymbol{C}$ such that:

$$H_n \vDash \boldsymbol{C}.$$

Semantically, any conformation function becomes satisfied by a hypothesis like $Hypothesis_i$ belonging to the n-component relational model [$Hypothesis_1$, $Hypothesis_2$, …, $Hypothesis_n$]. Note that $\boldsymbol{C}$ denotes the set of $\boldsymbol{C}_i$. So, $\boldsymbol{C}$ represents the transformed predicates into ideas and formally:

$$\boldsymbol{C}: P_n \rightarrow I_n \quad \text{(Result 2)}.$$

According to the results 1 and 2 we have:

$$(\boldsymbol{R}: I_n \rightarrow P_n) \quad \vDash \quad H_n \quad \vDash \quad (\boldsymbol{C}: P_n \rightarrow I_n)\ .$$

Then:

$$I_n \rightarrow P_n \quad \vDash \quad H_n \quad \vDash \quad P_n \rightarrow I_n\,.$$

In fact, the reflection transformations from ideas into predicates satisfy the hypotheses. And the hypotheses satisfy the inverse reflection transformations (or conformation transformations) from predicates into ideas.

4. CONCLUSIONS

Training machines based upon personal mental images of the world in the context of human-machine interactions shapes the skeleton of constructivist human-machine interactions. Schemata in constructivist training frameworks could demonstrate the trainer's realisations of the world. They conceptually represent the constituents of the trainer's thoughts for training concepts. Schemata support the trainer in developing her/his constructed concepts (that have been constructed with regard to her/his own realisation of the world). In this article, we have provided a logical and epistemological specification of concepts and we have seen the linkages between humans' mental images and their linguistic expressions as the origins of manifestation of concepts. Accordingly, we have logically specified the mental mappings from human into machine and, subsequently, we have focused on logical representations of transformations from human beings' conceptions into machines' knowledge bases relying on First-Order Predicate Logic. We have identified the transformations from humans' minds into machines' knowledge bases by 'reflection transformations' and, correspondingly, we have labeled the inverse cases by 'conformation transformations' in order to analyse the proposed logical descriptions. The reflection transformations from ideas into predicates satisfy the hypotheses. And the hypotheses satisfy the conformation transformations from predicates into ideas. In future research, we will employ the results in formal semantic analysis of concept transformations from minds into knowledge bases and in specifying their conceptualisations.

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Links

Link a: http://plato.stanford.edu/entries/schema

PAPER G. TOWARDS SEMANTIC ANALYSIS OF TRAINING-LEARNING RELATIONSHIPS WITHIN HUMAN-MACHINE INTERACTIONS

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ABSTRACT

In this article, First-Order Predicate Logic (FOL) is employed for analysing some relationships between human beings and machines. Based on FOL, we will be conceptually and logically concerned with semantic analysis of training-learning relationships in human-machine interaction. The central focus is on formal semantics and its role in the 'relationship' between human beings and machines. The analysed relationships between a human being and a machine will support our thoughts on and contemplations over the HowNess of establishing formal semantics within human-machine interaction.

1. INTRODUCTION AND MOTIVATION

Machine Learning is a subfield of Artificial Intelligence and Computer Science. A machine learning approach attempts to develop appropriate procedures and techniques that allow machines to improve the productivity of their performances concerning a given goal, see (Mitchell, 1997). In (Badie, 2016a), we have focused on conceptual analysis of human-machine interactions and we have provided a conceptual and epistemological junction between human beings' minds and machines' knowledge bases. According to (McIntyre Boyd, 2004) and relying on our epistemological approach, the multilevel interactions between a human being (as a trainer) and a machine (as a metaphorical learner) could be seen as a radical constructivist account of human cognition, realisation and comprehension. Let us bring up some fundamentals in order to clarify our conception and way of thinking about the metaphorical use of 'learning'. In the expression 'machine learning', the word 'learning' has been utilised as a binary predicate with the word 'machine'. Learning as a binary predicate has been asserted to be a role that is being performed by a machine. Thus, the act of 'learning' for a machine could be interpreted as a reflection of human learning (the phenomenon of 'learning') in machines. In fact, machine learning is a metaphor that attempts to simulate the phenomenon of 'learning' with regard to the ingredients, components and concepts that are concerned with effective and successful learning processes in the real world. Let us bring the notion 'concept' into our explanation and be more specific on this research's objectives. Machine concept learning approaches try to provide appropriate realisable logical descriptions for a human being's constructed concepts and their interrelationships after being transformed (from a human's mind into a machine's knowledge base) with regard to their structures and to their interrelationships to the world. Note that 'concept' is a complicated term. We see a concept as a linkage between a human's mental images of parts of reality (as things/phenomena), on the one hand, and a human's linguistic expressions and statements concerning those things/phenomena on the other hand, see (Götzsche, 2013). In (Badie, 2016a), we have explained that concepts can be transformed in order to be represented and expressed within a machine's knowledge base. For instance, concepts can be reflected in order to be represented in the form of the entities as classes of individuals and objects. In other words, a concept is understood and is seen as an idea that can be transformed into a hypothesis in order to correspond to a distinct entity [and, respectively, to a group of entities] or to its [and their] essential attributes, features, characteristics and properties. The hypotheses can describe multiple theories based on terminologies and world descriptions. Accordingly, they support inferential and reasoning processes and satisfy multiple conditions for definitions of truth with regard to interpretation functions.

In this article, we will employ First-Order [Predicate] Logic in order to focus on relationships between human beings and machines. FOL allows us to make arbitrarily complex relationships between different objects of a system. Based on FOL we will be conceptually and logically concerned with semantic analysis of training-learning relationships in human-machine interactions. We shall stress that our main focus is on the semantics of the ‘relationships’ between human beings and machines. The analysed relationships between a human being and a machine will support our thoughts about the HowNess of establishing a [formal] semantics concerning human-machine interactions. According to (Blackburn, 2008), semantics is the study of the meanings and the relation of signs to the objects to which the signs are applicable.

In the following sections, you will be offered the following: The Logical Specification of the Notion of Hypothesis, Preliminaries: Predicate Logic and Semantics in FOL, Formal Representation and Semantic Analysis of ‘Training-Learning’, and Conclusions and Future Work.

2. THE LOGICAL SPECIFICATION OF THE NOTION OF HYPOTHESIS

Based on Predicate Logic and focusing on Description Logics (Baader et al., 2010), a unary predicate is supposed to be logically equivalent to a concept. For instance, we can consider the unary predicate Set as a concept in order to employ it in concept learning (concept expression) processes. Additionally, a concept can be logically described by a hypothesis, see (Lehmann, 2010; Mitchell, 1997). For instance, the concept Set can be described as “a collection of the distinct things” in order to provide a foundation for a hypnosis. And, for instance,

- $\{a$, 5, Science, ∞, $\sum\}$ and {Book, ●} are two positive (constructive) examples of the proposed hypothesis, and
- ‘{3, T’ and ‘Y’ are two negative examples of the proposed hypothesis.

In our opinion,

i. analysing the supportive inferential processes on a hypothesis, and
ii. focusing on world descriptions using generated hypotheses relying on defined terminologies,

could collectively determine the applications of predicates and, subsequently, the applications of terms and statements. Conceptually and logically, the hypotheses focus on describing the predicates. Then they are expected to describe the same attributes, characteristics and properties. According to (Mitchell, 1997), a hypothesis as a logical description of a concept, arises during a machine learning process. Actually, it is a tentative explanation of why the objects are members (or non-members) of the concepts. A characteristic feature of most concept learning approaches is the use of background knowledge. In concept learning with background knowledge, a machine, with regard to the given set of training examples and background knowledge, will focus on hypothesis generation.

3. PRELIMINARIES: PREDICATE LOGIC

The Propositional Logic and its formulae (i.e., the formal and mathematical relationships or rules expressed in Propositional Logic's symbols) are constructed based on atomic objects. Note that the atomic objects and, accordingly, the propositional formulae, could only be either true or false. First-order Predicate Logic (FOL) is constructed over propositional logic by seeing objects as the elements of sets and by applying universal and existential quantifications (restrictions). That's why some logicians and mathematicians interpret FOL as Quantification Theory, see (Mendelson, 1987; Ohlbach; 1985). FOL allows us for making arbitrarily complex (specified) relationships between various objects. There are two kinds of symbols in FOL: (*i*) logical symbols and (*ii*) non-logical symbols. The set of logical symbols in FOL is {Conjunction ($\wedge$), Disjunction ($\vee$), Negation ($\neg$), Implication ($\rightarrow$), Bi-conditional ($\leftrightarrow$), Equality (=), Existential Restriction ($\exists$), Universal Restriction ($\forall$), Tautology ($\top$), Contradiction ($\perp$), Parentheses and brackets}. We shall stress that logical symbols always have the same meaning. It means that we are not allowed to interpret them and assign multiple values and definitions to them. The non-logical symbols are represented in the following forms:

- *Constant Symbols.* For instance, john, 0 and blue are constant symbols.

- *Unary Predicates.* In $P(x)$ and $Q(y)$, P and Q denote unary predicates. Also, x and y are variables (multiple constant symbols). These variables are the instances of P and Q. For instance, Person(john) denotes that 'John is a person'.

- *Binary Predicates* (*Relations*). $R(m,n)$ is a binary predicate and makes a relation between two variables m and n. For example, Equals(m,n) can represent the 'equality between m and n' (i.e., m equals n).

- *Function Symbols.* $f(x)$ is a function that f operates the variable x. For example, mother(john) can represent the 'mother of john'.

At this point, we shall draw your attention to the fact that the meanings of the non-logical symbols are dependent on human being' interpretations. So, we need to interpret the non-logical symbols to produce meanings and to clarify what we mean by them.

3.1. SEMANTICS IN FOL

In formal languages, semantics is the study and analysis of the meanings of symbols and signifiers. Semantics focuses on the relationships between the signifiers of any language. In fact, the formal semantics employs the products of the human beings' interpretations in order to produce meanings. In fact, we need to consider the interpretation I that consists of:

- the domain of interpretation (that is a non-empty set like D), and
- an interpretation function (like $\cdot^I$) that interprets the domain D in order to analyse the formal semantics of a term in FOL.

Example. A = {Bob, Mary, Julian} could be interpreted (A^I) to represent the list of three PhD researchers in Metaphysics. Obviously, a meaning has been produced.

Example. The interpretation function assigns to every atomic unary predicate P (e.g., Apple, Red), a set like $P^I \subseteq D^I$. For instance, the interpretation of Apple (= Apple^I) could express that "apple is a fruit and can be eaten".

Example. The interpretation function assigns to every atomic binary predicate R (e.g., Equals) a binary predicate $R^I \subseteq D^I \times D^I$. Accordingly, the interpretation of Equals (Equals^I) could express that "Equals describes a kind of alignment between its right-hand side and its left-hand side".

Here we feel the need to describe the logical conception of equivalence relationship between two predicates. Two unary predicates (either atomic or non-atomic) P and Q are equivalent ($P \equiv Q$), when for all interpretations I we have $P^I = Q^I$. On the other hand, they are not equivalent when there exists an (at least one) interpretation like J such that $P^J \neq Q^J$.

4. TRAINING-LEARNING: FORMAL REPRESENTATION

In this section the central focus is on conceptual and logical analysis of formal semantics within a training-learning relationship in the context of human-machine interactions. This research aims at investigating where the formal semantics come from and when it appears within a relationship between a human being and a machine. Considering the human being as the trainer and the machine as the metaphorical learner, accept the following axioms. These axioms focus on the non-logical symbols of our formalism. They are the main building blocks of this research.

- The symbols h and m denote 'human being' and 'machine' respectively. They both represent constant symbols.

- The most significant unary predicates in our formalism are *Learner* and *Trainer*. Also, *Learner*(m) and *Trainer*(h) represent two unary predicate assertions (world descriptions over unary predicates). They demonstrate that the constant symbol *m* is an instance of the unary predicate *Learner* and the constant symbol *h* is an instance of the unary predicate *Trainer*. In other words, *m* is a *Learner* and *h* is a *Trainer.*

- Considering the unary predicates *Learner* and *Trainer*, the binary predicates *TrainerOf* and *LearnerOf* are defined. Consequently, *TrainerOf*(*h*,*m*) and *LearnerOf*(*m*,*h*) are two binary predicate assertions (or relation assertions, or world descriptions over binary predicates). The first relation describes that the human being *h* is the trainer of the machine *m* and the second one describes that the machine *m* is the learner of the human being *h.*

- Two functions *trainer*(*m*) and *learner*(*h*) are defined in order to represent the 'trainer of machine' and the 'learner of human' respectively.

4.1. SEMANTIC ANALYSIS

According to the proposed axioms and to the non-logical symbols, we shall claim that the binary predicate *TrainerOf*(*h*,*m*) logically produces (implies) the equality *trainer*(*m*) = *h*. In fact,

$$\begin{gathered} \mathit{TrainerOf}(h,m) \qquad (i) \\ \Rightarrow \\ \mathit{trainer}(m) = h. \qquad (ii) \end{gathered}$$

The equation (*ii*) expresses the fact that the trainer of the machine *m* has been realised to be the person *h*. Note that this equality is produced with regard to our interpretation. In fact, it has been achieved based on the interpreted non-logical symbols. Therefore, the equation (*i*) as a binary predicate, describes the interpreted relation between *trainer*(*m*) and *h*. We may claim that this equality is the root of the formal semantics within a training-learning relationship. The binary predicate '=' (between *trainer*(*m*) and *h*) describes that the meanings of its right-hand side (or *h*) and its left-hand side (or *trainer*(*m*)) are the same. So, we shall emphasise that the achieved equality [as a binary predicate in FOL] aligns the meaning of *trainer*(*m*) and the meaning of *h.* Then we have:

$$\mathit{Equals}\,(\mathit{trainer}(m), h). \qquad (iii)$$

We shall maintain that the binary predicate (*iii*) has provided a supportive background for introducing the formal semantics. Considering this binary predicate, the function *trainer*(*m*) (as a non-logical symbol) and the individual *h* (as a constant symbol) have been supposed to have the same meanings. Additionally, regarding the commutative laws, 'the trainer of *m* is *h*' and '*h* is the trainer of *m*' are—logically—equivalent and, thus, they are—meaningfully—equal. Consequently, 'the trainer of *m* implies *h*' and '*h* implies the trainer of *m*'. Therefore:

$$[\ trainer(m) = h\ ] \ \Rightarrow\ [\ (\ trainer(m) \rightarrow h\ )\ \wedge\ (\ h \rightarrow trainer(m)\ )\ ]. \qquad (iv)$$

The logical term (*iv*) is inherently equal to:

$$(function \rightarrow constant) \wedge (constant \rightarrow function). \qquad (v)$$

We have already deduced that the term "a function symbol implies a constant symbol and a constant symbol implies a function symbol" supports the analysis of our formal semantics. Note that the term (*iv*) has been deduced based on the binary predicate *TrainerOf*(*h*,*m*) (or (*i*)). Then, there is a bi-conditional relation between (*i*) and (*iv*). Therefore:

$$TrainerOf(h,m) \ \leftrightarrow\ [\ (\ trainer(m) \rightarrow h\ )\ \wedge\ (\ h \rightarrow trainer(m)\ )\ ]. \qquad (vi)$$

Equivalently:

$$\mathbf{[}\ TrainerOf(h,m) \ \rightarrow\ [\ (\ trainer(m) \rightarrow h\ )\ \wedge\ (\ h \rightarrow trainer(m)\ )\ ]\ \mathbf{]} \qquad \&$$

$$\mathbf{[}\ [\ (\ trainer(m) \rightarrow h\ )\ \wedge\ (\ h \rightarrow trainer(m)\ )\ ]\ \rightarrow TrainerOf(h,m)\ \mathbf{]} \qquad (vii)$$

The logical term (*vii*) is structurally equal to:

$$\mathbf{[}\ Relation \ \rightarrow\ [\ (\ function \rightarrow constant\ )\ \wedge\ (\ constant \rightarrow function\ )\ ]\ \mathbf{]} \qquad \&$$

$$\mathbf{[}\ [\ (\ function \rightarrow constant\ )\ \wedge\ (\ constant \rightarrow function\ )\ ] \rightarrow\ Relation\ \mathbf{]}. \qquad (viii)$$

In Figure 1, this logical conclusion has been figured out.

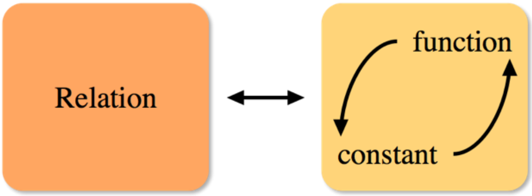

Figure 1. *The General Structure of Semantics within Relationships between Human Being & Machine*

Conceptually, taking the afore-mentioned conclusions into consideration, we need to focus on four fundamental relationships:

I. The training relationship between human and machine (from human into machine).

II. The learning relationship between machine and human (from machine into human).

III. The iterative loops between human and machine (from human into machine and from machine into human).

IV. The iterative loops between machine and human (from machine into human and from human into machine).

Therefore, the formal semantics of training-learning relationship in the context of human-machine interactions is definable over four constructive implications:

I. Implying the 'iterative loops between human and machine' from the 'training relation between human and machine'. Then:

$$TrainerOf(h,m) \rightarrow [\,(\,trainer(m) \rightarrow h\,) \wedge (\,h \rightarrow trainer(m)\,)\,]. \qquad (ix)$$

II. Implying the 'iterative loops between machine and human' from the 'learning relation between machine and human'. Then:

$$LearnerOf(m,h) \rightarrow [\,(\,learner(h) \rightarrow m\,) \wedge (\,m \rightarrow learner(h)\,)\,]. \qquad (x)$$

III. Implying the 'training relationship between human and machine' from the 'iterative loops between human and machine'. This item is the inverse of the item (I). Then:

$$[\,(\,trainer(m) \rightarrow h\,) \wedge (\,h \rightarrow trainer(m)\,)\,] \rightarrow TrainerOf(h,m). \qquad (xi)$$

IV. Implying the 'learning relationship between machine and human' from the 'iterative loop between machine and human'. This item is the inverse of the item (II). Then:

$$[\,(\,learner(h) \rightarrow m\,) \wedge (\,m \rightarrow learner(h)\,)\,] \rightarrow LearnerOf(m,h). \qquad (xii)$$

Therefore:

- Fundamental **I** expresses:
  [(*Training Relation*) → (*Training Function* ↔ *Learner Constant*)] **.**

- Fundamental **III** expresses:
  [(*Training Function* ↔ *Learner Constant*) → (*Training Relation*)] .

- Fundamental **II** expresses:

[(*Learning Relation*) → (*Learning Function* ⟷ *Trainer Constant*)] .

- Fundamental **IV** expresses:
  [(*Learning Function* ⟷ *Trainer Constant*) → (*Learning Relation*)].

According to the deduced results, we shall conclude that:

I. the training relations (from human into machine) support the interrelationship between 'the act of training' and 'the machine',

II. the learning relations (from machine into human) support the interrelationships between 'machine learning' and 'human',

III. the interrelationship between 'the act of training' and 'the machine' supports the training relation (from human into machine) and, finally,

IV. the interrelationship between 'machine learning' and 'human' supports the learning relation (from machine into human).

5. CONCLUSIONS

In this article, we have employed First-Order formalism in order to focus on relationships between human beings and machines. The context of this research has been the 'training-learning relation between human and machine'. The central focus has been on logical description and logical analysis of the training-learning relations within human-machine interactions. The analysed relationships between human beings and machines have supported analysing the HowNess of producing the formal semantics. We have concluded that four fundamentals conceptualise meanings and express the structure of the formal semantics within relationships. Subsequently, it's concluded that:

- the implications between 'relations' and 'the interrelationship of functions and constant symbols' support the formal semantics of the training-learning relationships.

The conclusion of this research has prepared a strong backbone for our future research. In future research, we will focus on semantic analysis of human concept learning with regard to the semantics of human's relationships with machines. The central focus will be on the formal semantics of concept transformations from humans' minds into machine's knowledge bases with regard to (Badie, 2016b). We will also work on semantic analysis of hypothesis generation. Human beings generate hypotheses in order to make them corresponded to distinct entities or to their essential attributes, characteristics and properties. Semantically, we will focus on the question of how hypotheses can determine the applications of the predicates.

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PAPER H. A CONCEPTUAL FRAMEWORK FOR KNOWLEDGE CREATION BASED ON CONSTRUCTED MEANINGS WITHIN MENTOR-LEARNER CONVERSATIONS

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ABSTRACT

Constructivism is a learning philosophy and an educational theory of learning. In the framework of constructivism, a human being with insights based on her/his pre-structured knowledge and on background knowings could actively participate in an interaction with another human being. The central focus of this article is on construction of conceptual knowledge and its development. This research localises the constructivist learning in the context of mentor-learner interactions. It will analyse meaning construction relying on my own conceptual framework that represents a semantic loop. The learner and the mentor as intentional participants move through this semantic loop and organise their personal constructed conceptions in order to construct meanings and produce their meaningful comprehensions. This research is concerned with definitions, linguistic expressions and meanings within the developmental processes of personal world constructions. It proposes a scheme for interpretation based on semantics and in conversations. I shall conclude that the outcomes of this research could be able to support smart education and smart learning environments.

1. INTRODUCTION

The dialogues and the conversational exchanges between mentors and their learners ask questions and give answers concerning their individual conceptions and realisations. I shall stress that what I will use and express under the label of 'concept' aims at providing comprehensible characteristics of human being's conceptions and conceptualisations. In the section 'related works and proposal' I will focus on the works related to realisations of concepts. Constructivism is a learning philosophy and an educational theory of learning that can be recognised as a model (and theory) of knowing with roots in philosophy as well as in psychology and cybernetics, see (Husén and Postlethwaite, 1989). A constructivist conversation could be seen as a radical constructivist account of the learner's and the mentor's realisations and understandings, see (Scott, 2001). This account is capable of enabling the mentor and the learner to develop their own understandings of the specified concepts with regard to their understandings of the more general concepts. Producing the personal understanding of a world (a universe of discourse) and developing it during the conversation could be said to be the most valuable product of a constructivist conversation. When a mentor and her/his learner start a conversation, then, based on their personal knowings and on personal pre-structured knowledge, they attain deeper realisations of the world. Conversation supports them to develop their understandings of the world (and of each other and of themselves). Jean Piaget, the originator of constructivism, believed that human being's mental objects (or schemata) gradually develop into more abstract (= general) and conceptual mental entities, see (McGawand Peterson, 2007; Moallem, 2001; Spiro et al., 1992). In fact, according to constructivist learning, the human being's mentality manifests itself in the form of schemata (Bartlett, 1932). I will explain some details in the next section. Considering learning in the framework of constructivism with reference to Conversation Theory (CT), which is conceived and elaborated by Gordon Pask, the enterprise begins with the negotiation of an agreement between the learner and the mentor to converse about a given domain and to learn/train about some particular topics and skills in that domain. It could work as an explanatory, heuristic and developmental framework, see (Pask, 1975; Pask, 1980). The CT is fundamentally an explanatory ontology combined with an epistemology, which has wide implications for psychology and educational technologies. Pask's main premise is that the reliable knowledge exists, is produced, and evolves in action-grounded conversations, see (McIntyre Boyd, 2004).

In this research, I will work on a conceptual framework that represents the process of meaning construction in constructivist conversational exchanges between mentors and learners. My main reference is my own semantics-based framework (see (Badie, 2015a; Badie, 2015b)) and this research attempts to develop my framework for analysing 'conceptual knowledge creation based on constructed meanings within conversational exchanges between mentor and learner'. According to (Badie, 2015a; Badie, 2015b), the learner and the mentor as intentional participants move through a semantic loop and organise their personal constructed conceptions in order to construct meanings, to improve the constructed meanings, and to produce their individual meaningful comprehensions. What could be offered by conceptual knowledge construction in the developed framework is 'a body of thought' and 'a semantic model to account for the emergence of the domain of the learner's and of the mentor's conceptual knowledge'. It can express how the produced meanings based on human being's constructed concepts could support her/him in constructing the personal worlds and in creating her/his conceptual knowledge.

2. RELATED WORKS AND PROPOSAL

The most momentous building block of this research is 'concept', thus I shall start with the realisation of concepts. The reference (Parker, 2008) identifies concepts as the furnitures of human beings' minds. According to this realisation, a well-furnished mind can be a source of successful knowledge acquisition (and learning). According to (Götzsche, 2013), a concept is a linkage between the human being's linguistic expressions and the mental images (of all kinds of perceptions) that she/he may have in her/his mind. Regarding (Götzsche, 2013) and taking (Bartlett, 1932) into consideration, the human being may represent a concept as a 'thing' under a specified label. Consequently, the labels are the constituents of propositions that mediate between thought, language, and referents. My conceptual approach has relied on this realisation and grasp of concepts with regard to (Bartlett, 1932; Götzsche, 2013). I assume that a concept can also be seen as an 'idea' that corresponds to a distinct entity or to its essential features, attributes, characteristics and properties. As mentioned, concepts are the conjunctions between the human being's linguistic expressions and her/his mental images. Focusing on conversational exchanges between mentors and learners, the mental images may be seen as the representation(s) of the aspects of the world (a universe of discourse). Taking this realisation into consideration, any conception of a leaner/mentor within a conversation can be identified as her/his act of visualising (in a broad sense) different concepts by linking her/his expressions (and specifications) to her/his own mental images [that have been visualised over various schemata). Let me be more specific. First, I focus on clarifying schema and schema-based knowledge constructions and, consequently, I will draw your attention to the interrelationships between 'specified characteristics of concepts in my approach' and 'the Kantian philosophy and Kantian account of schemata'.

In my opinion, knowledge can actively be constructed over human being's constructed concepts and conceptions. I believe that knowledge is built up (created) based upon the learner's/mentor's comprehensions of concept meanings with regard to their definitions. I shall stress that the definitions are highly influenced by the learner's/mentor's world descriptions based on their linguistic expressions. Also, the linguistic expressions have been assigned to their mental images of phenomena within the world (the universe of discourse). Reconsidering the introductory section, when a mentor and her/his learner start a conversation, then, based on their personal knowings and on personal pre-structured knowledge, they attain deeper realisations of the world. Conversation supports them to develop their understandings of the world [and of each other and of themselves]. As mentioned, in constructivist learning a human being's mentality manifests itself in the form of schemata. It's important to say that the schemata demonstrate the human being's realisation of the world. They conceptually represent the constituents of the learner's/mentor's thoughts for knowledge acquisition [and learning/mentoring] with regard to their realisations of the world.

Now, let me focus on the interrelationships between specified characteristics of concepts in my approach and the Kantian account of schemata. Regarding (Kant, 1999; Kant, 1790), Kant defines a non-empirical (or pure) concept as a category. According to Kantian philosophy, schemata are the procedural rules by which a category is associated with a sense impression. Kant claimed that the schema provides a reference to intuition in a way similar to the manner of empirical concepts. According to the Kantian account of schemata, empirical concepts (see (Link a)) are the most fundamental types of concepts that employ schemata. For instance, the concept 'Liquid' can be explained by a rule according to which a human being's imagination can visualise a general figure of a description like "A state and a distinct form that matter takes on" without being restricted and closed to any particular and specific shape produced by human being's experience. The empirical concepts provide the origin of what I have brought under the label of concepts. In fact, a human being proposes various linguistic expressions in order to describe her/his mental images of a phenomenon. This description is directly dependent on her/his own schemata, which have been designed and shaped over her/his experiences. Moreover, regarding the Kantian account of schemata, the human being is concerned with 'pure concepts of the understanding' see (Link b). According to a human being's realisation and grasp of the pure concepts of understanding, she/he focuses on characteristics, features, attributes, qualities and properties of an object, that are also other objects in general or as such. They all support her/him in producing better understanding of the things/phenomena in the world. At this point, I shall emphasise that my conceptual analysis may look a bit like Wittgenstein I (Wittgenstein, 1922), but I will make no use of Wittgenstein's approach.

3. CONVERSATIONAL LEARNING THEORY

According to (Laurillard, 1993; Laurillard, 2002) and (Link c), Laurillard's framework can be interpreted as a learning theory and as a practical framework for designing educational and pedagogical environments. This framework includes four important components:

i. Mentor's concepts (and conceptions)
ii. Mentor's constructed learning environment [that produces her/his constructed world]
iii. Learner's concepts [and conceptions]
iv. Learner's specific actions [that construct her/his world].

Regarding Laurillard's framework, we need to consider different forms of communication and associated mental activities. The main constructors of the associated mental activities are:

1. Discussion,
2. Adaptation,
3. Interaction, and
4. Reflection.

In fact, relying on Pask's Conversation Theory and focusing on Laurillard's framework, there are four kinds of human being's activities that take place in different kinds of flow between the components of Laurillard's framework:

- Discussion between the mentor and the learner on their conceptions, descriptions, realisations and reasonings.
- Adaptation of the learner's actions and of the mentor's constructed environment. There is a type of adaptation of the learner's world and of the mentor's world.
- Interactions between the learner and the environment which is defined by the mentor.
- Reflection of the learner's performance by both the mentor and the learner.

I have illustrated the above-mentioned characteristics in the framework represented in Figure 1. For instance, relying on the conversational learning framework and reconsidering the concept 'Liquid', the existed knowledge about the individual 'liquid' is strongly dependent on the interrelationships between learner and mentor,

i.e., 'learner → mentor → learner'.

In fact, the learner incorporates the internalised mentor and the mentor incorporates the internalised learner. Thus, the interactions and the discussions about 'liquid' (initially, over the empirical concept of Liquid) starts. Accordingly, regarding the pure concepts of understanding, learner and mentor focus on multiple characteristics, predicates, attributes, qualities and properties of the individual 'liquid'. Consequently, the learner and the mentor converse and exchange their conceptions in order to construct and develop their conceptual knowledge.

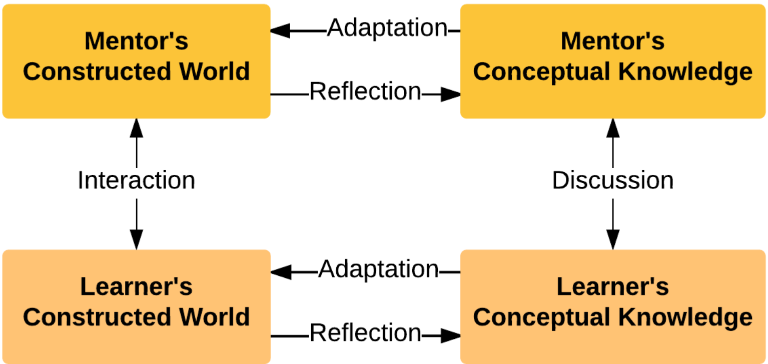

Figure 1. *Conversational Learning Framework*

4. MEANING CONSTRUCTION

I have assumed that the definitions and meanings based on a human being's conceptions strongly support her/him in knowledge construction processes in the framework of constructivist learning/mentoring and in the context of mentor-learner conversations. I also assume that an explanation is the actual act of explicating definitions and meanings. I shall emphasise that the main objective of the learner's (and the mentor's) explanations are to shed light on the produced personal realisations and understandings. In this section I focus on definitions and meanings, which are the main building blocks of my framework.

4.1. FROM DEFINITIONS TO MEANINGS

In a logical and semantics-based system a definition can be figured out as a kind of equivalency (and, semantically, as a kind of equality,) whose left-hand side is a concept (the concept that is going to be defined) and whose right-hand side is a description in the form of a number of expressions. For instance, a person defines Liquid by

Liquid ≐
"A state and a distinct form that matter takes on. Also, the matter in this state maintains a fixed volume, but has a variable shape that adapts to fit its container".

I shall stress that this definition has proposed a concept definition (a type of equivalence between the right-hand side and the left-hand side). It also presents a semantical equivalence between the right-hand side and the left-hand side. This semantical equivalence represents two concurrent implications;

i. one from the left-hand side into the right-hand side, and
ii. one from the right-hand side into the left-hand side.

In a constructivist conversation, any of the agents may define a concept based upon her/his individual conception. Subsequently, regarding the feedbacks of the conversant, she/he modifies and updates her/his definition. I have called it 'definition updating'. Let me point to the fact that that the learner and the mentor interact with each other in order to adapt and to conform their 'personally constructed worlds'. They also discuss in order to exchange their personal conceptual knowledge (their constructed knowledge over concepts and conceptions) and to develop it. Consequently, any proposed definition provides a supportive backbone for performing more developed and more well-organised concept descriptions. Additionally, a more developed concept description supports both the agent and the interlocutor in providing the more understandable explanations of meanings. At this point, considering the specified realisation of definitions, I focus on the expression 'meaning'. I have assumed the following descriptions to be comprehensible in the context and in my framework. Linguistically, meaning is, according to one approach, realised as a context-update function, see (Larsson, 2012). So, meaning is regarded as a function from a context into a context. Any context comprises different types of (general and specified) concepts. Therefore, I describe a meaning as a function from a concept into its updated form. In fact, in my conceptual and logical approach a meaning is seen as a 'concept-update function'.

4.2. A FRAMEWORK FOR MEANING CONSTRUCTION

In (Badie, 2015a; Badie, 2015b), I have analysed a conceptual and semantic loop that the learner and the mentor move through in order to organise their personal constructed concepts, in order to:

- describe their definitions,
- construct and formulate the meanings, and
- produce their individual meaningful comprehensions.

See the yellow sections of Figure 2. The proposed semantic loop is self-organising and can promote itself on higher conceptual levels and on higher levels of interaction and conversation. It has proposed a scheme for interpretation based on semantics and interactions. The most primary building block of learning in the framework of constructivist conversational learning can be built up over an asked question. Regarding empirical concepts and the human being's experimental conceptions, the one who undertakes to learn/train something and to produce her/his (deeper and deeper) understanding within a constructivist conversation, primarily becomes concerned with various general concepts related to that thing. Later on, she/he focuses on main characteristics, attributes, qualities, and properties of that

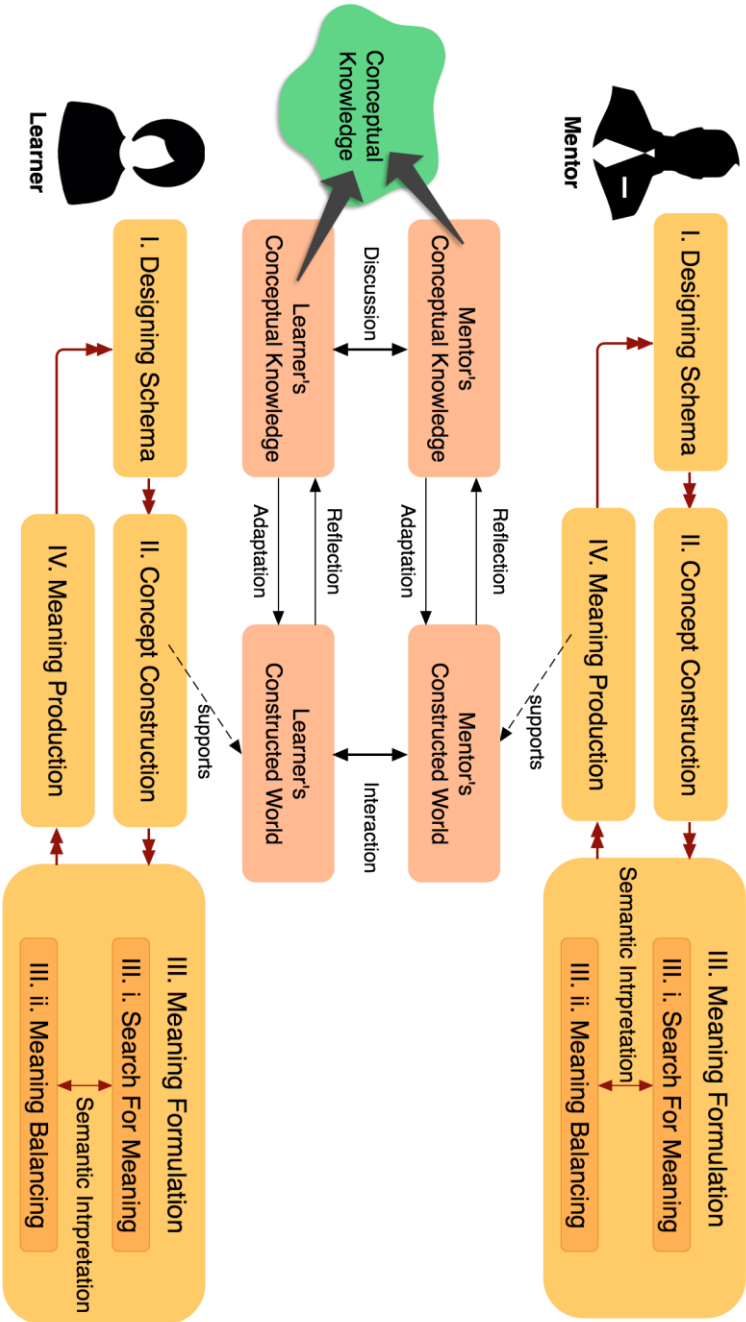

Figure 2. *Constructivist Conversation: Meaning Construction and Knowledge Creation*

thing in order to be concerned with realisation of that thing and to recognise its state and its condition within the world. More specifically, she/he focuses on forming (see Link d) [and reforming] concepts.

A mentor who has defined the concept 'Liquid' as "a state and a distinct form that matter takes on. Also, the matter in this state maintains a fixed volume, but has a variable shape that adapts to fit its container" is directly and indirectly concerned with a number of general concepts (e.g., State, Form, Matter, Volume, Shape, Container). Therefore, the learner needs to conceptualise these notions and to discuss her/his conceptions with the mentor. The preliminary understanding supports the learner in building various patterns in her/his mind and any of these patterns describes her/his thought over a conception. These patterns could be seen as the mental structures (consisting of mental objects or schemata). Accordingly, the learner relies on her/his mental structures to organise her/his conceptual knowledge and to provide strong backbones for her/his interpretations and explanations. In fact, the collection of the rules and processes that manage various linguistic expressions (and, respectively, definitions) based upon logical foundations do not (and cannot) have any meaning until the non-logical words[1] of the language are given interpretations. And the human being's mental structures support her/his personal interpretations. The collection of

1. concept formation,
2. concept transformation (from the mentor's mind into the learner's mind and vice versa), and
3. concept reformation

is the most significant matter in the development of the concept constructions within constructivist conversations. I have identified the process '1 → 2 → 3' as the main basis of 'Concept Construction' in my framework. Taking the constructed concepts into consideration, the reflection of the prior knowledge (what has been acquired or created before participating in the conversation) and the new knowledge (what is being acquired or created during the conversation) and the initial definitions must all be negotiated. So, the sequence

'Prior Knowledge → Definition → New Knowledge'

supports the learner in searching for the appropriate initial meanings of the class/classes of her/his constructed concepts. This phase is highly affected by 'interpretations'. In fact, the initial meanings are being interpreted in order to be balanced. The interpretation of a defined constructed concept is a function from a 'definition' into a 'meaning'. Logically, we may have iterative loops between 'definitions' and 'meanings' (consisting of functions from definitions into meanings

[1] The words like, e.g., and, or, not, since, then, so, all, every, any, have logical consequences and are identified as the logical words in a natural language with regard to classical symbolic logic and predicate logic.

and inverse functions from meanings into definitions). The agent could be able to balance and adjust the initial meanings based on the interrelationships between 'interpretation' and 'the inverse of the interpretation' (by comparing the subject of interpretation before and after being interpreted). The conclusions make an appropriate background for verifying the personally found meanings. Therefore, a meaning would be given a more appropriate shape after checking the balanced definitions based on the personal constructed concepts. The conclusions will support the agent in formulating the balanced meanings. Any formulated meaning is a basis for providing a supportive conceptual structure of meaning production. These conceptual structures are all personally formulated over personal constructed concepts and definitions. On the other hand, they induce new formulated meanings on higher conceptual levels and on higher levels of conversation. Finally, the supportive conceptual structures support the agent in producing meanings. Note that the produced meanings reinforce the meaningful comprehensions. I shall emphasise that the produced meanings support the construction of the individual mental worlds. Any produced meaning has been designed, shaped, balanced, formulated and produced based upon the learner's/mentor's formatted concepts, constructed concepts, expressed definitions and interpreted definitions.

Any produced meaning reflects in the constructor's self and supports her/him in re-shaping and developing her/his schemata on the next levels of her/his conversation. Thus, the learner's and the mentor's produced meanings are employed in the developmental processes of personal world construction. As you may have realised, at this point, I have entered Laurillard's conversational learning framework. The dashed arrows in Figure 2 show that my framework is getting connected with the simplified Laurillard's framework. As mentioned, regarding Piaget's developmental theory of learning, constructivist learning is concerned with how the individual human being goes about the construction of knowledge in her/his own cognitive apparatus. Therefore, the most important output of my 'meaning construction' framework is an input for the developmental processes of personal world construction.

5. THE OUTCOMES WITHIN SMART LEARNING SYSTEMS

I shall draw your attention to the relation between the main objectives of this research and the main concepts of smart education and smart learning. Considering the following items, I might claim that this research has proposed a conceptual analysis of a constructivist paradigm within smart learning systems.

a) The smart learning approaches (Link e) motivate higher levels of learners' understandings. Similarly, this research has focused on conceptual analysis of knowledge creation [based on produced understandings] in the framework of constructivism and in the context of mentor-learner conversations. Let me say that this research focuses on higher [and deeper] levels of learners' understandings with regard to their produced meanings and their generated meaningful comprehensions.

b) Through the various processes within the framework of smart learning, the learner initially activates background knowledge, identifies goals for her/his personal learning, focuses on information processing and focuses on self regulations. In a corresponding manner, this research is structured over the learner's [and the mentor's] background knowledge. It sees learning as the process of construction over personal background knowledge. It focuses on a human being's realisation of her/his objectives and specifies a self-organising process on the part of the learner.

c) Regarding the framework of smart learning, the learner transforms learning into demonstrations of understanding and, accordingly, reflects on her/his own learning. In a very similar way, the most significant matter in this research has been 'transforming learning phenomena into knowledge construction that has been achieved over constructed concepts and conceptions'. Also, this research focuses on transforming learning into the learner's/mentor's comprehensions of concepts' meanings with regard to their definitions. These two important matters could support learners in reflecting on their learnings [and on themselves].

6. CONCLUSIONS AND FUTURE WORK

This research has focused on constructivist conversational learning. A constructivist conversation has been seen as a radical constructivist account of the learner's and the mentor's comprehensions and understandings. Regarding Piaget's developmental theory of learning, constructivist learning is concerned with how the individual human being goes about the construction of knowledge in her/his own cognitive apparatus. I have proposed a conceptual framework that represents the process of meaning construction in constructivist conversational exchanges between mentors and learners. The proposed framework represents the processes of conceptual knowledge creation based on constructed meanings within conversational exchanges between mentor and learner. I have assumed that the definitions and meanings based on human being's conceptions strongly support her/him in the process of conceptual knowledge creation. I have also assumed the explanation as the actual explaining of definitions (and meanings). The backbone of this research is my proposed semantic loop. The learner and the mentor move through this semantic loop in order to organise their personal constructed concepts. Therefore, they will be supported in constructing meanings, to improve the constructed meanings, and to produce their individual meaningful comprehensions. This article's proposed framework is 'a body of thought' and 'a semantic model to account for the emergence of the domain of the learner's (mentor's) conceptual knowledge'. It expresses how the produced meanings based on the constructed concepts could support personal world constructions. The personal worlds are employed by Laurillard's conversational learning framework that is a learning theory and a practical framework for designing educational and pedagogical environments. It values discussions, adaptations, interactions and reflections. In future research, I will focus on logical and semantic analysis of conceptual knowledge creation based on constructed meanings within mentor-learner conversations relying on First-Order Predicate Logic and Description Logics.

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PAPER I. TOWARDS CONCEPT UNDERSTANDING RELYING ON CONCEPTUALISATION IN CONSTRUCTIVIST LEARNING

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ABSTRACT

This research works within the framework of constructivist learning (based on constructivist epistemology) and examines learning as an activity of construction. It also posits that knowledge acquisition (and learning) are transformative through self-involvement in some subject matter. I will mainly focus on conceptual and epistemological analysis of humans' conceptualisations based on their own mental entities (schemata). Subsequently, I will propose an analytical specification of humans' conceptualisations and understandings over their mental structures in the framework of constructivism and, accordingly, I will clarify my logical [and semantic] conceptions of humans' concept understandings. This research focuses on philosophy of education and on logics of human learning. It connects with the topics 'cognition in education' and 'mental models'.

1. INTRODUCTION

Perceived by a very general definition, the act of learning is identified as related to acquiring or modifying knowledge. Learning can be seen as the involvement of the self in increasing knowledge about a thing/phenomenon. We can interpret learning as a process that causes changes in a human's mind. The learner is someone who attempts to learn something and to acquire knowledge on that thing and the mentor is someone who opens the world to the learner and opens the learner to the world. All the characteristics mentioned are conducive to interpreting learning as the 'activity of construction'. In this article, I will focus on the constructional dimension of learning. At this point, I feel the need to focus on describing and specifying 'knowledge'. Regarding (Furst et al., 1956; Krathwohl, 2002), knowledge has a strong relationship with recognition [and understanding] of materials, ideas, methods, processes, structures and settings. Accordingly, a body of knowledge may cover (satisfy) multiple branches like, e.g., terminologies, ways and means, trends and sequences, classifications, methodologies, universals and abstractions, quantifications and qualifications, conditionings, principles and generalisations, and theories and structures. I may conclude that knowledge acquisition [and, respectively, learning] processes consist of a sort of 'Transformation functions' from reality into the sets and categories of various disciplines and systematic enterprises. I have focused on this subject in (Badie, 2016a; Badie, 2016b). According to these references, a human being has the ability to deal with different disciplines and systematic enterprises, and can transform them in her/his mind. I shall interpret these transformations as the outcomes of the self-involvement in increasing knowledge about a subject matter. In human systems, a learner is an intentional participant (agent) and attempts to know more about something in order to construct her/his knowledge about that thing. Any human has a background knowledge and tackles to carry on constructing knowledge over her/his existing knowledge. She/He attempts to develop her/his knowledge constructions and to get the opportunity to attain deeper comprehensions and understandings.

Constructivism is a philosophy that forms the backbone of this research. It is a learning philosophy and a pedagogical theory of learning that can also be realised as a model and a theory of knowing with separate roots in philosophy, psychology and cybernetics. According to the existential element of constructivism, the construction of knowledge is an active process, but the activity itself can be described in terms of individual cognition [and personal understanding] in different processes, see (Phillips, 1995). As for Piaget's developmental theory of learning[1], constructivist learning is concerned with how the individual learner goes about the construction of knowledge in her/his own cognitive apparatus. This article will, conceptually and epistemologically, analyse human understanding and conceptualisation based on the proper foundation that has been provided by the constructivist model of knowing (and constructivist epistemology). I will focus on the analysis of humans' schemata-based concept representations, where schemata form humans' mental structures. In my opinion, the central focus of constructivist knowledge acquisition and learning is on schemata-based conceptual representations and conceptualisations. Accordingly, this article will propose an analytical description of humans' schema-based understandings [of concepts] in the ground of their conceptualisations and in the framework of constructivism. Before offering specifications, I shall describe what I mean by the act and the process of constructivist knowledge acquisition and learning with regard to concepts. The following definition draws out the key elements of constructivist knowledge acquisition and learning, which have individual and social implications for humans, see (Watkins et al. 2002).

> "Knowledge acquisition is the reflective activity which enables the humans to draw upon their previous experiences [and background knowledge] to conceptualise [and, respectively, to realise and to understand in order to] evaluate the present, so as to build up and shape future actions and to construct [and, subsequently, to develop the construction of] new knowledge".

At this point, I shall emphasise that there is, obviously, no reason to claim that concept construction, as such, must be based on the processes described by constructivism. The aim of this study is that I will use constructivism to describe human concept construction as a kind of 'conditional reasoning' in a learning context and accordingly, I am trying to analyse concept construction in that context relying on constructivist epistemology. Therefore, in my opinion, constructivism could provide a proper base of description of the concept construction process, if it is seen as an individual's conditional reasoning in a learning context.

2. CONCEPTUALISATION AND CONCEPT UNDERSTANDING

According to constructivist knowing [and learning], human beings' mental structures manifest themselves in the form of mental objects (schemata[2]), see (Bartlett, 1932; Parker, 2008). Schemata conceptually represent the constituents of human's thoughts

[1] Jean Piaget (1896 - 1980) was the originator of constructivism. He was the first psychologist to make a systematic study of cognitive development and developmental theory of learning, see www.piaget.org/aboutPiaget.html.

[2] Piaget argued that all learning was mediated by the construction of mental objects that he called schemata. Schemata gradually develop into more conceptual mental entities.

for knowledge acquisition with regard to her/his perception of [parts of] the world. Regarding (Piaget and Cook, 1952), a schema is a "cohesive, repeatable action sequence possessing component actions that are tightly interconnected and governed by a core meaning". In constructivist learning and constructivist knowledge acquisition, schemata, in a broad sense, support humans in constructing concepts, developing (forming and reforming) their constructed concepts, in providing their semantic interpretations and in processing their meaning construction, for more details about concept formation see (Link a). I shall, therefore, conclude that a human's elucidation, explication and explaining abilities all become supported by her/his schemata. Subsequently, relying on semantic interpretations, humans become concerned with meanings of their mental entities associated with different objects/phenomena. In my opinion, schemata determine the locus of meanings and, thus, support world descriptions and reinforce the structural and descriptive analysis of mental entities. Moreover, the semantics as the scrutiny and the analysis of meanings could focus on various conditions for definitions of truth. Then, it connects with humans' inferences and reasonings which attempt to give satisfying conditions for definitions of truth. Accordingly, it seems possible to conclude that humans' inferences are given shapes over their designed schemata. In my conceptual approach, a concept is a linkage between the mental representations of linguistic expressions and the other mental images (e.g., representations of the world, representations of inner experiences) that a human has in her/his mind, see (Götzsche, 2013). Considering this idea of concepts, humans may be said to transform the collection of (i) linguistic expressions, (ii) images of the world, and (iii) their interrelationships in the form of [psychological] entities and utilising generic and specific labels. I could say that concepts might be understood to be representations of actualities and objectivities in humans' minds, and those representations can affect humans' reasoning processes.

2.1. UNDERSTANDING, EXPLANATION AND CONCEPTUALISATION IN THE FRAMEWORK OF CONSTRUCTIVISM

From the perspective of constructivist learning, human beings focus on knowledge construction and on developing the constructed knowledge over their background knowledge. The most significant objective of constructivism is producing one's own understanding of the world, see (Husén and Postlethwaite, 1989; McGawand Peterson, 2007; Keith Sawyer, 2014). Let me take into consideration the SOLO[3] taxonomy in order to focus on understanding in the framework of constructivism. According to SOLO, the sequence 'pre-structured knowledge → uni-structured knowledge → multi-structured knowledge → related knowledge → extended abstracts' represents a flow from shallow understanding to deep understanding, see (Burville Biggs, 1982). A shallow understanding of a thing/phenomenon may support humans in identifying some isolated facts and matters related to that thing/phenomenon. On the other hand, a deep understanding of a thing/phenomenon supports humans in linking lots of related facts (as conceptions) about that

[3] Structure of Observed Learning Outcomes (SOLO) taxonomy is a proper model that can provide a structured framework for who acquires knowledge in order to promote the efficiency of her/his knowledge acquisition.

thing/phenomenon and in linking those conceptions to other complicated conceptions. Additionally, deep understanding of a thing/phenomenon supports analysing, justifying, criticising, hypothesising and theorising about that thing/phenomenon.

Before getting into details I shall emphasise that 'understanding' is a very complicated term in philosophy, psychology and cognitive science. In my opinion, there cannot be any absolute and comprehensive description for understanding, but there can be acceptable descriptions of 'realisations of understanding'. Actually, there could be a very strong relationship between 'understanding' and 'explanation'. Explanation (i.e. the actual process of explaining something) can shed light on the produced personal understandings of that thing. I have assumed that humans' linguistic expressions and the produced meanings [based on humans' conceptions], strongly support knowledge construction processes and understanding in the framework of constructivist learning. Thus, an explanation could also be assumed to be the actual explanation of expressions and meanings. Therefore, humans rely on their own explanations in order to shed light on their produced personal comprehensions and understandings.

At this point, I focus on the term 'conceptualisation' in order to provide a supportive specification of understanding (see Appendix II: Concept Understanding and Conceptualisation). In (Badie, 2016b), I have described an understanding (of a concept) as "a local manifestation of a global (and universal) conceptualisation". I concluded that a specific concept understanding provides a local manifestation of a universal conceptualisation. Furthermore, a human's grasp of concepts provides a proper foundation for generating her/his own conceptualisations. So, the personal conceptualisation could be elaborated by the outputs of the processes of concept formation and reformation with regard to the basis that is provided by the individual realisation. When a human forms her/his conception (as an outcome of her/his constructed concept) from its attributes, qualities, properties and its relationships with other conceptions, she/he gets to know and to understand more than just some isolated facts about that conception (and of that concept). This qualifies deep knowledge acquisition rather than superficial knowledge acquisition over concepts. Note that (Parker, 2008) has also (from another point of view) focused on this subject in analysing inductive teaching strategies.

A person who understands something, directly or indirectly, gets concerned with the taxonomy of various concepts. I have focused on the last statement from the structuralist point of view. The structuralist description and analysis of understanding supports me in explaining a variety of facts about 'understanding' and 'understanding something'. The individual who understands something, needs to move through a chain of various related concepts. Then, we could see 'Concept' and 'Generality' as two significant aspects that support the structuralist account of understanding. According to (Kuczok, 2014), the notion of conceptualisation pertains to central terms in cognitive linguistics. According to (Langacker, 1991), it can be defined as the locus of meaning or even equated with meaning in lexical semantics, which should describe abstract entities like thoughts and concepts through structural analysis. So, considering the analysis of schemata and the humans' mental structures, I conclude that conceptualisations are highly dependent on schemata.

3. SEMANTICS OF CONCEPT UNDERSTANDING

In this section, I clarify my logical conceptions of ‘understanding’ and focus on logical description of understanding concepts through the lenses of semantics. The conclusions will be used for proposing a semantic expression of humans’ concept understanding. Suppose that a person undertakes to acquire knowledge about a concept and to understand it. Concept understanding, as a relation, could relate ‘the characteristics and attributes of a concept’ with ‘a description’. More specifically, understanding is a function (mapping) from a concept into some propositions (and statements) which could be interpreted as ‘concept descriptions’. In fact, the characteristics and properties of a concept by means of the understanding function could become mapped into concept descriptions (see Appendix II: Concept Understanding as a Relation).

I interpret an understanding as the limit (or as a type) of a conceptualisation, see (Link b). Considering understanding as a limited conceptualisation, it could be explained as a kind of process of forming [and reforming] concepts. So, an understanding focuses on concepts on the basis that is provided by a conceptualisation. Then, one who undertakes to understand something, needs to have that thing conceptualised.

As mentioned, an understanding (of a concept) is a local conceptualisation. Therefore, all understandings (of concept *C*) are conceptualisations (of concept *C*). Therefore, understanding *C* has been interpreted as the subset of conceptualising *C*. But not all conceptualisations are understandings. In fact, all conceptualised concepts may not be understood, but all understood concepts have been conceptualised. Considering the person *P* and the concept *C*,

“*P* understands *C*”, then: “*P* conceptualises *C*”.

Now I shall draw your attention to the concept formation process. Relying on concept formation processes, a person gets concerned with manipulating, formatting, classifying and structuring concepts. Accordingly, these processes all provide supportive foundations for her/his concept understanding process. Concept formation and concept reformation are the salient products of conceptualisation in constructivist learning. From the methodological point of view, the person needs to focus on the attributes, characteristics, qualities and properties of something in order to consider it as an instance of the concept *C*. One person may have focused on formation of *C* before acquiring knowledge about it, and thus, she/he reconsiders her/his initial formations after reconstructing her/his knowledge in order to reform *C* in her/his mind. On the other hand, another person may not have focused on *C* before the knowledge construction is processed, and then she/he can form *C* with insights based on acquired knowledge. I have identified the sequence ‘Concept Formation → Concept Transformation → Concept Reformation’ as the main foundation of the concept construction within meaning constructions, see (Badie, 2015a; Badie, 2015b).

Note that the formed concepts could be affected by acquired knowledge within constructivist discussions, dialogues and interactions in order to be transformed and to support concept reformations. Therefore, understanding the constructed (or reconstructed) concepts could be realised to be the limit of conceptualisation. Then, the person gets concerned with the attributes, characteristics, qualities and properties of concepts in order to distinguish them when they belong to different categories. In fact, she/he identifies, specifies and relates the generalised concepts. Subsequently, as a salient product of understanding, she/he could be able to make her/his personal labels and identifiers for identifying the [understood] concepts. These labels could be employed in categorising different things.

Moreover, by engaging the personal interpretation the person explicates what she/he means by the concept *C*. The interpretations make bridges between a person's 'expressions and explanations' and 'semantics and meanings'. Taking semantics into my analysis, it's possible to infer that someone who has focused on the concept *C*, needs to provide a manner of determining the truth values of her/his statements, expressions, theories and explanations concerning *C*. Consequently, I identify all understandings [of concept *C*] as the interpretations [of concept *C*]. Therefore, understanding *C* has been interpreted (and is expressed) as the subset of interpreting *C*. But, all interpretations are not understandings. In fact, all interpreted concepts could not be understood, but all understood concepts certainly have been interpreted. Considering the person *P* and the concept *C*,

"*P* understands *C*", then: "*P* interprets *C*".

More specifically, the collection of the rules and the processes that manage different terms and descriptions in linguistic expressions, do not (and cannot) have any meaning until the non-logical parts[2] and constructors of the language are given interpretations and are interpreted. The interpretations prepare the person for producing her/his personal meaningful [and understandable] concept descriptions. By learning and acquiring knowledge in the framework of constructivism, a human being attempts to provide a way to determine the truth values of the non-logical parts through her/his conceptions. Consequently:

I. The understanding (as a conceptualisation) focuses on the domain of conceptualisation. Actually, it conceptualises the objective of conceptualisation and, respectively, focuses on the objective of understanding. Therefore, the understanding focuses on the domain of understanding.

II. The understanding (as an interpretation) focuses on the domain of interpretation. In fact, it focuses on the objective of interpretation and, respectively, on the objective of understanding. Therefore, the understanding

[2] The words like, e.g., and, or, not, since, then, so, all, every, any, have logical consequences and are identified as the logical parts (words) in a natural language with regard to classical symbolic logic and predicate logic.

focuses on the domain of understanding.

III. The understanding (as a conceptualisation) conceptualises the concepts belonging to the domain of conceptualisation. Therefore, the understanding understands the concepts (works on the concepts).

IV. The understanding as an interpretation interprets the concepts belonging to the domain of interpretation. Therefore, the understanding understands the concepts (works on the concepts).

4. CONCLUSIONS

Constructivist learning is concerned with how the individual learner goes about the construction of knowledge in her/his own cognitive apparatus. Conceptually and epistemologically I have focused on analysis of human understanding and conceptualisation. I have been concerned with human concept representations over her/his schemata. Schemata support humans in constructing concepts, and developing their constructed concepts, in providing their semantic interpretations and in processing their meaning construction. According to this research, a concept understanding is a local manifestation of global/universal conceptualisations. Consequently, I have focused on a more specific logical description of conceptualisations and concept understandings based upon individual constructed concepts. The conclusions have been applied in a logical and semantic description and representation of 'concept understanding'. This research has formed a building block of my PhD research, which is dealing with semantic analysis of constructivist concept leaning. In future research, I will, logically and semantically (mainly based on Description Logics), focus on formalising and analysing humans' concept understanding and on proposing a semantic model for concept understanding in the framework of constructivism.

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Links

Link a: http://teachinghistory.org/teaching-materials/teaching-guides/25184

Link b: www.cocon.com/observetory/carlbereiter

PAPER J. A CONCEPTUAL FRAMEWORK OVER CONTEXTUAL ANALYSIS OF CONCEPT LEARNING WITHIN HUMAN-MACHINE INTERPLAYS

Farshad Badie

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ABSTRACT

This research provides a contextual description concerning existential and structural analysis of ‘Relations’ between human beings and machines. Subsequently, it will focus on conceptual and epistemological analysis of (i) my own semantics-based framework [for human meaning construction] and of (ii) a well-structured machine concept learning framework. Accordingly, I will, semantically and epistemologically, focus on linking those two frameworks for logical analysis of concept learning in the context of human-machine interrelationships. It will be demonstrated that the proposed framework provides a supportive structure over the described contextualisation of ‘relations’ between human beings and machines within concept learning processes.

1. INTRODUCTION AND MOTIVATION

Machine Learning as a subfield of Artificial Intelligence and Computer Science works on creating and developing appropriate procedures and techniques that allow machines to improve the productivity of their performances concerning a given goal, see (Mitchell, 1997). Regarding (Mitchell, 1997), a machine program is said to learn from an experience if:

i. there is a set of tasks for machine,
ii. there is a machine’s performance measure, and
iii. the machine’s performance at those tasks, as measured, improves with its experiences.

Concept Learning as a paradigm of machine learning has been structured over a set of theories and methodologies that focus on a kind of task in which a machine is trained [by a human being] in order to classify things (objects). Additionally, regarding Concept Training, the human being, as the trainer, shows the sets of example and non-example objects to the machine. Subsequently, the main machine’s task is a type of simplification. Actually, the machine compares the given thing with the provided examples. Thus, a characteristic feature of most concept learning approaches is the use of background knowledge. In concept learning with background knowledge, a machine, with regard to the given set of training examples and background knowledge, focuses on hypothesis generation. According to (Lehmann, 2010; Lehman et al., 2014), machine concept learning approaches lead to the construction of concepts (classes). The concept learning problems are tackled as a search through a space of candidate descriptions in the reference representation guided by exemplars of the target concepts. Also, the same algorithms can be adapted to solve the ontological problems. Note that these ontological problems are defined over the usage of ontologies in information and computer sciences. According to (Gangemi and Presutti, 2009), an ontology in information sciences is a formal and explicit specification of a shared conceptualisation on a domain of interest.

It shall be emphasised that the central focus of this research is on conceptual and epistemological issues of the expressions ‘concept’, ‘concept learning’ and ‘concept training’ within human-machine interplays. So, I will need to be careful with these terms. In this research, I will focus on epistemological analysis of concept learning while I will claim that “Machine Concept Learning approaches tackle to provide comprehensible logical representations for:

i. describing human beings’ constructed concepts, and
ii. describing the interrelationships between the constructed concepts after having been transformed (from humans’ minds into machines’ knowledge bases and ontological descriptions)”.

In (Badie, 2016a), I have focused on conceptual analysis of concept representation in the context of human-machine interactions. This paper has provided a conceptual and epistemological junction between human beings’ minds and machines’ knowledge bases (and ontological descriptions). In (Badie, 2016b), I have offered a description saying that the word ‘learning’ in machine learning has been utilised as a binary predicate with the word ‘machine’. ‘Learning’ as a binary predicate has been asserted (with regard to the language of descriptive logics) and has been described to be a role that is being performed by a machine. Thus, the act (role) of learning for a machine could be interpreted as a reflection (or mirroring) of the [human] learning phenomenon in machines. This research has been built on the basis of (Badie, 2016b; Badie, 2016c). So, I will, conceptually and epistemologically, summarise their conclusions in the framework of concept learning and in the context of human-machine interrelationships. Subsequently, I will show that they can provide a contextual description concerning existential analysis of ‘relations’ between human beings and machines. Later on, I will focus on conceptual analysis of my own semantics-based framework (for meaning construction by human beings) and a well-structured machine concept learning framework and on their junctions. Accordingly, it will be shown that this framework provides a supportive structure underpinning the described contextualisation of ‘relations’ between human beings and machines within concept learning processes.

2. BACKGROUND OF THOUGHT

First, I shall take up the notion of ‘concept’ and integrate it into my line of thought. The term ‘concept’ is an especially intricate term in philosophy, linguistics, psychology, epistemology, cybernetics and computer sciences. In my approach, concept has been understood to be, on the one hand, a linkage between a human’s mental images of parts of reality and, on the other hand, a human’s linguistic expressions and statements concerning those images, see (Götzsche, 2013). In machine concept training approaches, a concept is transformed into a [logically] equivalent form in order to be represented and be expressed in machines’ knowledge bases and ontological descriptions. For instance, relying on descriptive logical approaches, concepts can be reflected and, subsequently, be represented in the form of ‘entities’ and as the classes of objects, see (Baader et al., 2010; Hitzler et al., 2009a; Hitzler et al., 2009b). Regarding my research in (Badie, 2016c), human beings refine their constructed concepts into various conceptions that are representable in the form of hypotheses. Accordingly, the represented hypotheses could become corresponding to distinct entities or to their essential attributes, features, characteristics and properties. A hypothesis is a supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation, see (Link a). So, this supposition is capable of describing the multiple theories based on terminologies and in the form of world descriptions for a particular technical application and within the domain of interest. Consequently, the hypotheses created over the defined and analysed terminologies support inferential and reasoning processes and satisfy multiple conditions for definitions of truth with regard to interpretation functions.

This article focuses on (Badie, 2016b) that has been structured over Predicate Logic (PL), see (Mendelson, 1987). PL has supported me in the formal semantic[1] analysis of the term ‘relationships’ within human-machine interplays. Consequently, the conclusions supported my thoughts about the HowNess of establishing a formal semantics concerning human-machine interactions. Based on Predicate Logic and focusing on Description Logics (see (Baader et al., 2010)) a unary predicate is supposed to be logically equivalent to a concept. Relying on descriptive logical approaches we can translate a unary predicate into a concept in order to employ it in concept expression (so-called ‘concept learning’) processes. Furthermore, according to the afore-mentioned characteristics of hypotheses, a concept can be logically described by a hypothesis, see (Mitchell, 1997; Lehmann, 2010). Regarding (Mitchell, 1997), “a hypothesis, as a logical description of a concept, arises during a machine learning process”. It is a tentative explanation of ‘why the objects are members (or non-members) of the concepts’. Conceptually and logically, the hypotheses focus on describing the predicates. Then, they are expected to describe the same attributes, characteristics and properties. In my opinion, the outcomes of:

1. providing a strong logical description for amalgamation of mental representations of a thing’s linguistic expressions and its other mental images in the form of hypotheses,

2. analysing the supportive inferential processes on those hypothesis, and

3. focusing on world descriptions using generated hypotheses relying on defined terminologies

could determine the applications of predicates and, subsequently, the applications of terms and statements.

[1] Semantics is the study of the meanings, and the relation of signs to the objects to which the signs are applicable. In formal languages semantics is the study and analysis of the meanings of symbols and signifiers. Semantics focuses on the relationships between the signifiers of any language. In fact, the formal semantics employs the products of the human beings’ interpretations in order to produce meanings.

3. FORMAL SEMANTIC ANALYSIS

The following axioms in Predicate Logic focus on defining the outcomes of my interpretations. They will be employed for formal semantic analysis of relationships between 'human' and 'machine':

a. The terms '*human*' and '*machine*' are constant symbols.

b. The most significant unary predicates in the formalism are *Learner* and *Trainer*. According to this interpretation, *Learner*(*machine*) and *Trainer*(*human*) represent two world descriptions over unary predicates. They represent the facts that 'the machine is a learner' and 'the human is a trainer' respectively.

c. Considering the unary predicates *Learner* and *Trainer*, the binary predicates *TrainerOf* and *LearnerOf* can be defined. Consequently, *TrainerOf*(*human*,*machine*) and *LearnerOf*(*machine*,*human*) are two world descriptions over binary predicates. For instance, the relation *TrainerOf*(*human*,*machine*) describes that the human is the trainer of the machine.

d. The functions *trainer*(*machine*) and *learner*(*human*) can be defined in order to represent the 'trainer of machine' and the 'learner of human' respectively.

Based on these four axioms, we may conclude that:

$$\mathit{TrainerOf}(\mathit{human},\mathit{machine}) \rightarrow$$
$$[\ (\mathit{trainer}(\mathit{machine}) \rightarrow \mathit{human})\ \&\ (\mathit{human} \rightarrow \mathit{trainer}(\mathit{machine}))\ ]$$

AND

$$[\ (\mathit{trainer}(\mathit{machine}) \rightarrow \mathit{human})\ \&\ (\mathit{human} \rightarrow \mathit{trainer}(\mathit{machine}))\ ] \rightarrow$$
$$\mathit{TrainerOf}(\mathit{human},\mathit{machine}).$$

This logical term is structurally equal to:

$$\mathit{Relation} \rightarrow$$
$$[\ (\mathit{function} \rightarrow \mathit{constant})\ \&\ (\mathit{constant} \rightarrow \mathit{function})\ ]$$

AND

$$[\ (\mathit{function} \rightarrow \mathit{constant})\ \&\ (\mathit{constant} \rightarrow \mathit{function})\ ] \rightarrow$$
$$\mathit{Relation}.$$

Obviously, this logical description (let me name it '*RFC*') has been structured based on four fundamentals:

1. The *training* relationship between *human* and *machine* (from *human* into *machine*).
2. The *learning* relationship between *machine* and *human* (from *machine* into *human*).
3. The iterative loops between *human* and *machine* (from *human* into *machine* and then from *machine* into *human*).
4. The iterative loops between *machine* and *human* (from *machine* into *human* and then from *human* into *machine*).

Therefore, the formal semantics of training-learning relationship in the context of human-machine interactions is definable and analysable over two constructive bi-conditions:

I. Indicating the existence of the 'iterative loops between *human* and *machine*' from the '*training* relation between *human* and *machine*'. Inversely, indicating the truth (or the existence) of the '*training* relationship between *human* and *machine*' from the 'iterative loops between *human* and *machine*'.

II. Indicating the existence of the 'iterative loops between *machine* and *human*' from the '*learning* relation between *machine* and *human*'. Inversely, indicating the truth (or the existence) of the '*learning* relationship between *machine* and *human*' from the 'iterative loops between *machine* and *human*'.

According to the deduced results, the followings are valid:

I. The process of machine training (as a relation between human and machine) supports the interrelationships between 'the act and the process of training' and 'the machine (as the learner)', and inversely, the interrelationships between 'the act of training' and 'the machine (as the learner)' support the machine training (as a relation between human and machine).

II. The process of machine learning (as a relation between machine and human) supports the interrelationships between 'the act and the process of learning' and 'the human (as the trainer)', and inversely, the interrelationships between 'the act and the process of learning' and 'the human (as the trainer)' support the machine learning (as a relation between machine and human).

More specifically, by taking 'concepts' into consideration and limiting the 'Learning' task to 'Concept Learning', we can come up with the following consequences:

A. The concept training relations (from human into machine) support the interrelationships between 'the act and the process of concept training' and 'the machine (as the learner)'. Subsequently, interrelationships between 'the act and the process of concept training' and 'the machine (as the learner)' support the concept training relations (from human into machine).

B. The concept learning relations (from machine into human) support the interrelationships between ‘the act and the process of concept learning’ and ‘the human (as the trainer)’. Subsequently, the interrelationships between ‘the act and the process of concept learning’ and ‘the human (as the trainer)’ support the concept learning relations (from machine into human).

Proposition. The combination of (A) and (B) provides a contextual description concerning existential analysis of ‘relations’ between human beings and machines. This contextual description has been supported by the fact that the formal semantics of ‘relations’ between human beings and machines is describable over six components (i.e., two constants, two functions, and two relations) as can be seen in the logical description *RFC*.

4. A CONCEPTUAL FRAMEWORK

The central focus of this section is on building and analysing a conceptual framework over the proposed contextual and conceptual analysis of ‘relations’ between human beings and machines. This framework expresses “the junction of humans’ meaning construction processes and machines’ concept learning over humans’ constructed concepts”. This framework provides a supportive structure over the described contextualisation of ‘relations’ between human beings and machines within concept learning processes. It has been built over the separated components [a] and [b]. The component [b] represents the process of concept learning (by machine) and the component [a] represents the process of meaning construction (by human being). It shall be claimed that the interrelationships between a human being (as a trainer) and a machine (as a learner) could be represented as the reflection of [a] in [b], and as the conformation of [b] on [a]. In (Badie, 2016c), I have focused on logical characterisation of ‘reflection’ and ‘conformation’ transformations.

4.1. COMPONENT [b]: CONCEPT LEARNING BY MACHINE

As mentioned, a very important and determinative feature belonging typically to all machine concept learning approaches is using background knowledge. Generally, the background knowledge could be classified as concept descriptions in knowledge bases and in ontological descriptions. However, there are strong dependencies between concept descriptions and ontological descriptions. For instance, in semantics-based technologies the background knowledge could be categorised as:

i. Ontology Languages, e.g., OWL (Link b), that represent specified knowledge about various things and objects,

ii. internal knowledge bases,

iii. Question-Answering and querying endpoints [that focus on querying a knowledge base using the querying languages, e.g., SPARQL (Link c)], and

iv. the collection of the interrelated datasets, e.g., Linked Data (Link d).

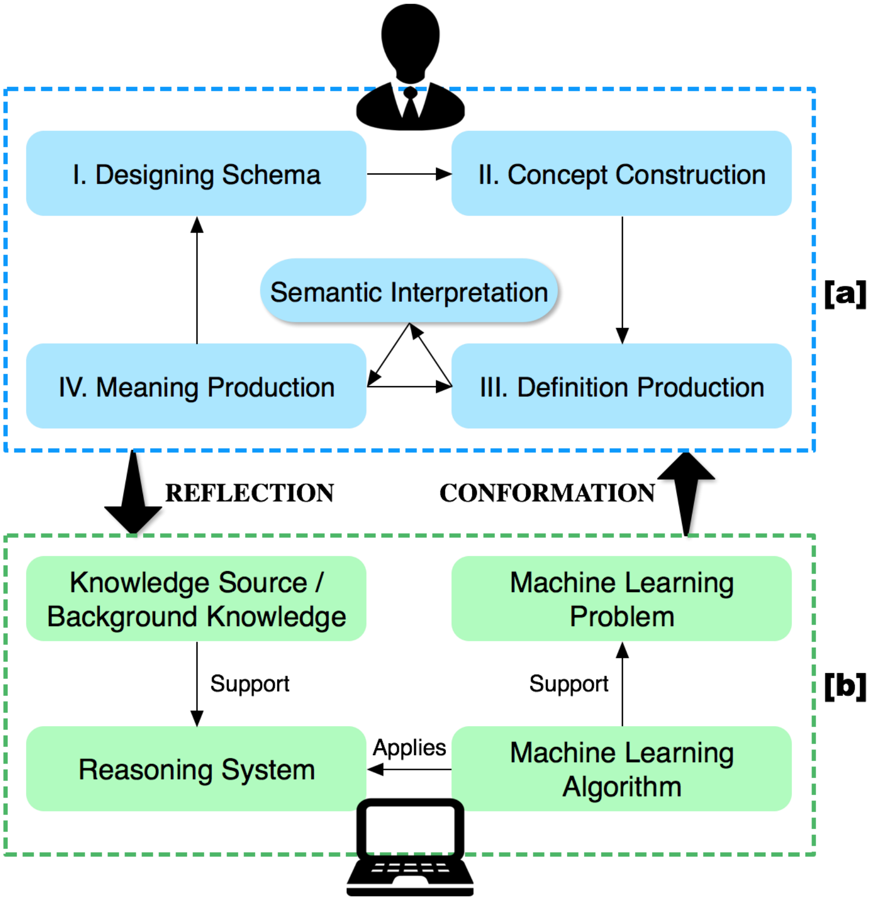

Figure 1. *A Conceptual Framework for Concept Learning within Human-Machine Interplays*

(Lehmann, 2009) has focused on analysing a successful component-based framework for concept learning in Description Logics and in OWL, see (Link e). The architecture of this component-based framework has been structured over four components. Actually, besides the 'Knowledge Source' component (see Figure 1), the 'Machine Learning Problem' component (e.g., definitions of atomic concepts by more specified descriptions, providing subsumption and equality axioms in terminological knowledge, providing positive and negative examples of classes of objects), the 'Machine Learning Algorithm' component, and the 'Reasoning System' component are related to each other, see Figure 1.

I shall stress that the architecture of this framework could clarify what ingredients (as the building blocks) are effective within concept learning processes and how those ingredients could be connected to each other. You will see the interconnections of this Framework and my developed Framework in the next section.

4.2 COMPONENT [a]: MEANING CONSTRUCTION BY HUMAN BEING

For analysing meaning construction (and production), I rely on my own semantics-based framework, see (Badie, 2015a; Badie, 2015b; Badie, 2016d). I shall start with this conception that "a human being, before becoming a trainer of a machine, is the developer of her/his personal conceptions (of her/his constructed concepts) over the individual designed schemata (or mental entities)", see 'Designing Schema' in Framework 1. Schemata are cohesive, repeatable action sequence possessing component actions that are tightly interconnected and governed by a core meaning, see (Piaget and Cook, 1952) and (Link f, Link g). In my semantic approach, schemata (or mental entities) support humans in constructing their own concepts (and in linking their mental images of the world and their linguistic expressions). Furthermore, schemata support humans (who are the trainers of machines) in developing their constructed concepts and in producing their own conceptions in order to employ them in generating hypotheses, see 'Concept Construction' in Framework 1. The constructed concepts provide strong backbones for humans' semantic interpretation and meaning construction processes within their concept training. I shall claim that the most significant importance of the proposed contextual and conceptual description of concept learning [and concept training] regarding interrelationships between humans and machines is that "they have been analysed over schemata-based conceptualisations and conceptual representations". Considering (Badie, 2015a; Badie, 2015b; Badie, 2016d), the trainer needs to employ inductive rules in order to expand her/his constructed general concepts into more specified ones. Additionally, a human being during her/his interactions (with the environment, with her/his experiences and with machines) investigates and forms new concepts (or reforms the old ones) and applies them in her/his concept construction processes, and accordingly, in her/his hypotheses generation. This fact clearly shows that the trainer has become concerned with 'definitions' and 'concept descriptions' in his training process, see 'Definition Production' in Framework 1.

Note that I already have discussed these descriptions in the 'Machine Learning Problem' component within machine concept learning process (i.e., the component [b]). Furthermore, the generalisation of various specified concepts support the trainer in discovering [and constructing] new concepts. Accordingly, she/he focuses on the characteristics, attributes and properties that can be used to distinguish exemplars (of various concepts) from non-exemplars. Again, these products are employed in the 'Machine Learning Problem' component within the machine concept learning process [b]. In fact, the trainer is highly concerned with identifying, establishing, indicating [and relating] the induced examples of her/his conceptions. Subsequently, she/he can make her/his personal labels of the concept categorisations in order to manage different classes of concepts in the form of hypotheses. Note that these hypotheses are considered as the reflections of her/his created concepts in various classes of objects, and as analysed earlier, they are used in constructing the background knowledge in the 'Knowledge Source' component (see Figure 1). According to the afore-mentioned characteristics of humans' conceptions and definition productions within schemata-based concept training processes, the sequence

'PriorKnowledge → Definition → New Knowledge'

supports the trainer in searching for the initiative meanings of the class/classes of constructed concepts and their relationships, see 'Meaning Production' in Framework 1. It's only possible by means of semantic interpretations, see 'Semantic Interpretation' in Framework 1. From the logical point of view, the interpretation of a defined constructed concept is a function. This function has turned a definition into a meaning. The Interpretation functions operate the trainer's definitions over her/his constructed concepts. Therefore, the interpretations 'activate' the meanings of her/his constructed concept. Consequently, the productions have made appropriate backgrounds for verifying the personally found meanings based on personal defined constructed concepts. I shall stress that "a meaning would be given a better shape after checking the balanced definitions based on personal constructed concepts". So, there could be a number of loops between Definitions and Meanings. Methodologically, meanings are the supportive backbones for providing meaningful conceptual structures. The meaningful conceptual structures are all personally organised based on individual constructed concepts and definitions. On the other hand, the meaningful conceptual structures could induce more developed meanings on higher conceptual levels (and after gaining more experience within interactions with environment, self, and the machine). The produced meanings could be employed in developing the individual schemata through higher levels of conceptions.

5. CONCLUSIONS

Concept Learning as a paradigm of machine learning has been structured over a set of theories and methodologies that focus on a kind of task, in which machines are trained [by human beings] in order to classify various things (objects). A characteristic feature of most concept learning approaches is the use of background knowledge. In concept learning with background knowledge, a machine, with regard to the given set of training examples and background knowledge, focuses on hypothesis generation. Through the lenses of logics and relying on descriptive logical approaches, the machine concept learning paradigm tackles to provide comprehensible logical representations in order to (a) describe humans' constructed concepts, and (b) describe the interrelationships between the constructed concepts after having been transformed (from humans' minds into machines' knowledge bases and ontological descriptions). The central focus of this research has been on conceptual and epistemological issues of the expressions 'concept', 'concept learning' and 'concept training' within human-machine interplays. Regarding my previous researches on:

i. concept representation analysis in the context of human-machine interactions,

ii. conceptual analysis of concept representation in the context of human-machine interactions,
iii. epistemological junctions between human beings' minds and machines' knowledge bases and ontological descriptions, and
iv. meaning construction over humans' constructed concepts,

I have initiated this research with the conception that 'a human being, before being a trainer of a machine, is the developer of her/his personal conceptions over her/his designed schemata'.

This research has provided a contextual description concerning existential and structural analysis of 'Relations' between human beings and machines. It has been suggested that the most significant importance of provided contextual [and conceptual] description of concept learning (as well as concept training) regarding the interrelationships between humans and machines, has been established over schemata-based conceptualisations and conceptual representations. Subsequently, I have focused on conceptual and epistemological analysis of:

i. my own semantics-based framework for meaning construction based on human's constructed concepts, and
ii. a well-structured component-based machine concept learning framework that works based on Description Logics and within ontological languages.

Accordingly, I have focused on linking those two frameworks for logical analysis of concept learning in the context of human-machine interrelationships. It has been demonstrated that the proposed framework provides a supportive structure over the described contextualisation of 'relations' between human beings and machines within concept learning processes. This research has formed a building block of my PhD research, which is dealing with Semantic Analysis of Constructivist Concept Leaning.

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Links

Link a: http://www.oxforddictionaries.com/definition/english/hypothesis

Link b: www.w3.org/2001/sw/wiki/OWL

Link c: www.w3.org/wiki/SparqlEndpoints

Link d: www.w3.org/standards/semanticweb/data

Link e: http://dl-learner.org/development/architecture/

Link f: http://plato.stanford.edu/entries/schema

Link g: https://global.britannica.com/topic/schema-cognitive

PAPER K. TOWARDS SEMANTIC ANALYSIS OF MENTORING-LEARNING RELATIONSHIPS WITHIN CONSTRUCTIVIST INTERACTIONS

Farshad Badie

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ABSTRACT

The multilevel conversational exchanges between a mentor and a learner could be seen as a radical constructivist account of their comprehensions. The process of knowledge construction could be realised to have a significant importance in the context of mentor-learner interactions. The most important fundamental is that 'the conversational exchanges between mentors and learners ask questions and give answers concerning their individual conceptions, comprehensions and reasonings'. These questions and answers are the main building blocks of the 'Relations' between mentors and learners. In this article, I will employ Predicate Logic in order to focus on the relationships between learners and mentors. This research will, conceptually and logically, be concerned with [formal] semantic analysis of mentoring-learning relationships in the context of constructivist interactions. The conclusions will shed light on how a [formal] semantics for constructivist interactions is established.

1. INTRODUCTION

The multilevel interactions and conversational exchanges between a human being (as a mentor, trainer, teacher) and another human being (as a learner, student) could be seen as a radical constructivist account of their cognitions, realisations and comprehensions, see (Scott, 2001). First, I shall draw your attention to some fundamentals that will clarify my conception of, and my way of thinking about, the use of the term 'learning' in this article. Relying on First-Order Predicate Logic (FOL), in the expression 'human learning', the word 'learning' has been utilised in the form of a predicate with the word 'human'. Thus, learning can describe something about a human being who is being discussed and interpreted. In other words, learning in human learning has been described in the form of a predicate to model a role that is being performed by a human being. I shall emphasise that I am fully aware of the basic principle in linguistic analysis that there is a fundamental difference between the phrase human learning and a sentence/clause like the human is learning. In fact, relying on linguistic analysis, assertions [and statements] can only be made by sentences (and not by phrases), but the central focus of this research is on logical analysis of the term 'human learning' and its formal semantics. According to the proposed logical conception, a subject could be related to Human by means of Learning. Through the lens of cognition, 'learning' (as a human's act) could be interpreted and explained from different perspectives. In this research learning is recognised as the act of [knowledge] construction. And the most central assumption of this research is 'considering learning as the activity (and the developmental process) of knowledge construction'. Knowledge construction (that is a salient product of constructivist learning) could be realised to have a significant importance in the context of mentor-learner interactions and in their conversational exchanges; for more details about constructivist learning[1] see (Piaget, 1964; Phillips, 1995;

[1] Jean Piaget is the originator of the theory of constructivism. He argues that humans generate knowledge and meaning from an interaction between their experiences and their ideas, see (Piaget, 1946; Piaget, 1967). Piaget's developmental theory of learning contemplates that the constructivist learning is concerned with how a human being goes about constructing her/his individual knowledge structures. In philosophy of

McGawand Peterson, 2007). The most important fundamental is that 'the conversational exchanges between mentors and learners ask questions and give answers concerning their individual conceptions, comprehensions and reasonings'. In my opinion, these questions and answers are the main building blocks of the [constructivist] relations between mentors and learners. In the following section I will be more specific about the concept of 'relation'. The processes of knowledge acquisition, knowledge construction and knowledge development could work illustratively, interpretively, instructively and heuristically in the framework of Conversation Theory (CT), which is conceived and elaborated by Gordon Pask, see (Pask, 1975; Pask, 1980). Pask's main premise is that reliable knowledge exists, and it evolves in action-grounded conversations, see (McIntyre Boyd, 2004). Relying on CT and regarding my achievements in meaning construction, the framework of conversational learning is inherently a semantic model that accounts for the emergence of the domain of the learner's and of the mentor's constructed knowledge. Semantics as the study of the meanings can express how the produced meanings based upon learners' and mentors' constructed concepts could support them in constructing their universal knowledge. Furthermore, producing one's own understanding of world and developing it during the interaction could be said to be the most valuable product of a constructivist interaction. A constructivist interaction supports the mentor and the learner in:

i. exchanging their own conceptions (based on their conceptualisations) with each other, and
ii. conceptualising their own and the interlocutor's produced comprehensions.

In this article, the interactions will be analysed with respect to developmental processes[2] of humans' world constructions over their conceptions and concept constructions (I will explain more about concepts). I contemplate that the developmental processes of personal world constructions [by mentors and learners] provide supportive backbones for a semantic model that accounts for the meeting of their constructed knowledge. In this article, First-Order Predicate Logic (FOL) will be employed in order to analyse the relationships between learners and mentors. Predicate Logic allows us to make arbitrarily specified relationships between different objects of the conversational learning system. This research will, conceptually and logically, be concerned with [formal] semantic analysis of mentoring-learning relationships in the context of constructivist interactions. The conclusions will shed light on how a [formal] semantics for constructivist interactions is established.

2. ABOUT CONCEPTS

education, Constructivism as a theory and as an epistemology-based model focuses on knowledge construction in human beings' own cognitive apparatus.

[2] Developmental Processes of Learning could be seen as the product of Developmental Theory of Learning and Cognitive Development, see (Link a).

The notion of 'concept' is a very sensitive term that must be used with caution, but I assume the use of concept to be comprehensible in this context and in the following logical formalisms. Concepts play fundamental parts in the use of reasons and languages, see (Simpson and Weiner, 1989). According to (Götzsche, 2013), a concept is an interrelationship between humans' mental images of [parts of] reality/fiction and their linguistic expressions (and descriptions). Also, my conceptual approaches in (Badie, 2015a; Badie, 2015b; Badie, 2016a) are based on this theoretical notion of 'concept'. In this research, I consider the collection of (i) mental images [of an object/phenomenon], (ii) linguistic expressions [about that object/phenomenon], and (iii) the interrelationships between (i) and (ii) as a conceptual entity and name it Concept. This article is tackling to propose a logical description of concept analysis in the context of mentor-learner interaction, so I shall propose that the concepts could be represented by hypotheses in order to correspond to a distinct entity or to its essential attributes, features and properties. Subsequently, assessed by logics, the entities determine the applications of terms and phrases, and can manifest themselves in the form of [unary] predicates (you will see about them in the next section). Therefore, FOL-based predicates could be employed in logical and formal analysis of this research's objectives.

3. PREDICATE LOGIC AND PREDICATION

Propositional Logic and its formulae[3] are constructed based on atomic objects. The statements involving atomic objects and, subsequently, the statements involving non-atomic formulae could only be either true or false. Predicate Logic (as the logic of predication) is constructed over propositional logic by considering objects as the elements of sets, and by applying universal and existential quantifications (restrictions) on different objects, see (Barwise, 1977; Mendelson, 1987). The fundamental symbols in Predicate Logic are divided into logical and non-logical symbols. The logical symbols are: Conjunction ($\wedge$), Disjunction ($\vee$), Negation ($\neg$), Implication ($\rightarrow$), Bi-conditional ($\leftrightarrow$), Equality ($=$), Existential Restriction ($\exists$), Universal Restriction ($\forall$), Tautology ($\top$), Contradiction ($\perp$) and Parentheses. Any of the logical symbols have the same meaning in different contexts and conditions. This statement means that we are not allowed to interpret them and assign multiple values and definitions to them. Besides logical symbols, the non-logical symbols are interpreted and represented in the following forms (obviously, we need to interpret the non-logical symbols of a logical system in order to produce meanings and to clarify what we mean by those symbols):

A. Constant Symbols (beginning with a lower-case letter): For instance, martin, apple, yellow, and $\prod$ denote constants.

B. Unary Predicates (beginning with an upper-case letter): In $P(a)$ and $Q(b)$, P and

[3] The logical relationships and rules expressed in Propositional Logic's symbols are known as Propositional Logic's Formulae.

Q, respectively, denote unary predicates. Also, *a* and *b* are variables (multiple constant symbols) and are the instances of *P* and *Q* respectively. For example, Fruit(apple) denotes that "apple is a fruit". Also, one may claim that it denotes that "all apples are fruits". So, what it actually means is that a specific individual denoted by the constant apple is a Fruit and the individual 'apple' belongs to the set (and class) of Fruits. As mentioned, the unary predicates could represent concepts. So, the unary predicate Fruit denotes a concept and the constant apple denotes an instance of the concept Fruit.

C. Binary Predicates or Relations: $R(m,n)$ is a binary predicate and represents a relation between two variables m and n. For example, FatherOf(john, mary) can represent the fact that 'John is the father of Mary'.

D. Function Symbols (beginning with a lower-case letter): $f(x)$ is a function that has operated the variable x. For example, father(john) can represent the 'father of john'.

4. CONVERSATIONAL LEARNING THEORY

According to (Laurillard, 1993; Laurillard, 2002), Laurillard's framework can be interpreted as a learning theory and as a practical framework for designing educational and pedagogical environments. This framework includes four important components:

i. Mentor's concepts (and conceptions),
ii. Mentor's constructed learning environment (that produces the mentor's constructed world),
iii. Learner's concepts (and conceptions), and
iv. Learner's specific actions (that construct the learner's world).

Regarding Laurillard's framework, we need to consider different forms of communication and associated mental activities. The main constructors of the associated mental activities are discussion, adaptation, interaction, and reflection.

In fact, relying on Pask's Conversation Theory and focusing on Laurillard's framework, there are four kinds of human activities that take place in different kinds of flows between the components of Laurillard's framework:

1. Discussion between the mentor and the learner [on their conceptions, descriptions, comprehensions, and reasonings].
2. Adaptation of the learner's actions and of the mentor's constructed environment. There is a kind of adaptation of the learner's mental universe and of the mentor's mental universe.
3. Interactions between the learner and the environment that is defined by the mentor.
4. Reflection of the learner's performance by both the mentor and the learner.

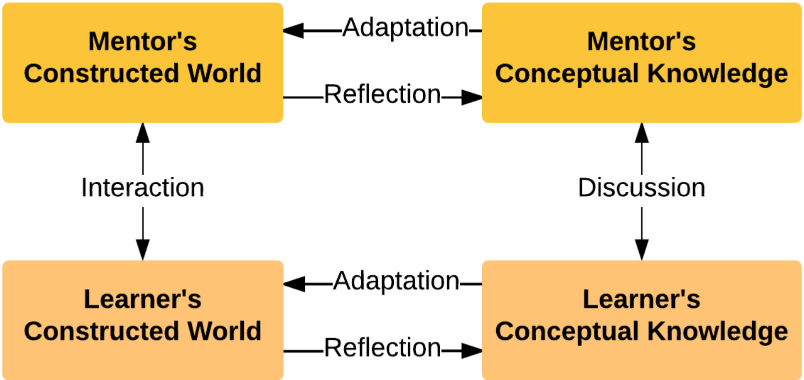

Figure 1. *The Conversational Learning Framework*

I have illustrated the above-mentioned characteristics in the framework represented in Figure1. I have analysed meaning construction over this framework in (Badie, 2016b). In this framework, the learner and the mentor are incorporated. They converse and exchange their conceptions in order to construct and develop their knowledge. This framework provides a contextual ground for semantic analysis of mentor-learner interactions and relationships.

5. SEMANTIC ANALYSIS OF M-L RELATIONSHIPS

This research aims at investigating where the formal semantics come from and when it appears within a relationship between a mentor and a learner. I shall draw your attention to four fundamental axioms that clarify what could be offered by the non-logical symbols within my formalism. Defining axioms is the most important prerequisite for bridging the gap between formalism and the meanings.

1. The symbols *m* and *l* denote mentor and learner respectively. They both represent constant symbols.

2. The atomic concepts in logical analysis of interactions between learner and mentor are *Learner* and *Mentor*. Transforming them into predication, the unary predicates *Learner* and *Mentor* are the fundamental unary predicates in the logical system. Also, *Learner*(*l*) and *Mentor*(*m*) represent two world descriptions over unary predicates. They demonstrate that the constant symbol *l* is an instance of the unary predicate *Learner* and the constant symbol *m* is an instance of the unary predicate *Mentor*. In other words, *l* is a *Learner* and *m* is a *Mentor*.

3. Considering the unary predicates *Learner* and *Mentor*, the binary predicates *MentorOf* and *LearnerOf* are definable. Consequently, *MentorOf*(*m*,*l*) and

LearnerOf(l,m) are two world descriptions over relations that express the facts that "the individual m is the mentor of the individual l" and "the individual l is the learner of the individual m" respectively. These two world descriptions have described the most fundamental (atomic) relationships between two human beings in the form of binary predicates.

4. The functions *mentor*(l) and *learner*(m) are defined for representing the concepts of "the mentor of the individual l" and "the learner of the individual m" respectively. These functions are introduced in order to make a logical linkage between an individual and the role of her/his interlocutor.

According to the proposed axioms, I shall claim that the binary predicate *MentorOf*(m,l), logically, implies that "the mentor of the individual l is (equality) the individual m". In fact, a world description over the relation between a mentor and a learner has supported the conclusion that there is an alignment between an agent (mentor and learner) and her/his role (mentoring and learning) for her/his interlocutor (learner and mentor).

Let me focus on the mentor's perspective in order to provide the formal analysis:

$$MentorOf(m,l) \quad (i) \;\Rightarrow$$

$$mentor(l) = m. \quad (ii)$$

The equality (*ii*) is the product of interpretation and the binary predicate (*i*) describes the interpreted relation between *mentor*(l) and m. I shall claim that this equality is the root [and the origin] of the formal semantics within all mentoring-learning relationships including the relationships in the context of constructivist interactions. The equality in the form of a binary predicate describes that the meanings of *mentor*(l) and m are the same. Then we have:

$$Equals\ (mentor(l)\ ,\ m). \quad (iii)$$

Considering the binary predicate (*iii*), the function *mentor*(l) that is a non-logical symbol, and the person (individual) m that is a constant symbol, have been expressed to have the same meanings. Relying on commutative laws, *mentor*(l) = m and m = *mentor*(l) are logically and meaningfully equivalent. Therefore:

$$Equals\ (m\ ,\ mentor(l)). \quad (iv)$$

Accordingly, (*iii*) and (*iv*) are the conclusions of (*ii*). So,

- the mentor of l implies m, and
- m implies the mentor of l

are the conclusions of "the mentor of l is m". Therefore:

$$[mentor(l)= m\ ] \;\Rightarrow\; [\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ]. \qquad (v)$$

The logical term (*v*) is inherently equal to:

$$(function \rightarrow constant)\ \ \&\ \ (constant \rightarrow function). \qquad (vi)$$

I have already deduced that my central focus has been on the interrelationships between function symbols and constant symbols. Note that there is a bi-conditional relation between (*i*) and (*v*). Therefore:

$$MentorOf(m,l) \leftrightarrow [\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ]. \qquad (vii)$$

Equivalently:

$$MentorOf(m,l) \rightarrow [\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ]$$
$$\&$$
$$[\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ] \rightarrow MentorOf(m,l). \qquad (viii)$$

The logical term (*viii*) is structurally equal to:

$$Relation \rightarrow [\ (\ function \rightarrow constant\ ) \wedge (\ constant \rightarrow function\ )\ ]$$
$$\&$$
$$[\ (\ function \rightarrow constant\ ) \wedge (\ constant \rightarrow function\ )\ ] \rightarrow Relation. \qquad (ix)$$

In Figure 2, this logical conclusion has been figured out.

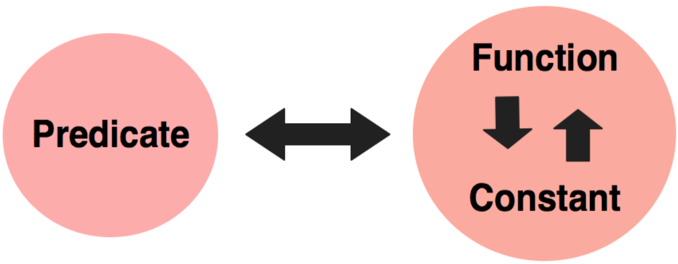

Figure 2. *The General Structure of the Formal Semantics within Mentor and Learner Relationships*

Regarding (*viii*) in the form of (*ix*), the logical system of (*i*) has been constructed over four fundamental relationships:

a. The mentoring relationships [from mentor into learner].

b. The learning relationships [from learner into mentor].
c. The iterative loops between mentor and learner (beginning from mentor into learner).
d. The iterative loops between learner and mentor (beginning from learner into mentor).

Therefore, the formal semantics of mentoring-learning relationship in the context of mentor-learner conversational exchanges is definable over four constructive implications as follows (that are all extracted from *viii*).

I. Implying (c) from (a). Formally:

$$MentorOf(m,l) \rightarrow [\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ]. \qquad (x)$$

It expresses:

[*Mentoring Relation* → (*Mentoring Function* ↔ *Learner Constant*)].

II. Implying (d) from (b). Formally:

$$LearnerOf(l,m) \rightarrow [\ (\ learner(m) \rightarrow l\ ) \wedge (\ l \rightarrow learner(\mathrm{m})\ )\ ]. \qquad (xi)$$

It expresses:

[*Learning Relation* → (*Learning Function* ↔ *Mentor Constant*)] .

III. Implying (a) from (c). Formally:

$$[\ (\ mentor(l) \rightarrow m\ ) \wedge (\ m \rightarrow mentor(l)\ )\ ] \rightarrow MentorOf(m,l). \qquad (xii)$$

It expresses:

[(*Mentoring Function* ↔ *Learner Constant*) → *Mentoring Relation*].

IV. Implying (b) from (d). Formally:

$$[\ (\ learner(m) \rightarrow l\ ) \wedge (\ l \rightarrow learner(m)\ )\ ] \rightarrow LearnerOf(l,m). \qquad (xiii)$$

It expresses:

[(*Learning Function* ↔ *Mentor Constant*) → *Learning Relation*].

According to the deduced results, the unification of the following four items gives meaning (based on formal logic) to the relationship between mentor and learner, see Figure 3.

I. The mentoring relations (from mentor into learner) support the interrelationships between 'the act of mentoring' and 'the learner'.

II. The learning relations (from learner into mentor) support the interrelationships between 'the act of learning' and 'the mentor'.

III. The interrelations between 'the act of mentoring' and 'the learner' support the mentoring relations (from mentor into learner).

IV. The interrelations between 'the act of learning' and 'the mentor' support the learning relations (from learner into mentor).

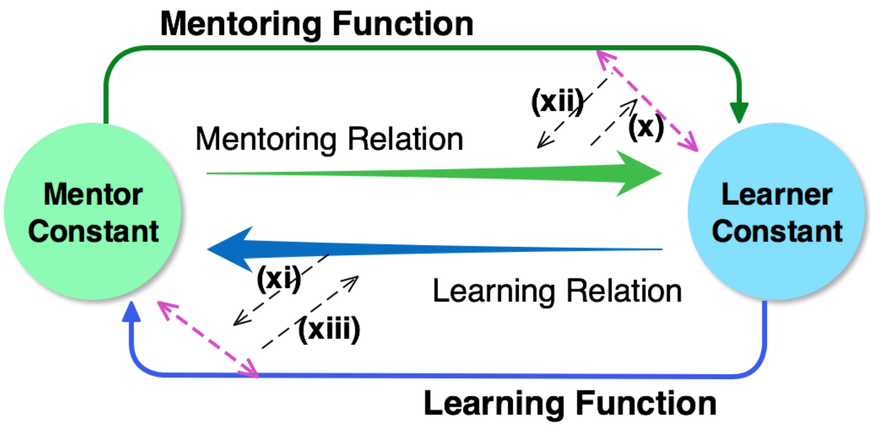

Figure 3. *Establishing Meaning in the Relationship between Mentor and Learner*

Consequently,

- by merging (I) and (III),
- by merging (II) and (IV), and
- focusing on constructivist interactions,

we have the following conceptual conclusions:

A. The constructivist mentoring relations (from mentor into learner) support the interrelationships between 'the act of constructivist mentoring' and 'the learner'. Subsequently, the interrelations between 'the act of constructivist mentoring' and 'the learner' support the constructivist mentoring relations (from mentor into learner).

Taking the conversational learning framework into account, the constructivist mentoring relations are adapted [and provided] by the mentor's conceptual knowledge. Then the mentor's conceptual knowledge supports the interrelationships between 'the act of constructivist mentoring' and 'the learner'. Additionally, the interrelations between 'the act of constructivist mentoring' and 'the learner' support the mentor's conceptual knowledge.

B. The constructivist learning relations (from learner into mentor) support the interrelationships between 'the act of constructivist learning' and 'the mentor'. Subsequently, the interrelations between 'the act of constructivist learning' and 'the mentor' support the constructivist learning relations (from learner into mentor). Relying on the conversational learning framework, the constructivist learning relations (from learner into mentor) are adopted [and provided] by the learner's conceptual knowledge. Then, the learner's conceptual knowledge supports the interrelationships between 'the act of constructivist learning' and 'the mentor'. Moreover, the interrelationships between 'the act of constructivist learning' and 'the mentor' support the learner's conceptual knowledge.

Summary

The mentor's conceptual knowledge and the learner' conceptual knowledge are being transformed and exchanged during the constructivist interaction. Furthermore, conversational learning's main premise is that the reliable knowledge exists and evolves in action-grounded interactions and conversations. So, the discussions between the mentor's and the learner's conceptual knowledge make a balance in their united [and global] reliable conceptual knowledge. This reliable knowledge affects both A and B. Accordingly, A and B have interconnections with each other. The results of the interactions between A and B are reflected in reliable united conceptual knowledge.

Regarding the proposed conceptual analysis, there are strong interrelationships between two classes of concepts. And in fact, in a broad sense, the multiple interconnections between these classes give meaning to (and establishes the semantics of) the mentor-learner relationships within constructivist learning systems.

CLASS 1

- The mentor's conceptual knowledge.
- The learner's conceptual knowledge.
- The reliable united conceptual knowledge.
- The adaptation of the reliable united conceptual knowledge in the mentor's world.
- The adaptation of the reliable united conceptual knowledge in the learner's world.

CLASS 2

- The reflection of the constructivist mentoring relations (from mentor into learner) in the reliable united conceptual knowledge and, respectively, in the mentor's conceptual knowledge.
- The reflection of the interrelationships between 'the act of constructivist mentoring' and 'the learner' in the reliable united conceptual knowledge and, respectively, in the mentor's conceptual knowledge.
- The reflection of the constructivist learning relations (from learner into mentor) in the reliable united conceptual knowledge and, respectively, in the learner's conceptual knowledge.
- The reflection of the interrelationships between 'the act of constructivist learning' and 'the mentor' in the reliable united conceptual knowledge and, respectively, in the learner's conceptual knowledge.

6. CONCLUSIONS

In the expressions 'human learning' and 'human mentoring', the words 'learning' and 'mentoring' are utilised as binary predicates (roles) with the word 'human'. Knowledge construction as the most important objective of constructivist learning is a significant matter in the context of mentor-learner interactions. The most central focus of this article has been on 'Relations' between mentors and learners. I have focused on the process of knowledge construction in the framework of Conversation Theory (CT) and have assumed that conversational learning is inherently a semantic model to account for the emergence of the domain of the learner's and of the mentor's conceptual knowledge. The developmental processes of personal world constructions [by mentors and learners] provide supportive backbones for that semantic model. Furthermore, I have considered the fact that conversational learning could express how the produced meanings based upon constructed concepts could support mentor and learner in constructing their universal conceptual knowledge. In this article, Predicate Logic has been employed in order to provide a logical basis for relationships between learners and mentors. Relying on Predicate Logic, this research, conceptually and logically, has been concerned with semantic analysis of mentoring-learning relationships in the context of constructivist interactions. I have thought of the establishment of meaning (in a broad sense) in the relationships between mentors and learners. Taking the results into consideration and reconsidering the framework of constructivist conversational learning, it has been checked how the produced meanings could be related to the discussions between the mentors' and the learners' conceptual knowledge. Consequently, relying on the semantic ground of conversational learning framework, it has been realised that the produced meanings are interrelated in order to get reflected (mirrored) in the [mentor's and learner's] united conceptual knowledge. This research has formed a building block of my PhD research which is dealing with Semantic Analysis of Constructivist Concept Leaning.

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Links

Link a: https://psychohawks.wordpress.com/2010/09/05/theories-of-cognitive-development-jean-piaget/

PAPER L. KNOWLEDGE BUILDING CONCEPTUALISATION IN SMART CONSTRUCTIVIST LEARNING SYSTEMS

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ABSTRACT

This chapter focuses on the meeting of Constructivism (as a learning theory) and Smart Learning and, thus, theorises Smart Constructivist Learning. The main field of research is Smart Learning Environments. Relying on the phenomena of 'meaning construction' and 'meaningful understanding production' in the framework of smart constructivism, we will focus on analysing Smart Constructivist Knowledge Building. Accordingly, we analysed Learning-and-Constructing-Together as a smart constructivist model. The outcomes of this chapter could support the developments of smart learning strategies.

1. INTRODUCTION AND MOTIVATION

The process of knowledge building leads to changes in humans' minds. In the context of cognitive developmental psychology, conceptual change is a type of process that focuses on the conversion of a human's conceptions and the relationships between her/his old and new conceptions, see (Chi, 1992; Limon, 2002). Thus, the most salient effect of knowledge building could be recognised to be on conceptual change of the learners'/mentors' conceptions over the course of time. We shall begin this chapter with our special focus on the fact that knowledge acquisition (that is the most determinative process within knowledge building processes) is a reflective activity that enables learners and mentors to draw upon their previous (and accumulated) experiences and reflect on their background as well as existing knowledge, see (Watkins et al., 2002). The reflective activity of knowledge acquisition supports learners and mentors in reflecting on themselves, on their society and on their environment. Knowledge acquisition enables learners and mentors to conceptualise and understand. Subsequently, it enables learners to evaluate both their present and past, so as to build up and shape their future actions (i.e., operations, practices, proceedings, movements, contributions and manners) as well as to construct and develop the construction of their latest pre-structured and pre-constructed knowledge. As described, 'understanding' has been recognised as the consequence of 'conceptualisation'. Our research (in (Badie, 2016a; Badie, 2016d)) has concluded that "an understanding could be realised to be a local manifestation of a global conceptualisation".

It is important to account for the fact that human beings become concerned with various construction processes over their pre-formed knowledge in order to obtain the opportunities to develop their constructed knowledge and to produce their deeper understandings (i.e., meaningful comprehensions). Constructivist Learning (based on constructivist epistemology and constructivist models of knowing) has become the central framework of this research. Relying on this framework, our supposedly theoretical model of learning deals with how knowledge can assumedly be built by a learner/mentor. Through the lens of cognitive psychology, Piaget's developmental theory of learning says that constructivist knowledge acquisition (and knowledge building) is concerned with how an individual goes about the construction of knowledge in her/his own mental apparatus, see (Piaget, 1936; Piaget, 1952). Accordingly, for any learner or mentor, knowledge acquisition could be recognised as the activity of seeking knowledge regarding different objects, processes, events and phenomena with regard to her/his background knowledge. As for the structural and existential characteristics of constructivism, the construction of knowledge is conceived of as a type of dynamic process. It can be informally described in terms of personal understanding in multiple actions. Consequently, constructivist learning is highly concerned with the active generation of personal understandings, see (Badie, 2016d; Badie, 2016c).

This chapter focuses on ‘Smart Constructivist Learning Systems’, which are a specific sub-class of constructivist learning systems where ‘constructivism’ meets ‘smart learning’. In accordance with the subject of this book, though, we look at the area of ‘Smart Learning Environments’. According to (Uskov et al., 2016a), “smart education represents an integration of smart objects and systems, smart technologies, smart environments, smart features (smartness levels), smart pedagogy, smart learning and teaching analytics systems”. Relying on the framework of smart education, section we will focus on the development of a conceptual framework for analysing knowledge building in the framework of smart constructivism and over the flow of the learners’ understandings. Correspondingly, we will characterise the main components of a smart constructivist pedagogy (and a smart constructivist model of learning). It may justifiably be assumed that the outcomes of this chapter will support designing and developing innovative learning and mentoring strategies as the products of smartness. We will conceptualise and prove that there are strong interrelationships between ‘smart constructivist model of learning’ and ‘collaborative learning strategy’.

According to (Uskov et al., 2016a; Uskov et al., 2016b), research in the area of smart learning systems should not only focus on software/hardware/technology features, but also on smart ‘features’ and the ‘functionality’ of smart systems. Furthermore, in order for smart learning systems to be effective and efficient for different learners, or mentors, there are certain smartness levels (smart distinctive features). The most significant feature in this research is analysing the phenomenon of ‘smart learning’ in the framework of constructivism. So, it is concerned with acquiring new knowledge and building knowledge over existing (and background) knowledge and over the accumulated experiences by agents. Also, this research aims at identifying and recognising the concept of ‘understanding’ toward awareness of learners. Therefore, this chapter will be highly concerned with ‘learning/self learning’ feature. Additionally, our approach will rely on logical descriptions using, e.g., assumptions, implications and different logical rules over conceptual analysis of the phenomenon of ‘smart learning’ and concept of ‘understanding’. So, this research is—essentially—structured over logical reasoning processes and could support researchers’ thoughts for developments of inferential and logical reasoning processes within smart learning systems.

Note that this research has been designed over our own approaches in the analysis of ‘meaning construction’ and ‘understanding production’ processes, see (Badie, 2015a; 2015b; 2016c; 2017a). Our ideas have been based on a new scheme for interpretation based on semantics and interaction. Interaction consists of:

a. interactions between learner and her/his self,
b. interactions between learner and other agents (e.g., mentors, other learners and smart programs), and
c. interactions between learner and her/his environment.

2. BACKGROUND OF THOUGHT

In this research, our conceptualised scheme for interpretation (based on semantics and interaction) will be analysed in the framework of smart constructivism. In our opinion, learning in the framework of constructivism is highly concerned with the active generation of personal meaningful understandings. This is based on personal constructed meanings and over personal mental objects. More specifically, we believe that the phenomenon of ‘understanding’ could be valid (and meaningful) based on learners’ constructed meanings. In fact, this belief is the main building block of this research. This means that this chapter is specially concerned with ‘meaning construction’, ‘meaningful understanding production’ and ‘knowledge construction’ in the framework of smart constructivism. We strongly believe that there is a bi-conditional relationship between ‘understanding production’ and ‘meaning construction’ in the framework of smart constructivism. Accordingly, it shall be claimed that the phenomenon of ‘understanding’ could be valid (and meaningful) based on learners’ and mentors’ constructed meanings in the framework of constructivism and, in the context of smart learning environments.

3. CONSTRUCTIVIST LEARNING SYSTEMS: LITERATURE REVIEW

This section conceptualises the most significant and supportive characteristics of constructivist learning. In our opinion, the following items are the most fundamental. They can be shared by Constructivism and SmartNess:

a. Understanding the learner’s understanding;
b. Respecting the learner’s background knowledge;
c. Paying attention to the learner’s understanding of personal learning; and
d. Focusing on the learner’s and mentor’s reliable universal knowledge in the context of their interactions.

In fact, these items could, conceptually and epistemologically, relate the concept of ‘Constructivism’ to the phenomenon of ‘SmartNess’. This section totally focuses on the concept of ‘Constructivism’. The next section, subsequently, will focus on the actuality of the junction between constructivist learning and smart learning systems.

3.1. UNDERSTANDING THE LEARNER'S UNDERSTANDING

The most fundamental point is the concept of 'understanding'. This concept is very complicated and sensitive in psychology, neuroscience, cognitive science, cybernetics, philosophy, and epistemology. There has not been any absolute, decisive, or independent description and specification of 'understanding'. Anyhow, it shall be stressed that (i) we can potentially describe our grasp (and our conceptions) of the concept of 'understanding', e.g., (Foerster, 2003). This is relying on the fact that it is possible to support the understanding (and the realisation) of understanding within various specific areas. Furthermore, (ii) we could describe 'understanding' in order to support its representation (e.g., (Peschl and Riegler, 1999; Webb, 2009)). Finally, (iii) some descriptions could focus on specifying the components and constituents of understanding (i.e., from the perspectives of cognition and affects), see (Chaitin, 1987; Kintsch et al., 1990; Di Pellegrino et al., 1992; MacKay, 2003; Zwaan and Taylor, 2006; Uithol et al., 2011; Uithol and Paulus, 2014). We believe that the first item is the most crucial one. In addition, it shall be claimed that (ii) and (iii) could logically be subsumed under (i).

Let us focus on our own realisation of the concept of 'understanding'. Assessing from the epistemological point of view, it could be concluded that there has always been a very strong bi-conditional relationship between 'understanding something' and 'explaining something'. The dependency between understanding and explanation is considerable in analytic sciences (e.g., mathematics, physics, chemistry, biology, computer science) as well as in the humanities and social sciences. The explanation or the actual explaining of a phenomenon [and of an object, event and process] can shed light on the produced personal understanding of that thing. The relationship between understanding and explanation is bi-directional. Therefore, there is also a path from understanding to explanation. In fact, the well-understood phenomena [and objects, events, processes] could be explained more properly in order to be interpreted and realised by other agents (mentors and other learners). It is worth mentioning that there have been some descriptive models that focus on the concepts of explanatory proofs and explanatory systems along with their interrelationships with the concept of 'understanding', see (Grosholz and Breger, 2013).

In our opinion, "a human being who tackles to understand something—directly or indirectly—becomes concerned with the taxonomy of various concepts relevant for that thing and, thus, she/he needs to move toward the chain of various related concepts in order to approach to the more specified concepts", see (badie, 2015b). Additionally, she/he must be able to propose strong explanations of those related concepts. We shall, therefore, say that 'concept' and 'generality' could be interpreted as the most significant ideas that could support the structuralist account of understanding and could support understanding the concept of 'understanding'. Consequently, constructivist learning (based on a constructivist epistemology and constructivist models of knowing) is highly concerned with an individual's knowledge building processes based on her/his own produced understandings. The constructivist learning systems make the learners and the mentors concerned with the understanding of more specific concepts with regard to the special focuses on their understanding of more general concepts. In fact, the constructivist learning systems focus on developing the concept of ‘understanding of more specific concepts’.

3.2. THE IMPORTANCE OF THE LEARNER’S BACKGROUND KNOWLEDGE

Any background knowledge, by activation, becomes actualised and directed to the more-developed construction of knowledge. Living and experiencing different things are the first metaphorical teachers of all human beings. Additionally, in the context of learning environments, background knowledge could be defined as knowledge that learners have. This may come either from their previous learning environments and learning materials or from their own life experiences, see (Marzano, 2004; Fisher and Frey, 2009). Constructivist learning systems focus on knowledge building over learners’ background knowledge. In fact, through the lens of constructivism, the concept of ‘learning’ is seen as the ‘process of construction’ over personal background knowledge. Furthermore, constructivism focuses on the individual learners’ comprehensions (in the shadow of their constructed meanings) of their own objectives with regard to insights based on their background knowledge. The theory of constructivism could also focus on the individual mentors’ comprehensions of their own objectives with regard to insights based on their background knowledge and on knowledge of what will be taught.

3.3. THE LEARNER’S UNDERSTANDING OF PERSONAL LEARNING

Here, the focus is on learners’ conceptualisations and realisations of the phenomenon of ‘learning’ (e.g., (Säljö, 1979; Rossum and Schenk, 1984; Rossum and Hame, 2010)). More clearly, learners are concerned with:

i. their own conceptions of the phenomenon of ‘learning’, as well as their conceptions of their personal learnings, and
ii. the reflection of their personal learning on themselves and society.

It shall be stressed that the most significant matter in constructivist learning is transforming the phenomenon of ‘learning’ into the constructions of knowledge. In fact:

- Constructivism focuses on transformation of the phenomenon of ‘learning’ into the learners’ comprehensions of their personal constructed meanings.
- Constructivism focuses on transformation of the mentors’ comprehensions of their personal constructed meanings into the phenomenon of ‘mentoring’.

3.4. THE LEARNER’S AND MENTOR’S RELIABLE UNIVERSAL KNOWLEDGE IN THE CONTEXT OF THEIR INTERACTIONS

Constructivist learning could work as an explanatory, heuristic, and developmental framework. It must be considered that there exists a kind of reliable global and universal knowledge between constructivist learners and constructivist mentors. It is constructed and developed by both groups. For example, this knowledge evolves in learners' and mentors' action-grounded conversational exchanges, see (Pask, 1980; Boyd, 2004). According to our research in (Badie, 2016c, p.174), the produced meanings by learners and mentors support the constructions of their own worlds. Subsequently, regarding Laurillard's conversational learning framework (see (Laurillard, 1993; Laurillard, 2002)), the learners' and the mentors' constructed worlds become interacted and the learner-mentor interactions manifest themselves between their constructed worlds. The outcomes of these interactions become reflected in the learners' and the mentors' conceptual knowledge that support their reliable universal knowledge. These processes express how the constructed meanings (by learners and mentors) could be reflected in their constructed reliable universal knowledge.

4. SMART CONSTRUCTIVISM: RESEARCH PROJECT OBJECTIVES

This section, based on the identified concepts in the last section, investigates some conceptual and epistemological linkages between constructivist learning and smart learning systems. The conclusions could potentially express how educationalists and educators in smart learning environments could benefit from constructivist learning systems.

4.1. SMART CONSTRUCTIVISM: UNDERSTANDING THE LEARNER'S UNDERSTANDING

As mentioned, we believe that comprehending the learner's understanding is the most crucial conception relevant to the concept of 'understanding'. According to (Hwang, 2014), 'learning behaviour and learning pattern analysis' could be one of the most significant research issues of smart learning. It shall be taken into consideration that these outcomes are applicable to understanding learners' behaviours and learning patterns in the integrated real-world and virtual-world environments. Comprehension of learners' understandings, as the consequences, could support educationalists and educators in designing and developing more effective learning strategies. In fact, this issue is a very good example of grasping the idea of learners' understanding within smart learning environments.

In addition, we interpreted the concepts of 'concept' and 'generality' as the most significant concepts that could support the structuralist account of understanding (and understanding the concept of 'understanding'). Taking into consideration the concept of 'generality', the smart learning approaches must motivate deeper (and more complicated) levels of learners'/mentors' understandings. Accordingly,

- supporting any individual learner in producing her/his own deeper understanding of the world, and

- supporting any mentor in producing her/his own deeper understanding of the learners' understandings and of the problems of the learners

could be considered to be the most important objectives of smart learning systems. The most salient characteristic of smart constructivist learning systems is their special attention to the learners' understandings (and, respectively, to the mentors' understandings) with respect to their own produced meanings and with regard to their own generated meaningful understandings. An individual's understanding of more specific concepts could be achieved with regard to her/his understanding of more general concepts. For example, a learner's understanding of the concept of 'InductiveLogicProgramming' is absolutely dependent on and supported by her/his understanding of the concept of 'LogicProgramming'. Therefore:

i. Smart constructivist learning systems must focus on explaining more general concepts and, inductively, move toward explaining more specific concepts.

Also, similar to what (Hwang, 2014) suggests for recording the details of the students' learning behaviours, we can conclude that:

ii. Since smart constructivist learning systems respect the learners' and mentors' produced understandings of the world, these learning systems can record the individuals' understandings of the world in order to provide good opportunities for educationalists to achieve valuable understandings of the learners' understandings. Note that the educationalists, educational psychologists, and learning theorists could also achieve valuable understandings of the learners' and the mentors' understandings. Furthermore, the long-term analysis of multiple levels of the learners'/mentors' understandings can definitely support researchers in knowing more about the efficiencies and productivities of any smart educational system.

4.2. SMART CONSTRUCTIVISM: THE LEARNER'S BACKGROUND KNOWLEDGE

In the framework of smart learning, any learner must be informed about the learning program's objectives. Subsequently, she/he could be able to identify her/his personal objectives. Accordingly, she/he

- activates her/his background knowledge,
- compares her/his own objectives with the program's objectives,
- focuses on processing different kinds of information, and
- works on self regulating and organising her/his self.

We shall claim that activating the background knowledge is the most crucial process within these processes. Furthermore, referring to (Spector, 2014) and relying on constructivist theory of learning, one of the most important characteristics of an effective, efficient, and engaging smart learning environment is one that can adapt to the learner/mentor and can personalise instruction and learning support. This characteristic is highly relevant to:

i. the wide variety of learners with different levels of prior knowledge, different psychological backgrounds, and different interests, and
ii. the attitudes and policies of mentors with their background knowledge of any learning environment, their background knowledge of any learner, and their knowledge of what they are going to teach/train.

It shall be concluded that:

- Smart constructivist learning systems must respect the learners' and the mentors' background knowledge and attempt to construct, as well as develop, knowledge over their existing background knowledge. These systems do not destruct or destroy the pre-constructed knowledge of learners. Rather, they only focus on repairing, mending, and developing.
- Smart constructivism must produce and develop a kind of self-organisation process for any learner with respect to her/his own insights (based upon her/his life experiences, her/his previous learning experiences and her/his identified personal objectives).
- Smart constructivist learning systems can adapt to any learner in order to support her/his learning process by suggesting her/him the right learning strategies with regard to her/his background knowledge. The outcomes could, to a very high degree, support and advance the learners' lifelong learning.
- Smart constructivist learning systems can be adapted to the mentors (also, to the adaptive teachers and smart programs) and personalise their own instruction and teaching strategies with regard to the personalised learning environments and the conceptualised learners.

4.3. SMART CONSTRUCTIVISM: THE LEARNER'S UNDERSTANDING OF PERSONAL LEARNING

In smart learning environments, any learner transforms the phenomenon of 'learning' into 'demonstrations of understanding of what she/he is learning'. Accordingly, the learner reflects on her/his own learning strategy and promotes it over time.

Smart constructivism must consider the transformation of the phenomena of 'learning' and 'mentoring' into knowledge constructions. Smart constructivist learning systems must support learners/mentors in reflecting their own conceptions of 'what they assume they have to do as learners/mentors' on their learning/mentoring processes and, respectively, on their knowledge constructions and, consequently, on themselves and on their society.

4.4. SMART CONSTRUCTIVISM: THE LEARNER'S AND MENTOR'S RELIABLE UNIVERSAL KNOWLEDGE IN THE CONTEXT OF THEIR INTERACTIONS

Smart constructivist learning systems must aim at supporting learners and mentors in developing their universal conceptual knowledge. By taking into consideration

i. the learners' constructed worlds,
ii. the mentors' constructed worlds,
iii. the learners' conceptual knowledge, and
iv. the mentors' conceptual knowledge,

smart constructivist learning systems must support the development of their reliable universal knowledge. It shall be stressed that any learner and any mentor can try to adapt the universal conceptual knowledge to her/his own constructed world. This means there is always a bi-directional relationship between 'own constructed worlds' and 'the universal conceptual knowledge' in the form of 'reflections' and 'adaptations', respectively.

5. SMART CONSTRUCTIVISM: METHODS USED IN RESEARCH PROJECT AND THEIR OUTCOMES

5.1. LEARNERS' DEVELOPING CONCEPTIONS OF LEARNING IN SMART CONSTRUCTIVISM

This section focuses on learners' conceptions of the phenomenon of 'learning' and, subsequently, on their conceptions of the phenomenon of 'smart learning'. Note that any learner's conception(s) of the phenomenon of 'learning' play(s) a fundamental role in her/his study behaviour, see (Pratt, 1992; Rossum and Hame, 2010). Regarding behavioural and cognitive analysis of human beings' qualitative interpretations of the phenomenon of 'learning', any learner observes, interprets, and evaluates the world through the lenses of her/his own conceptions. In fact, the amalgamation of her/his mental images of the concept of 'smart learning' and her/his mental representations of the words 'smart' and 'learning' in 'smart learning' are manifested in the form of her/his conceptions. Accordingly, they are expressed in her/his actualisations and interpretations that all support her/his own understandings of smart learning. (Götzsche, 2013) provides information about the amalgamations of 'mental images' and 'linguistic expressions' regarding the philosophy of mind and language.

Note that the design and development of any smart constructivist learning system must be learner-centered, see (Coccoli et al., 2014). Considering the significant importance of learner-centered analysis of the concept of 'smart learning', and in order to propose more analytic descriptions of smart learning, we need to put ourselves into the learners' shoes and observe the phenomenon of 'learning' from their perspective. Regarding this requirement, we take into account the significant products of (Säljö, 1979; Rossum and Schenk, 1984; Rossum and Hame, 2010). The model sketches in Säljö's seminal studies on the concept of 'learning' in the learners' perspective. In more proper words, this model, qualitatively, focuses on adult learners' experiences of (and thoughts about) the phenomenon of 'learning'. This model could be interpreted as a layered model, see Figure 1. Any of this model's inner (deeper) layers is supported by its outer (shallower) ones.

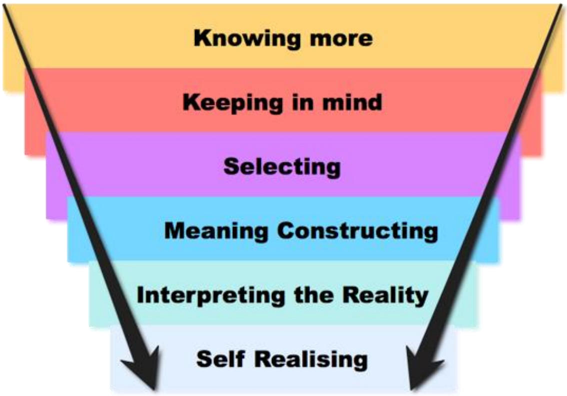

Figure 1. *Learners' Developing Conceptions of the Phenomenon of 'Learning'*

Let us describe and analyse them:

1. On the shallowest layer, the learner recognises that the phenomenon of 'learning' is equivalent to 'knowing more and knowing new things'. Such a learner is strongly dependent on her/his learning environment, learning materials, and teachers (trainers, instructors, tutors and mentors). This learner heavily needs to to be expressed, be explained, and be imparted. Furthermore, she/he needs her/his teacher/mentor to isolate and classify the flow of (as well as well-structured) information into separated and individual facts. Such a learner needs the teacher/mentor to break down the procedures into isolated facts. On this layer, the learner needs to know more (and more) isolated and realisable facts. This learner needs to attain the abilities of 'naming' and 'identifying'. Identifying multiple facts (and, respectively, primary procedures) prepare the learner for 'describing' facts and primary procedures.

2. The second layer could be identified by the concept of 'keeping in mind'. The concept of 'keeping'—indirectly—relates the learner to 'reusability' (the ability of reusing) and 'reproduction' (the ability of reproducing). In fact, she/he aims to memorise an acquired and known fact in order to apply and activate it regarding her/his own requirements and tasks. The one who attempts to keep something in mind is still trying to know more. Reusing and reproducing prepare the learner for 'describing' and 'combining' various facts [and, respectively, procedures].

3. The third layer is identified by the concept of 'selecting'. The ability of selection (and refinement) prepares the learner for pragmatism and for practical approaches.

The learner expects her/his teacher (trainer, instructor, tutor and mentor) to motivate her/him through selection processes. Selection and refinements connect the learner with 'comparing', 'contrasting', 'relating', and 'explaining'. Additionally, it—indirectly—makes connection with 'justifying' and 'analysing'.

4. The fourth layer is identified by 'meaning construction'. She/he has become concerned with interpretation, analysis, justification, primary reasoning, and primary criticising. In our opinion, this layer is the most crucial one due to the fact that it makes a linkage between learners' fundamental and their advanced conceptions of the concept of 'learning'. We shall extremely emphasise that identifying this level by 'meaning construction' is not equivalent with ignoring the fact that meaning construction is an infinite process of any learner. The fourth layer is identified by 'meaning construction', because the process of meaning construction reaches its highest point and finds its most extreme significance on this layer. This layer provides a crucial interval for signifying the phenomenon of 'learning' (by any individual learner) within her/his learning processes.

5. The fifth layer makes the learner concerned with 'interpreting the reality'. Learning as an interpretative, explanatory and expository process must be capable of supporting the learner in 'interpreting', 'explaining' and 'understanding' the reality of the world; So, she/he has become concerned with explaining the causes and reasons, criticising, formulating and theorising. On this layer, many learners characterise 'learning' as the process of self development.

6. The sixth layer is identified by 'self realisation'. The learner has become concerned with 'creation', 'generation' and 'reflection/mirroring (on her/his self and on society)'. It's very important to know that learning as the transcendental process of self realisation and self organisation is continual, successive and concatenated.

Regarding the described layers of learners' conceptions, we can realise that any shallower (outer) conception—logically, conceptually, and cognitively—supports its deeper (inner) layer. Assessed by logics, the conjunction of the outer layers is subsumed under their inner ones. For instance, the conjunction of the concepts of 'knowing more', 'keeping in mind' and 'selecting' are subsumed under 'meaning construction'. Then, through the lens of [formal] semantics, the provided logical model by 'meaning constructing' satisfy 'knowing', 'keeping in mind' and 'selecting'. Informally, the one who has become concerned with meaning construction, has already been concerned with 'knowing', 'keeping in mind' and 'selecting'. Accordingly, the succession of the layers' contents from 'knowing more' to 'self realising' could represent the flow of the concept of 'understanding' in learners' perspectives. In fact, there is a succession which could be considered as a flow of understanding regarding the expressed model. The succession could be described as:

(1) knowing new isolated facts …
(2) identifying them …

(3) keeping them in mind …
(4) describing them …
(5) reusing them …
(6) combining them …
(7) selecting them …
(8) comparing them with each other …
(9) relating them to each other …
(10) explaining them and explaining by applying them …
(11) interpreting them and interpreting by using them …
(12) analysing them and analysing other things using them …
(13) justifying for their existences and justifying by employing them …
(14) reasoning for [and based on] them …
(15) criticising for [and based on] them …
(16) theorising for [and based on] them …
(17) developing them and developing other things based upon them …
(18) reflecting on selves and on society (with regard to them).

An important question is "How could we establish a connection between a flow of understanding with regard to the learners' developing conceptions of the phenomenon of 'learning' and the phenomenon of 'smart learning'?". In other words, "how could we characterise the concept of 'understanding' with regard to the learners' conceptions of learning within smart learning environments?". In answer to these questions we shall stress that any smart learning environment should be filled with available and well-organised learning materials and should be aesthetically pleasing. Any smart learning environment must be 'effective', see (Spector, 2014; Bates, 2008; Merrill, 2013). What is likely to make a learning environment effective, efficient and engaging for a wide variety of learners with different levels of background knowledge, psychological backgrounds and interests, is one that can adapt to the learner and to personalise instruction and learning support. This suggests that appropriate adaptation is a hallmark of smart behaviour. According to (Hwang, 2014), the concept of 'smart learning environments' has been presented as one "… that make(s) adaptations and provide appropriate support (e.g., guidance, feedback, hints or tools) in the right places and at the right time based on 'individual learners' needs, which might be determined via analysing their learning behaviours, performance and the online and real-world contexts in which they are situated. …". Furthermore, (Hwang, 2014) states that a smart learning environment is able to offer adaptive support to learners by immediate analyses of the "needs of individual learners from different perspectives". It shall be taken into consideration that any smart learning environment meets the personal factors (e.g., learning styles and preferences) and learning status (e.g., learning performance) of individual learners. In fact, all individual learners and their needs are the most central components and incorporators of smart learning environments. It is worth mentioning that IBM (see (IBM)) has also recognised smart educations as student-centric education.

Taking all these characteristics of smart learning into consideration, any smart learning system, as a student-centric system, must prepare a background for the learners' flow of understanding and support them within different aspects of their understandings. Also, as mentioned earlier, the most central focus of constructivist smart learning systems is on learners' understandings with regard to their own produced meanings and their generated meaningful comprehensions. At this point, we shall state that smart constructivist learning systems must be developed over the individual learners' conceptions and requirements. These developments must be supported by the special focus on the flow of understanding of learners.

Let us take into consideration some significant results of our discussions with undergraduate students. A number of students wanted to know which facts would be required and helpful for them? We can transform this requirement into (i) 'How could a learner know the required and helpful facts?'.

Also, a few students told us that they know that they need to select some facts in order to conceptualise them and to have a better understanding of them, but they don't know which facts must be selected. Again, we can transform this requirement of learners into (ii) 'How could a learner find the ability to select the rightful and beneficiary facts in order to construct meaning over them?'.

Also, a student wanted to know how she could let her mentor know about her constructed meanings? (Actually she wanted to know how she could express her meanings). This question could be translated into (iii) 'How could a learner announce her/his constructed meanings' to her/his mentor or to other learners?'.

Questions such as (i), (ii), and (iii) are prevalent to any learner. Smart constructivist learning systems must be able to provide a kind of requirement analysis and to suggest rightful choices to individual learners. This could be actualised by standing beside the learners. In fact, in the beginning, the learning system, the learner (and the mentor) should not look at each other, but should look at the same point and discover the appropriate facts together. Accordingly, the conceptions of the mentor could influence the learner and vice-versa. Furthermore, the learner's and the mentor's conceptions could be influenced (and modified) with regard to what the system has suggested to them. Smart constructivism must be capable of locating the learner in her/his best position to go toward her/his production of meaningful comprehension. Respectively, the mentor must be guided to find her/his most appropriate position in relation to the learners' positions.

In order to express and analyse the concepts of 'meaning', 'meaning construction', and 'meaningful comprehension', our theoretical model needs to be supported by a proper educational and pedagogical model. This can provide an organised framework for representing different levels of learners' understandings. We need to employ a model of learning concerned with various complexities of understanding at its different levels/layers in order to support the conceptualised idea of 'understanding', to analyse the flow of understanding (in experts'/educationalists' points of view), and to model it in smart constructivist learning systems.

5.2. SMART CONSTRUCTIVISM AND THE STRUCTURE OF OBSERVED LEARNING OUTCOMES

The Structure of Observed Learning Outcomes (SOLO) taxonomy is a proper model that represents multiple layers of learners' understandings within learning and knowledge acquisition processes, see (Biggs and Collis, 2014). SOLO provides an organised framework for representing different levels of learners' comprehensions. It is concerned with various complexities of understanding at its different layers. In the framework of SOLO, learners are concerned with five levels of understanding, see Figure 2.

As an analytic example, we focus on a learner, Martin, who is learning Java Programming:

- Pre-structured knowledge: Martin does not really have any knowledge about Java. This kind of knowledge about Java has been constructed over his mental backgrounds and from his previous experiences, e.g., experiencing different products that are developed in Java, meeting Java's official and related websites, discussing with Java programmers. The most important fact is that Martin does not have any special constructed knowledge about Java.

- Uni-structured knowledge: Martin has a limited knowledge about Java and may know few isolated facts. Thus, he mainly focuses on identifying those isolated facts. For example, he knows that Java works based on classes (of objects) and that Java is an object-oriented language. He may know that Java derives its syntax from C. Based on this, Martin has a very shallow understanding of Java. The known facts are isolated and he is not able to either relate them together or apply them.

- Multi-structured knowledge: Progressing from the previous level to this level simply means that Martin knows a few facts about Java, but he is still unable to find logical and conceptual linkages between them. Martin (i) has extended the domains of his factual knowledge about the isolated facts, (ii) has become concerned with combinations of various isolated facts (but not on relating them), and (iii) has become concerned with descriptions of the results of those combinations. For example, he knows about object-oriented languages, he knows that object-oriented programming is a paradigm based on the concept of 'objects' and 'things', and he knows that object-oriented programming languages focus on 'objects' rather than 'subjects' and 'actions'. Martin has produced some mental combinations of these facts. He is preparing himself for producing logical and relational models based on his produced combined facts.

- Related Knowledge: Martin has started to move towards higher levels of conception about Java. He has also begun moving towards deeper levels of understanding of Java. At this level, he is able to link different facts together and to explain several conceptions of Java. The important fact is that he has become concerned with analysis, argumentation, explanation, justification, comparison, and applications relevant to Java. Now, Martin can explain and analyse the elements of the set of his factual knowledge and can relate them together. Now, he is able to relate the characteristics of object-oriented systems and Java programming. He knows why object-oriented paradigms are in favour of 'objects' and not in favour of other phenomena. He is able to explain and analyse the characteristics of Java as well as apply different methods to them.

- Extended Abstract: This layer is the deepest and the most complicated level of Martin's understanding. Here, Martin is not only able to link a huge number of related conceptions together, but he can also link them to other specified and complicated conceptions. Now, he is able to link multiple explanations and justifications in order to produce more complicated extensions relevant to Java. Martin has become concerned with theorising, hypothesising, creating, and criticising.

According to Figure 2, the extended abstracts are the products of deeper realisations and understandings of relational structures (and constructed related knowledge). Relational structures are the products of deeper comprehensions of multi-structures (and constructed multi-structured knowledge). In a similar manner, the multi-structures are the products of deeper comprehensions of uni-structures (and constructed uni-structured knowledge). Finally, the uni-structures are the products of deeper pre-structures (and pre-structured background knowledge).

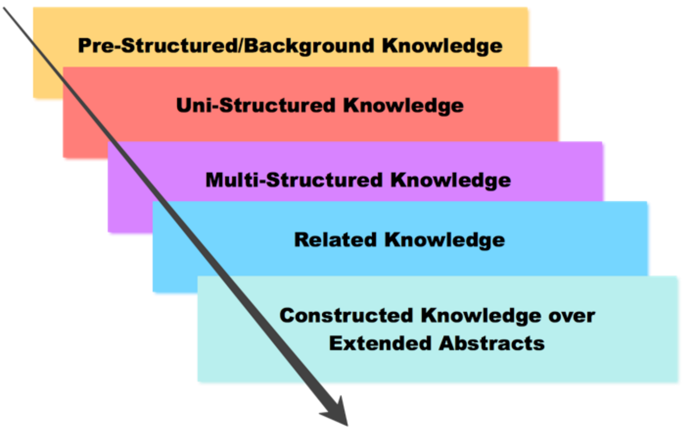

Figure 2. *SOLO Taxonomy: Levels of Constructed Knowledge and Levels of Produced Understanding.*

At this point, we need to focus on the HowNess of satisfaction of the flow of understanding from 'pre-structured and background knowledge' to 'constructed knowledge over extended abstracts' by smart learning development and design. Smart constructivist learning systems must be able to support the development of knowledge constructions over any learner's background and pre-structured knowledge. The central idea is that smart constructivism must generate a kind of self-updating process for any learner (with respect to her/his own insights based on her/his background knowledge) in order to prepare her/him for her/his individual processes of semantic interpretation and meaning construction and, subsequently, for understanding production. Let us be more specific on the concepts of 'semantic interpretation' and 'meaning construction'.

As characterised earlier, one of the most significant features of smart constructivist learning systems is their special focus on the learners' understandings with regard to their own produced meanings and generated meaningful comprehensions. In addition, we have mentioned that there is a bi-conditional relationship between 'understanding production' and 'meaning construction'. Therefore, we shall stress that the following items have a logical bi-conditional relationship:

- The process of knowledge construction as "pre-structured knowledge ⟶ uni-structured knowledge ⟶ multi-structured knowledge ⟶ related knowledge ⟶ knowledge over extended abstracts".

- The learners' meaning construction.

At this point, we employ a linguistic approach to explain and analyse this bi-conditional relationship. This approach, in dynamic semantics, has considered meaning as a context-update function, see about this feature in dynamic semantics in (Chierchia, 2009; Gabbay et al., 2010). You can also find one of its particular applications in (Larsson, 2012). Considering meaning as a context-update function, the input of the Meaning function is a context and the output is its updated form. Any context comprises different types and different numbers of conceptions. Terminologically, we can consider conceptions as the sub-class of contexts. Therefore, we describe any 'meaning' as a conception-update function like

$$\textit{Meaning}: \textit{Conception} \longrightarrow \textit{Conception}'.$$

This function iteratively organises itself in multiple loops and repetitions. It shall be claimed that the constructed meanings of any learner, based on her/his constructed knowledge over extended abstracts, are the updated forms of her/his constructed meanings within relational structures. Also, the constructed meanings in the ground of her/his related knowledge (on mental relational structures) are the products of her/his constructed meanings based on her/his multi-structured knowledge (on mental multi-structures). In a similar manner, the constructed meanings based on multi-structured knowledge (and on mental multi-structures) are the updated products of the constructed meanings based on uni-structured knowledge (on mental uni-structures). Finally, the constructed meanings on uni-structured knowledge (on mental uni-structures) are the updated constructed meanings over mental pre-structures and pre-conceptions.

When it comes to semantic interpretations, our approach recognises the learner's semantic interpretation as the connector of her/his various levels of constructed meanings, see 'interpretation' in (Simpson and Weiner, 1989). In other words, the interpretations, semantically, support the succession of the [updated] meanings. Relying on this conception, an interpretation could be known as the continually adjusted relationship between two things. It is quite important to consider the following when it comes to smart constructivism:

1. The learner's intention behind her/his conceptions.
2. The learner's actual mental universe of her/his conceptions, which are based on her/his accumulated experiences.

As concluded earlier, smart constructivism must consider the fact that any individual learner—by doing learning—transforms 'what she/he is learning' into "uni-structures of knowledge, multi-structures of knowledge, related structures of knowledge, and constructed knowledge over extended abstracts". In fact, any learner, based on her/his tasks and roles as a learner, increases the complexities of her/his constructed meanings in order to be closer to her/his own deepest understanding. Smart constructivist learning systems must be capable of supporting learners in reflecting their own multiple conceptions of a phenomenon (and object, process, event) and in mirroring the concatenation of their produced conceptions on their own learning as well as on different levels of their constructed knowledge.

6. KNOWLEDGE IN SMART CONSTRUCTIVIST LEARNING SYSTEMS: ANALYSIS OF METHODS' OUTCOMES

Relying on the framework of constructivism, the current theoretical analysis of smart learning is not focusing on ontologies or the existence of knowledge. The central focus, though, is on the tenets of humans' knowledge construction and development. This involves the creation of mental models when encountering new, unusual, or otherwise, unexplained experiences, see (Spector, 2014). We have taken into account that learners create their own mental representations in order to make sense of their experiences and learning tasks. By interpreting the phenomenon of 'learning' as the process of knowledge construction, we need to put any individual learner at the center of the proceeding of knowledge construction. The personal characteristics of any learner, the mental backgrounds, personal experiences, and the pre-structured and uni-structured knowledge support the foundations of knowledge construction. This section deals with how multiple categories of knowledge can be assumed to be constructed in the framework of smart constructivism.

6.1. CATEGORIES OF KNOWLEDGE IN SMART CONSTRUCTIVISM

We adopt Bloom's taxonomy in order to clarify what we mean by 'categories of knowledge'. Bloom's taxonomy is a framework for classifying educational and pedagogical objectives, which could be interpreted as the statements of what educators and educationalists expect the learners to have dealt with, see (Bloom, 1956;

Krathwohl, 2002). Considering Bloom's taxonomy and taking into account the constructivist theory of learning, we could express the view that the concept of 'knowledge' has a strong relationship with 'recognition' of multiple phenomena (as well as objects, processes, and events). In fact, knowledge construction is supported by any individual's insights, based on her/his own recognition of various materials, methods, procedures, processes, structures, and settings [in the form of her/his conceptions]. According to (Cambridge Dictionary, 2017), having knowledge about something or about some phenomenon could be realised as being related to the following items:

(i) Having a piece of knowledge about that thing/phenomenon, and
(ii) judging about that thing/phenomenon based on personal experiences and information.

We shall claim that we are allowed to divide knowledge into separated classes (for example, into $Class_1$, $Class_2$, …, $Class_n$) if and only if we have aimed at clarifying and specifying the humans' conceptions of any of them (e.g., $Class_i$) and, respectively, of all of those separated classes (i.e., $Class_1$, $Class_2$, …, $Class_n$). In the end, we must consider the union of all classes as the phenomenon of 'knowledge'. Let us focus on analysing how Bloom has dealt with the phenomenon of 'knowledge'. Bloom's taxonomy categorises knowledge into multiple classes, e.g., distinct classes for knowledge of terminologies, knowledge of ways and means, knowledge of trends and sequences, knowledge of classifications and categorisations, knowledge of criteria, knowledge of methodologies, knowledge of quantifications, knowledge of principles, knowledge of generalisations and specifications, and knowledge of theories and structures. Since then, (Krathwohl, 2002) has proposed a knowledge dimension in the revised version of Bloom's taxonomy. The revised taxonomy consists of (i) factual knowledge (e.g., terminological knowledge), (ii) conceptual knowledge (e.g., knowledge of theories, models and structures), procedural knowledge (e.g., knowledge of methods and algorithms), and (iv) meta-cognitive knowledge (e.g., contextual knowledge, conditional knowledge).

We strongly believe that these four classes could support us in clarifying and analysing the interconnections between the phenomena of 'learning (and knowledge acquisition)' and 'knowledge building'. We shall, therefore, claim that the phenomenon of 'learning' consists of a sort of transformations from constructed knowledge in the world (e.g., by experts, by theoreticians) into the sets of 'facts', 'procedures', and 'concepts' in different 'contexts'. We believe that procedures are constructed over the chain of separated, connected, and related facts. In our opinion, any procedure is just the concatenation of a number of facts. Therefore, learning provides multiple functions from constructed knowledge into 'facts' and 'concepts'. Learners need to deal with those facts and concepts and, subsequently, they need to construct their own knowledge with their insights based on what they construct over those facts and concepts. In (Badie, 2017b), we have argued as following: "… In our opinion, there is a concept behind every fact. Then any factual knowledge can be supported by a conceptual knowledge. For instance, according to a fundamental characteristic of terminological knowledge (as a type of factual knowledge), we can represent terminologies by means of taxonomies. A taxonomy could be constructed based upon concepts. Then a terminological knowledge has been supported by a conceptual knowledge. Also, as another instance, we can define a body of the related elements and interpret it as a set of constructors for denoting various concepts and their interrelationships. That's how the concept languages and descriptive languages appear. Then, we could be able to represent knowledge over concepts, their instances and their relationships …". Thus, we shall claim that everything is translatable into (and [mentally] representable in the form of) a concept. Accordingly, concepts are manifested in the learners' conceptions and, respectively, they could be declared in learners' hypotheses. A concept might be interpreted to be a linkage or interconnection between the mental representations of linguistic expressions and the other mental images (e.g., representations of the world, representations of inner experiences) that a learner has in her/his mind, see (Götzsche, 2013).

It shall be concluded that the phenomenon of 'smart learning' must provide multiple transformations from 'knowledge', either 'received from outside' or 'experienced within inside', into concepts. Learners represent those concepts in their minds and, subsequently, propose their own conceptions of those concepts. Consequently, learners construct their own knowledge with insights based on their produced conceptions. It is a fact that learners' conceptions could elucidate others (other learners and mentors) and could be shared with them through internet, social networks, virtual classes and media. Learners can propose/announce their own conceptions of what they have constructed in the form of texts, voices, videos, etc. The collection of these processes could be identified by 'construction of own packages of knowledge by learners' in smart constructivist learning systems.

6.2. A CONCEPTUAL FRAMEWORK FOR KNOWLEDGE BUILDING IN SMART CONSTRUCTIVISM

The main objective of this section is to propose a conceptual framework for representing the stream of understanding within knowledge construction processes in smart constructivist learning systems. First, we shall refer the readers to our research in (Badie, 2016b), which is dealt with formal semantic analysis of interrelationships between multiple categories in learners' developing conceptions of the phenomenon of 'learning' (as modelled in (Säljö, 1979; Rossum and Schenk, 1984; Rossum and Hame, 2010)). We need to employ the results of that research. More particularly, that research has focused on conceptualisation of the phenomenon of 'learning' within the top-ontology of adult learners' developing conceptions of learning. Self realisation (and self awareness) is the most excellent conception of learners. It can conclude all other conceptions within its lower categories. Assessed by logics, all conceptions of learners (within lower categories of conceptions) are subsumed under 'self realisation'. Relying on Description Logics, (Badie, 2016b) has focused on discovering the main constructive concepts and their interrelationships under 'self awareness'. Subsequently, a semantic representation of adult learners' developing conceptions has been sketched out. Figure 3 represents a network that has been developed over an important piece of the proposed semantic representation in (Badie, 2016b). Figure 3 is sketched out for structural analysis of 'smart learning' on the highest conceptual level and from the perspective of the most excellent learning conceptions; this semantic representation is meaningful in the context of 'conceptualisations'.

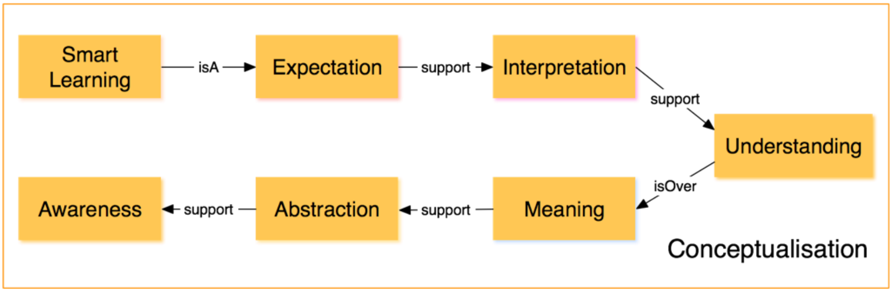

Figure 3. *A Semantic Representation of Concept of 'Understanding' in Smart Learning Environments*

This network shows that the concept of 'smart learning' is a kind of expectation. In some cases, it is an 'outlook'. Smart learning as an expectation, supports learners' interpretations and understandings of the world. In fact, this expectation—relying on individuals' interpretations—produces a strong belief that the phenomenon of 'smart learning' will be valid and meaningful. Furthermore, humans' interpretations support their personal understandings; it is possible to say that any personal understanding is a kind of limited interpretation in the context of conceptualisations. Learners—relying on their conceptualisations and by engaging their interpretations—explicate what they mean by classifying a thing, process, event, or phenomenon as an instance of a concept. The interpretations prepare learners for producing their personal meaningful (and understandable) descriptions over their own conceptions, and, in fact, over their constructed concepts. Therefore, an 'understating' could be realised to be the sub-process of an 'interpretation'.

On the other hand, though, all interpretations are not necessarily understandings. In fact, all the interpreted concepts may not be understood, but all the understood concepts certainly have been interpreted, see our research in (Badie, 2016d). Then, understanding, in the framework of smart constructivism, is produced over 'interpretations' of things, processes, events, and phenomena as well as within smart learning environments. Additionally, as analysed, understanding could be considered as constructed over individuals' constructed meanings. Meanings on the deepest layers of understanding, as well as on highest floor of the constructed knowledge, support 'abstractions' and 'production of knowledge over the extended abstracts' by learners. These abstractions support individual meaningful comprehensions over individual constructed meanings. Figure 4 is structurally and conceptually supported by Figure 3. Figure 4 represents a conceptual framework for 'knowledge creation' over the stream of learners' understandings within smart constructivist learning systems. It represents a conceptual description of 'knowledge building' toward 'deepest understanding levels of learners' within smart learning environments.

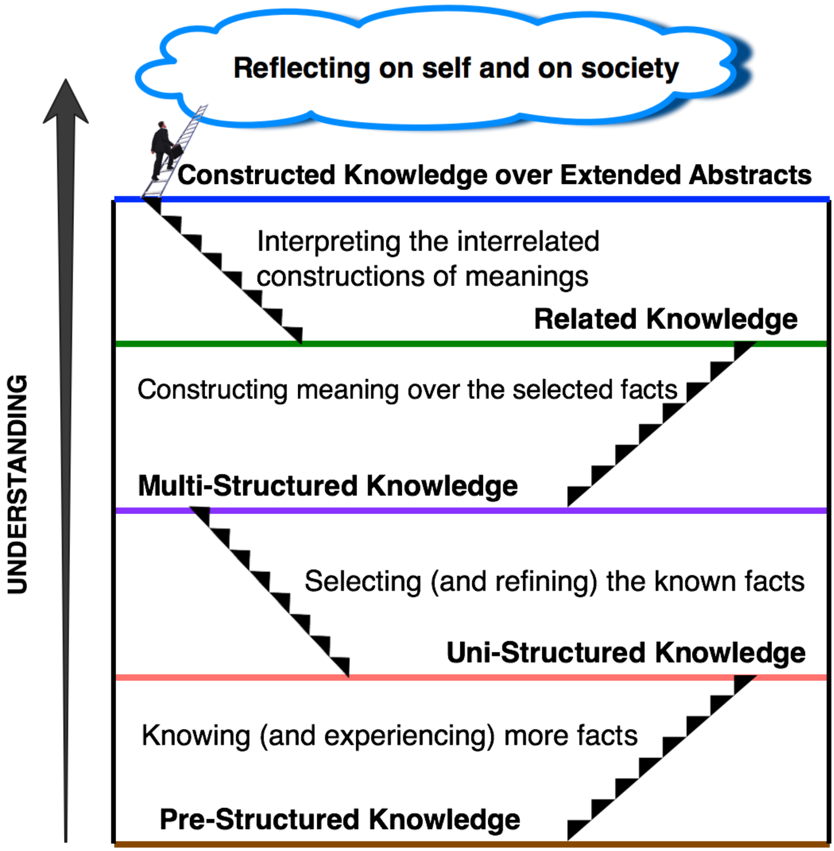

Figure 4. *A Conceptual Framework for Knowledge Building within Smart Constructivist Learning Systems*

7. CONCEPTUALISING A SMART CONSTRUCTIVIST PEDAGOGY: TESTING OF RESEARCH OUTCOMES

This section employs the outcomes of (Glasersfeld, 1995; Wasson, 1996; Fosnot, 1996; Boethel and Dimock, 2000; Fox, 2001; Maclellan and Soden, 2004; Yilmez, 2008) in order to conceptualise a smart constructivist pedagogy based on the proposed model of knowledge building. According to Figure 3, the phenomenon of 'smart learning' in the framework of constructivism is an expectation that is supported by any individual's interpretations and meaningful understandings. Consequently, both learners and mentors are interpreters, organisers, and constructors within the process of smart constructivist learning and in the context of smart learning environments. In fact, they are the developers of a collaborative process of constructing. Therefore, it shall be emphasised that smart constructivism doesn't assess the phenomenon of 'learning' as an outcome of a development. It does, however, recognise it as a development. Here, learners are inventors. They must be allowed to generate their hypotheses based on their own conceptions of the world. The main characteristics of the conceptions are as follows:

1. Conceptions are learner-centered (individual-centered).
2. Conceptions are central-organising.
3. Conceptions are generalised across experiences and direct observations.
4. Conceptions require re-organisable pre-conceptions.
5. Conceptions make sense to communities by becoming shared.

In the framework of smart constructivist pedagogy, learners must have opportunities to announce their pre-conceptions, their presuppositions, their hypotheses based on their presuppositions, and their possible suggestions over them. Learners, as constructors of meanings, need to organise their experiences and, correspondingly, generalise and specialise the experiences into their personal hypotheses. Furthermore, mentors, adaptive teachers, and smart programs must be able to:

i. work on conceptual and logical analysis of learners' hypotheses,
ii. check the validity and definability of learners' hypotheses, and
iii. find reasonable descriptions and specifications for denying and refusing the learners' hypotheses.

The third item could be done deductively (based on rules) or inductively (based on different cases of study). In other words, learners' hypotheses must be illuminated and explored in order to be disclaimed. Any kind of error, mistake, or inaccuracy would be assessed as an outcome of learners' misconceptions. The learners' misconceptions could be found and organised. Thus, their mistakes would—evidentially—be explored for themselves. Note that counterexamples are quite efficient in resolving learners' misconceptions and errors. It shall be concluded that smart constructivist mentoring focuses on:

i. discovering conceptions/misconceptions of any individual learner,
ii. discovering the common conceptions/misconceptions among a group of learners,
iii. conceptualising learners' conceptions/misconceptions,
iv. conceptualising and attempting to understand learners' understandings over their conceptions/misconceptions, and
v. motivating proper conceptions and resolving misconceptions.

It shall be stressed that smart constructivism could consider 'improvable and re-organisable conceptions of learners' as the main building blocks of its knowledge building pedagogy. (Caswell and Bielaczyc, 2001) is in line with the conceptualised theory and has had a special focus on the learners' productive use of the principle of improving their conceptions within their relationships with their 'constructed knowledge'. At this point, we shall conclude that the presented conceptualisation of knowledge building has had a special attention to 're-organisable conceptions of learners within their connections with their collaborative constructed knowledge'. Table 1 is presented in order to itemise the most important components of Smart Constructivist Pedagogy and its significant characteristics. Later on, Figure 5 schemes the conceptual interrelationships between those components and the phenomena of 'knowledge' and 'conception'.

Table 1. *Main Components of Smart Constructivist Pedagogy*

Components	Characteristics
Smart Constructivist Learning	• The phenomenon of 'learning' in the framework of smart constructivism is interpreted as a process of knowledge construction. The constructed knowledge is idiosyncratic. • Smart constructivist learning is strongly concerned with self-regulation, auto-organisation, self-development and, finally, with self-learning. • Learning in the framework of smart constructivism is an active and dynamic (not passive) process. • In the framework of smart constructivism, the constructed knowledge [by any individual learner] is not innate, passively absorbed, or invented, but it is 'constructed' and developable. • In the framework of smart constructivism, learners interpret their world and, correspondingly, construct their own versions of the world based on their personal conceptions. • The most significant objectives of smart constructivist learning are 'meaning construction' and 'meaningful understanding production'. • Smart learning in the framework of constructivism proceeds toward developing constructed structures. • Experiences and prior understandings of learners play fundamental roles in smart constructivist learning. • Smart constructivist learning encourages and motivates any individual learner to explore and discover the world by her(him)self. • Smart constructivist learning encourages any individual to make her/his own sense of the world. • In the framework of smart constructivism, the phenomenon of 'learning' is situated in the context in which it occurs. • Smart constructivist learning is strongly supported by social interactions and conversational exchanges.

Smart Constructivist Mentoring (by Human Beings, Adaptive Mentors, Smart Programs)	• The phenomenon of 'mentoring' in the framework of smart constructivism is a process of knowledge construction. • Mentoring in the framework of smart constructivism is an active and dynamic (not passive) process. • Smart constructivist mentoring conceptualises learners' conceptions of the world. • In the framework of smart constructivism, the constructed knowledge by mentors is not innate, passively absorbed, or invented, but it is 'constructed' and developed by the mentor with regard to the learners' opinions, actions, transactions, questions, and answers. • In smart constructivist learning systems, the mentor is an expert and advanced learner and has a special respect for learners' choices. • In smart constructivist learning systems, the mentor is an organiser around significant conceptions that could motivate learners. • In smart constructivist learning systems, the mentor must get to know about any individual learner and her/his backgrounds. • In smart constructivist learning systems, the mentor assists learners and links them with their background knowledge. • In smart constructivist learning systems, the mentor mainly focuses on (i) constructing meanings for her(him)self, (ii) giving feedbacks to learners with regard to their constructed meanings, (iii) developing meaningful understandings for her(him)self. • In the framework of constructivism, smart mentoring conceptualises learners' understandings based on their conceptions of the world. • In the framework of constructivism, smart mentoring builds a world of developed understandings. • In the framework of constructivism, smart mentoring proceeds toward developing constructed knowledge structures. • In the framework of smart constructivism, any learner must be driven (by her/his mentor) to understand the world and to change her/his understanding with regard to her/his misconceptions. In fact, smart mentoring discovers/recognises learners' misconceptions, mistakes, and errors. • In the framework of constructivism, smart mentoring focuses on making senses. It's highly affected by the learners' senses of the world. • In the framework of constructivism, smart mentoring is situated in the context in which the phenomenon of 'smart learning' occurs. • In the framework of constructivism, smart mentoring is strongly supported by social interactions and conversational exchanges. • In the framework of constructivism, an effective smart mentoring aims at presenting open-ended identifiable, describable, specifiable, justifiable, and analysable problems to learners.

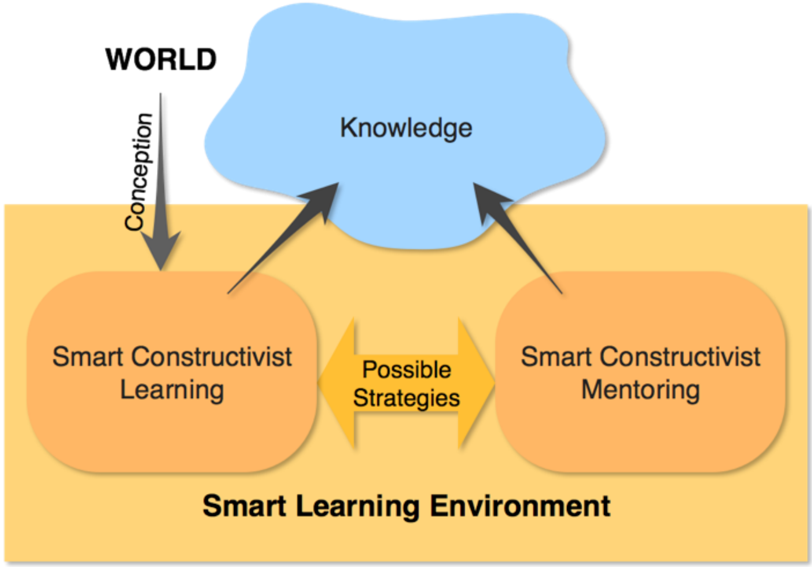

Figure 5. *The Conceptual Relationship between Main Components of Smart Constructivist Pedagogy*

8. LINKING SMART CONSTRUCTIVISM AND COLLABORATIVE LEARNING STRATEGY: VERIFICATION OF RESEARCH OUTCOMES

This section picks up the collaborative learning strategy (that is highly relevant to smart education) in order to focus on explaining its possible connections with 'the smart constructivist model of learning'. This section describes why 'collaborative learning strategy' could cope with and could be furnished by the presented and conceptualised approach.

The central focus of this research has been on knowledge building. This means we need to take into consideration the phenomenon of 'knowledge building' in order to check the validity and reliability of the constructivist model of learning in junction with 'collaborative learning strategy' and smart learning environments. First, it seems useful to take a look at Popperian epistemology (Popper, 1972) in order to work on conceptual analysis of knowledge building in smart learning. More specifically, the concept of 'knowledge building' could derive from an epistemology that treats conceptions of human beings as entities in their own right that can have properties, connections, and potentialities. Consequently, it's quite important to be concerned with the concepts of 'pervasive knowledge building' and 'knowledge of community'. In fact, we need to focus on the fact that in collaborative learning, or 'Learning-and-Constructing-Together', the constructed knowledge must be capable of becoming spread widely throughout a group of learners. In the context of collaborative learning, any individual learner constructs her/his own knowledge. This means she/he attempts to construct the universal knowledge and also develop the construction of the knowledge of her/his community. This section

- relies on (Baker et al., 2013) and its conceptual analysis of the phenomenon of ‘togetherness’ in learning environments,
- follows the analysed policies of (Scardamalia et al., 1994; Bereiter and Scardamalia, 2012; Scardamalia and Bereiter, 2014), and
- uses the methodological notions of (Sorensen, 2005),

to focus on conceptualisation of ‘Learning-and-Constructing-Together’ while it’s concerned with knowledge building within junctions of ‘smart constructivism’ and ‘collaborative learning’.

8.1. ESSENTIAL VALUE 1: THE STATE OF KNOWLEDGE

Creative knowledge work could be interpreted as a work that advances the state of knowledge of a community. The ‘state of knowledge’ is an emergent collective phenomenon and might be interpreted for a group of people. According to the concept of ‘state of knowledge’, knowledge building pedagogy is supported by the premise that authentic creative knowledge work can take place in any learning environment and, respectively, in any smart learning environment. The state of knowledge of a group of learners within a smart learning environment only indirectly reflects the knowledge of individual learners. This conclusion could be implicated by smart constructivism. In fact, relying on smart constructivism, the state of knowledge of an individual learner, based on her/his constructed meanings, could highly reflect the knowledge of the community. Also, inversely, the state of knowledge of the community could—only indirectly (and not directly)—reflect the knowledge of the individual learners. Then, learners could re-organise and update their constructed meanings. Therefore, it is reasonable to expect that individuals’ achievements go along with developments and advancements of community knowledge. This conclusion seems to be in parallel with the proposed approach of Zhang and colleagues in (Zhang et al., 2009). This characteristic, based on the state of knowledge, could highly affect course-by-course, program-by-program, and semester-by-semester changes in plans and strategies of any smart learning environment. Note that the mentor, adaptive mentor, or the smart program, is another member of any learning community and, therefore, her/his/its constructed meanings reflect the knowledge of the community. In addition, it shall be considered that the mentor’s knowledge is, regarding the feedbacks and transactions of learners, developable.

8.2. ESSENTIAL VALUE 2: THE PHENOMENON OF ‘DISCOURSE’

According to (Scardamalia and Bereiter, 2014), discourse could come from sharing knowledge and subjecting conceptions to criticism. For example, in online meetings, web conferences, webinars, and Massive Open Online Courses (MOOC), any individual learner could become concerned with a kind of discourse which could be interpreted as ‘a filter that determines what could be accepted into the canon of justified beliefs’, see (Latour and Woolgar, 1979). However, it could be argued that modern learning strategies must support any individual learner and, also, any individual mentor, in playing her/his own creative roles in order

i. to improve her/his own conceptions, and
ii. to judge and to make decisions more rationally beside her/his manners of criticism.

We shall claim that this kind of discourse-based judgement and decision making is the consequence of any individual’s and, consequently, of a community’s construction of factual [and, respectively, conceptual] knowledge. It can be labelled as ‘Social Constructivism in the Framework of Smart Constructivism’. Relying on practical and empirical approaches, this kind of social constructivism would be more concerned with shared goals of advancing understanding beyond what is currently interpreted and understood. In fact, the practices could support the processes of meaning construction. Consequently, the produced social meanings, in the context of interactions and conversational exchanges between individuals within a smart learning environment, could be updated and be more-organised.

8.3. ESSENTIAL VALUE 3: AUTHORITATIVE INFORMATION AND THEIR RELIABILITY

Smart constructivism in collaborative learning supports learners in:

i. using their own authoritative information that are achieved based on their own experiences, explorations, studies, etc. and
ii. bringing other authoritative information (e.g., from other individuals, from e-books and e-references, from learning applications) as evidences of their own authoritative information.

The latter supports the development and re-organisation of all individuals’ constructions based on received authoritative information from others within their social interactions. It shall be claimed that the interconnections between (i) and (ii) elaborates the ‘state of knowledge of community’ in the long term. Accordingly, the interrelationships between (a) and (b) increases the state of knowledge of the community:

a. a learner’s constructions based on her/his own authoritative information.
b. a learner’s development of her/his constructions with regard to others’ authoritative information.

8.4. ESSENTIAL VALUE 4: EXPLANATION AND UNDERSTANDING

The Organisation for Economic Co-operation and Development (OECD) has emphasised the importance of conceptual understanding as a basis for creative knowledge work of all kinds: “Educated workers need a conceptual understanding of complex concepts, and the ability to work with them creatively to generate new ideas, new theories, new products, and new knowledge”, see (OECD, 2008, p. 1). It might be assumed that any individual learner has to understand appropriately in order to develop her/his own knowledge constructions. Similarly, as discussed earlier, learners’ understandings are strongly supported by explanations. Accordingly, it must be stressed that the development of knowledge building in smart learning societies is highly related to the phenomena of ‘explanation’ and ‘understanding’.

Smart constructivism, as a theory of learning, must support conceptual understanding of learners in different communities and organisations. Special attention must be given to guiding, instructing, and mentoring any individual learner. Any learner, in such a framework, must be guided in order to construct her/his own meanings and to support her/his society with her/his constructed meanings. In addition, the smart constructivist theory of learning within collaborative strategies focuses on developing the communities’ understandings. In our opinion, a proper strategy must follow the conceptual framework presented in Figure 4. Besides them, smart constructivism must focus on developing ‘knowing HowNess combined with knowing WhyNess’ as ‘explanatorily coherent practical knowledge’. A similar principle for practical knowledge has been analysed in (Bereiter, 2014).

9. SMART CONSTRUCTIVIST LEARNING COMMUNITIES: VALIDATION OF RESEARCH OUTCOMES

According to (Adamko et al., 2014), smart learning communities must be sensible, connectable, accessible, ubiquitous, sociable, sharable, and visible/augmented. We shall claim that our research has interconnections with the features of ‘being connectable’, ‘accessibility’, ‘being sharable’, and ‘visibility’.

At this point, we shall draw your attention to Vygotsky’s theory of social constructivism, see (Vygotsky, 1978a; 1978b; 1987). We shall claim that Vygotsky’s ideas are quite helpful in conceptualising smart constructivist learning communities. Vygotsky’s theory, based on his ideas in human cultural and bio-social development, has supported the development of social constructivism. Vygotsky believed that ‘social interaction’ plays a fundamental role in the process of humans’ cognitive development. In his opinion, an individual who has stronger understandings and higher abilities in particular domains could be a so-called ‘teacher’. He specified the concept of ‘teacher’ by defining the notion as an MKO (i.e., More Knowledgable Other). Additionally, Vygotsky defined ZPD (i.e., the Zone of Proximal Development) in order to express the concept of ‘learning’ by an individual learner under MKO’s supervisions and/or in her/his collaborations with other individuals. Vygotsky believed that learners could learn (could do ‘learning’) in this zone. It shall, therefore, be concluded that we can have a similar conception of smart learning communities. In fact:

i. A mentor, an adaptive mentor, or a smart program, as a more knowledgable other and as an individual who has stronger understandings and higher abilities in particular domains, supervises learners.

ii. Learners have interactions and conversational exchanges with each other and develop their personal constructions of knowledge.

iii. The phenomenon of ‘smart learning’ occurs over actions, transactions, questions, and answers between any learner and mentor as well as between any learner and other learners.

9.1. CONCEPTUALISING SMART CONSTRUCTIVIST LEARNING COMMUNITIES

The Fundamental Characteristics of Smart Constructivist Learning Communities are as follows:

- Smart learning communities are communities using a discourse engaged in activity, reflection, interaction, and conversation.

- The main goals of any smart learning community are (i) Learning-and-Constructing-Together and (ii) producing the Collaborative Understanding.

- The main belief of any smart learning community is that the phenomena of ‘smart learning’ and ‘development’ are integrally tied to any individual’s communicative and social interactions with other individuals.

- The second important belief of smart learning communities is that the use of information technologies (IT) and information communication technologies (ICT) is more likely to make a constructivist perspective towards the phenomenon of ‘smart learning’.

- Smart learning communities must be given senses by (i) learners’ made senses of the world based on their own experiences, explorations, and discovered key concepts and by (ii) their shared conceptions of the world.

- In the context of smart learning communities, any individual learner must be permitted to express, explain, defend, prove, and justify her/his conceptions of the world. Subsequently, all learners must be allowed to communicate their conceptions to each other as well as to their smart learning community.

- Smart learning communities must involve instructed interactions that guide any individual learner to recognise and resolve her/his conceptual inconsistencies and to modify conceptions through her/his interactions and conversational exchanges.

- In the context of smart learning communities, interactions and conversational exchanges between two agents support bi-directional meaning constructions and collaborative understanding developments.

- In the context of smart learning communities, any constructed knowledge by an individual learner supports collaborative knowledge construction.

9.2. SMART CONSTRUCTIVIST LEARNING COMMUNITIES AND KNOWLEDGE BUILDING TECHNOLOGIES

In the context of smart constructivist learning communities, any conception is a building block of a knowledge construction. Any conception of an individual learner must be connected to and related to all others' conceptions. For example, any conception of a learner could be expressed in the form of her/his notes, paintings, sound clips, and video clips. Accordingly, the conceptions can be recorded and archived in the digital library of the relevant smart learning environment. Subsequently, the smart learning environment must record a huge collection of conceptions. These could be represented by, e.g., data models, conceptual models, graphical models, statistical models, and concept maps. This can be seen in Figure 6. Consequently, any conception would be viewable in multiple views as well as from different perspectives. For example, John's conception could be viewed from the perspective of Bob's and Mary's conceptions or from the perspective of their mentor's conception. In addition, there could be different possible interpretations for any linkage between two conceptions. These all could be recorded in the digital library. For example, Elizabeth may have an interpretation of John's conception while she has observed and conceptualised John's conception from the perspective of Mary's conception. Accordingly, Elizabeth's interpretation, over the arc/line between John's and Mary's conceptions, could produce a new conception that could be recorded in the digital library.

The 'Knowledge Forum' is a proper knowledge building environment, see (Scardamalia et al., 1994; Caswell and Bielaczyc, 2001; Scardamalia, 2004; Scardamalia and Bereiter, 2014). This multimedia knowledge building environment could be recognised as a kind of smart learning environment. Such a smart learning environment focuses mainly on knowledge building. Knowledge Forum becomes organised by all of its users. All users are the constructors and developers of a huge collaborative knowledge construction. It might be assumed that such an environment can be an appropriate developable environment for 'knowledge building within smart constructivist learning communities'. Such a smart system can represent the advancing knowledge of any individual and of any community.

It's undeniable that smart learning communities are dependent on discourse engaged in activity, reflection, and interaction. We cannot deny that the most important objective of a modern learning community like Knowledge Forum is Learning-and-Constructing-Together. It must be taken into consideration that a smart constructivist learning community believes that 'smart constructivist learning' and 'knowledge development' are integrally dependent on any individual's interactions and collaborations with other agents. Furthermore, we cannot ignore the importance of Collaborative Meaning Construction and Understanding Production in smart constructivist learning communities.

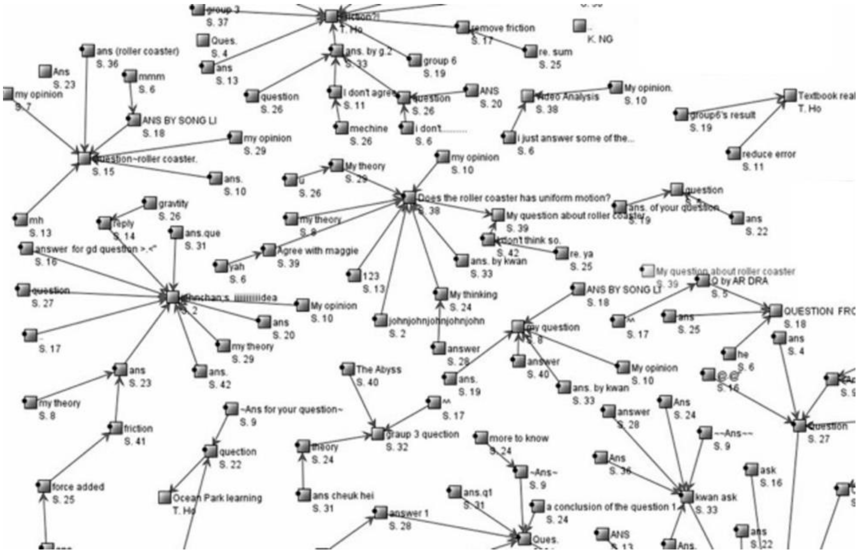

Figure 6. *Knowledge Building View in Knowledge Building Communities*

10. DISCUSSION AND CONCLUDING REMARKS

This research is focused on the area of Smart Learning Environments. Our theory has been presented (i) based on traditional constructivist theory of learning and (ii) by considering new requirements of learners in the digital age. It has, with special focus on ‘constructivist epistemology’ and ‘constructivist models of knowing’, conceptualised Smart Constructivist Learning Systems.

In this research, knowledge acquisition has been recognised as the process of seeking knowledge [by human beings] about different phenomena, objects, processes and events with regard to their personal background knowledge. The concepts of ‘knowledge building’ and ‘understanding production’ have been the most sensitive terms in this article. More clearly, our theoretical model deals with:

i. how knowledge may reasonably be assumed to be built by an individual, and
ii. how her/his meaningful understandings could be assumed to be produced.

The constructivist theory of smart learning and, respectively, the smart constructivist theory of learning is a modern learning theory that is conceptualised over the phenomenon of ‘smartness’. What we have offered has been a ‘conceptual, logical and epistemological description’ which has justified the importance of Smart Constructivist Knowledge Building Strategies. More specifically, this research has presented a specification of conceptualisation of

a. smart constructivism,
b. smart constructivist learning,
c. meaning construction and understanding production in the framework of smart constructivism,
d. knowledge building in the framework of smart constructivism,
e. smart constructivist collaborative learning,
f. smart constructivist learning communities,
g. smart knowledge building environments, and
h. collaborative meaning construction and understanding production in the framework of smart constructivism.

As for the structural characteristics of smart constructivism, knowledge construction is conceived of as a type of active process, and it can be informally described in terms of personal understanding in multiple actions. Furthermore, it has been theorised that the phenomenon of 'understanding' could be valid and meaningful based on learners' [and mentors'] constructed meanings in the framework of constructivism and in the context of smart learning environments. Accordingly, the concept of 'knowledge building' is interpreted as the consequence of 'meaning construction', 'understanding production', and 'sense making' by any individual learner.

Subsequently, this chapter has worked on designing a conceptual (and logical) framework for analysing knowledge building in the framework of smart constructivism and over the flow of learners' understandings. Considering that framework, we have identified the most significant characteristics of a smart constructivist pedagogy. It has been assumed that the conceptualised theory must be able to support other learning/mentoring strategies as the products of the phenomenon of 'smartness'. Accordingly, we have—relying on the characterised concept of 'smart learning communities'—picked up the 'collaborative learning strategy' and worked on checking the validity of Learning-and-Constructing-Together (as a model of learning) within smart learning communities. Subsequently, the most fundamental characteristics of knowledge building within smart learning communities are conceptualised. We shall claim that smart constructivism, besides Learning-and-Constructing-Together, could support some strategies like, e.g., Learning-and-Constructing-by-Doing and Learner-based programs of study with variable structures adaptable to types of learners.

We strongly believe that the theory of smart constructivism and, subsequently, the constructivist model of learning within smart learning environments can support subsequent developments of smart learning strategies. This theory could support renewed qualitative developments of knowledge building and understanding production within smart learning environments.

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PAPER M. A FORMAL SEMANTICS FOR CONCEPT UNDERSTANDING RELYING ON DESCRIPTION LOGICS

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ABSTRACT

In this research, Description Logics (DLs) will be employed for logical description, logical characterisation, logical modelling and ontological description of concept understanding in terminological systems. It's strongly believed that using a formal descriptive logic could support us in revealing logical assumptions whose discovery may lead us to a better understanding of 'concept understanding'. The Structure of Observed Learning Outcomes (SOLO) model as an appropriate model of increasing complexity of humans' understanding has supported the formal analysis.

1. INTRODUCTION

The central focus of this research is on concepts. My point of departure is the special focus on the fact that there is a general problem concerning the notion of 'concept', in linguistics, psychology, philosophy and computer science. This research aims at providing a logical description (and analysis) of the use of concepts in terminological knowledge representation systems and, thus, I need to assume concepts' applications in order to be comprehensible in the context and in my logical formalism. Taking into consideration (Baader et al., 2010) and (Rudolph, 2011), a concept might be correlated with a distinct 'entity' or to/with its essential features, characteristics and properties. Note that an entity's properties express its relationships with itself and with other entities. Through the lens of Predicate Logic, a concept might be considered to be equivalent to a [unary] predicate. It shall be emphasised that this remark is not about language, but this is how concepts are perceived by logicians. Accordingly, it could be claimed that predicates could, logically, express the characteristics of concepts in terminological systems. More specifically, predicates assign characteristics, features and properties of concepts into some subjects. It's believed that predicates may determine the applications of logical descriptions. As all logicians know, predicates play fundamental roles in reasoning processes and in giving satisfying conditions for definitions of [logical] truth. By taking into consideration that 'a predicate expresses a condition that the entities referred to may satisfy, in which case the resulting sentence will be true (see (Blackburn, 2016))', predicates can be applied in expressing meanings within formal semantics. Subsequently, the formal semantics could focus on multiple conditions through definitions of truth (and falsity). The central objective of formal semantics can be said to be formalising and manipulating the relationships between the signifiers of a description and what the signifiers do [or have been designed to do], see (Jackendoff, 1990; Gray et al., 1992; Barsalou, 1999; Resnik, 1999).

As mentioned, the central focus of this research is on concepts (and through the lens of Predicate Logic). Concepts and their interrelationships will be used to establish the basic terminology adopted in the modelled domain regarding the hierarchical structures. My logical descriptions will have a special focus on my methodological assumption that expresses that

- human beings' grasps of their constructed concepts (in the form of their conceptions) provide foundations for producing their own conceptualisations. Accordingly, they can find out that an individual thing/phenomenon is an instance of that concept.

This article will focus on describing and characterising humans' concept understandings and will deal with a formal-semantic model for figuring out the underlying logical assumptions of 'concept understanding'. The term 'understanding' will be observed from multiple perspectives and, subsequently, the expressiveness of the semantic model's descriptions will be improved. The Structure of Observed Learning Outcomes (SOLO) taxonomy is an appropriate model of increasing complexity of humans' understanding. SOLO as a descriptive model of knowing and understanding can support my formalism. Additionally, its taxonomical structure could be expressed in the form of some logical inclusions.

In this research, the formal semantic analysis of [concept] understanding is based on Description Logics (DLs). I believe that DLs can support me in proposing an understandable logical description for clarifying concept understanding. DLs as the profound formalism are used for representing predicates and for formal reasoning over them. They mainly focus on terminological knowledge. It is of a terminological system's particular importance in providing a logical formalism for knowledge representation systems and, also, for semantic representations and ontologies (as formal and explicit specification of a shared conceptualisation on the domains of interest), see (Davies et al., 2003; Staab and Studer, 2009).

The main contributions of this research are:

i. providing a formal semantics (relying on DLs) for conceptual analysis of concept understanding, and analysing a knowledge representation formalism for expressing concept understanding, and
ii. designing and formalising an ontology that provides a structural representation of concept understanding within the analysed semantic model.

2. DESCRIPTION LOGICS

First, I shall mention that (Baader et al., 2010) is my main reference to Description Logics. Description Logics (DLs) represent knowledge in terms of individuals (objects, things), concepts (classes of things), and roles (relationships between things). Individuals correspond to constant symbols, concepts to unary predicates, and roles to binary (or any other *n*-ary) predicates and relations in Predicate Logic. Reconsidering the predicate P in Predicate Logic, we have [possibly specified] concept C in DLs. There are two kinds of atomic symbols, which are called atomic concepts and atomic roles. These symbols are the elementary descriptions from which we can inductively (by employing concept constructors and role constructors) build the specified descriptions. Considering N_C, N_R and N_O as the sets of atomic concepts, atomic roles and individuals respectively, the ordered triple $\langle N_C, N_R, N_O \rangle$ represents a signature. The set of main logical symbols in ALC (Attributive Language with Complements: the Prototypical DL, see (Schmidt-Schauss and Smolka, 1991)) is: {Conjunction ($\sqcap$: And), Disjunction ($\sqcup$: Or), Negation ($\neg$: Not), Existential Restriction ($\exists$: There exists ...), Universal Restriction ($\forall$: For all ...)}. We also have Atomic Concepts (A), Top Concept ($\top$: Everything) and Bottom Concept ($\bot$: Nothing) in *ALC*.

In order to define a formal semantics, we need to apply terminological interpretations over our signatures. More particularly, any [terminological] interpretation consists of

i. a non-empty set Δ (that is the interpretation domain and consists of any variable that occurs in any of the concept descriptions), and
ii. an interpretation function $.^I$ (let me call it 'interpreter').

The interpreter assigns to every individual (like a) a '$a^I \in \Delta^I$'. Also, it assigns to every atomic concept A (or every atomic unary predicate) a set $A^I \subseteq \Delta^I$, and to every atomic role P (or every atomic binary predicate) a binary relation $P^I \subseteq \Delta^I \times \Delta^I$. Table 1 reports the syntax and the semantics of *ALC*.

Syntax	**Semantics**
A	$A^I \subseteq \Delta^I$
P	$P^I \subseteq \Delta^I \times \Delta^I$
$\top$	Δ^I
$\bot$	$\varnothing$
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\neg C$	$(\neg C)^I = \Delta^I \setminus C^I$
$\exists R.\ C$	$\{ a \mid \exists b.(a,b) \in R^I \wedge b \in C^I \}$
$\forall R.\ C$	$\{ a \mid \forall b.(a,b) \in R^I \supset b \in C^I \}$

Table 1. *The Prototypical Description Logic*

A knowledge base in DLs usually consists of a number of terminological axioms and world descriptions (so-called 'assertions'), see Table 2. The terminological interpretation I is called a 'model' of an axiom (or a model of a basic world description) if and only if it can semantically satisfy it, see Tables 2 and 3. In the following Tables P is an atomic role, R and S are role descriptions, A is an atomic concept, and C and D are concept descriptions.

Name	Syntax	Semantics
Concept Inclusion Axiom	$C \sqsubseteq D$	$C^I \subseteq D^I$
Role Inclusion Axiom	$R \sqsubseteq S$	$R^I \subseteq S^I$
Concept Equality Axiom	$C \equiv D$	$C^I = D^I$
Role Equality Axiom	$R \equiv S$	$R^I = S^I$
Concept Assertion	$C(a)$	$a^I \in C^I$
Role Assertion	$R(a,b)$	$(a^I, b^I) \in R^I$

Table 2. *Axioms and World Descriptions in DLs*

Over Concept	Over Role
$A^I \subseteq \Delta^I$	$P^I \subseteq \Delta^I \times \Delta^I$
$\bot^I = \varnothing$	$\bot^I = \varnothing$
$(\neg C)^I = \Delta^I \setminus C^I$	$(\neg R)^I = (\Delta^I \times \Delta^I) \setminus R^I$
$(C \sqcap D)^I = C^I \cap D^I$	$(R \sqcap S)^I = R^I \cap S^I$

Table 3. *Inductive Concept Descriptions*

Let me start the logical analysis with two examples:

Example 1

Mary has verified that 'there is a young student' and 'there is a non-old student' are expressing the same matter. Her verification between these two propositions is expressible in DLs by:

$$\exists \text{hasStudent.Young} \equiv \exists \text{hasStudent.}\neg\text{Old}.$$

It's realisable that Mary has assumed the axiom stating that Young and Old are two disjoint concepts and, in fact, the logical term '$\text{Young} \sqcap \text{Old} \sqsubseteq \bot$' has formed a terminological axiom for Mary. It's obvious that Mary's interpretation over:

i. $\text{Young} \sqcap \text{Old} \sqsubseteq \bot$ (meaning that Young and Old are disjoint concepts), and
ii. $\text{Person} \sqsubseteq \text{Young} \sqcup \text{Old}$ (meaning that every person is either young or old)

has played crucial roles here. In fact, Mary has interpreted and, respectively, understood that these two sentences ('there is a young student' and 'there is a non-old student') have the same meanings. More specifically, Mary's terminological interpretation (over i and ii) has produced her understanding of an equivalence between the concept descriptions $\exists$hasStudent.Young and $\exists$hasStudent.¬Old. We can see that Mary's interpretation has been restricted (limited) to her understanding of disjointness of the concept descriptions $\exists$hasStudent.Young and $\exists$hasStudent.¬Old.

At this point, I shall claim that the concepts (concept descriptions) *C* and *D* are logically and semantically equivalent, when 'for all' possible terminological interpretations like *I*, we have: $C^I = D^I$. In this example, if one person, say John, does not assume the axioms stating that 'Young and Old are two disjoint concepts' and 'every person is either young or old', then there will not be an equivalence relation between $\exists$hasStudent.Young and $\exists$hasStudent.¬Old.

Let me conclude that Mary's and John's understandings are dissimilar, because they have had different terminological interpretations in their minds (and it is because of their different conceptions and concept formations). For example, regarding John's terminological interpretation, the proposition 'there is a middle-aged student' could be added beside 'there is a young student' and 'there is a non-old student'. In fact, John could have the axiom

- 'Person $\sqsubseteq$ Young $\sqcup$ MiddleAged $\sqcup$ Old (meaning that every person is young or middle aged or old)'

in his mind. Consequently, John by taking this axiom (based on his own conception) into consideration doesn't understand '$\exists$hasStudent.Young' and '$\exists$hasStudent.¬Old' as equivalent concept descriptions.

Example 2

Mary has verified that 'Anna has a child who is a philosopher' and 'Anna has a child who is a painter' could be jointly expressed by 'Anna has a child who is a philosopher and painter'. Translated into DLs we have her expression as followings:

$$\exists \text{hasChild.Philosopher} \sqcap \exists \text{hasChild.Painter}$$
$$\equiv$$
$$\exists \text{hasChild.(Philosopher} \sqcap \text{Painter).}$$

Suppose that Anna has two children and one is a philosopher and the other one is a painter. Then, $\exists$hasChild.(Philosopher $\sqcap$ Painter) is not equivalent to $\exists$hasChild.Philosopher $\sqcap$ $\exists$hasChild.Painter. Actually, Mary has not proposed a correct description, and this is because of her inappropriate terminological interpretation. Accordingly, her understanding has followed her inappropriate interpretation. In fact, she incorrectly (semantically: False) has understood that the proposition 'Anna has a child who is a philosopher and painter' expresses the same matter. Reconsidering the proposed formalism, $\exists$hasChild.Philosopher $\sqcap$

$\exists$hasChild.Painter and $\exists$hasChild.(Philosopher $\sqcap$ Painter) are not—semantically—the same and there should not be an equivalence symbol between them. Thus, Mary's interpretation has not been satisfactory. Subsequently, her understanding is not satisfactory and appropriate.

3. A SEMANTIC MODEL FOR CONCEPT UNDERSTANDING

In this section I clarify my logical conceptions of 'concept understanding'. The term 'understanding' is very complicated and sensitive in psychology, neuroscience, cognitive science, philosophy and epistemology. There has not been any complete model for describing this term, but there are some proper models of:

a) understanding of understanding, see (Foerster, 2003),
b) understanding representation, see (Peschl and Riegler, 1999; Webb, 2009), and
c) specification of the components of understanding (i.e., from the cognition's and from the affects' perspectives), see (Chaitin, 1987; Kintsch et al., 1990; di Pellegrino et al., 1992; MacKay, 2003; Zwaan and Taylor, 2006; Uithol et al., 2011; Uithol and Paulus, 2014).

This research, by analysing a formal semantics, focuses on the junctions of 'understanding of concept understanding' and 'concept understanding representation' in terminological systems and, more specifically, it focuses on logical analysis of concept understanding and its terminological representation.

3.1. CONCEPT UNDERSTANDING AS A RELATION (AND FUNCTION)

I shall claim that 'concept understanding', as a relation, could relate 'the characteristics and attributes of a concept' with 'a description'. More specifically, *understanding* is a function (mapping) from a concept into some propositions (and statements) which could be interpreted as 'concept descriptions'. In fact, the characteristics and properties of a concept by means of the *understanding* function become mapped into concept descriptions. Let me be more specific:

A. A human being—by concept understanding—attempts to map the significant characteristics of concepts into some concept descriptions. For example, 'breathing', as a biological and psychological process, is a characteristic and trait of all animals and, thus, breathing (that is a role) is the characteristic of the concept Animal. Then,

i. knowing the fact that the individual horse is an instance of the concept Animal (Formally: Animal(horse)), and
ii. drawing the [concept subsumption] inference 'Horse $\sqsubseteq$ Animal',

collectively, lead us to knowing and to understanding that 'horses breathe' (or equivalently: 'horses do breathing'). The role breathing could be manifested in the concept Breath. Therefore, (i) and (ii) collectively lead us to expressing the concept description 'Animal(horse) $\sqcap$ $\exists$hasTrait.Breath' for the individual horse (as an instance of the concept Animal) and, respectively, for the concept Horse (as a sub-concept of Animal).

B. A human being—by concept understanding—attempts to map the concepts' properties and their interrelationships with themselves into some concept descriptions. For example, the one who knows that 'male horses breathe', by taking the terminological and assertional axioms

$$\{ \\ \mathrm{Animal(horse)}, \\ \mathrm{Horse} \sqsubseteq \mathrm{Animal}, \\ \mathrm{MaleHorse} \sqsubseteq \mathrm{Horse}, \\ \mathrm{FemaleHorse} \sqsubseteq \mathrm{Horse} \\ \}$$

into consideration, can know and understand that 'female horses breathe' as well.

C. A human being—by concept understanding—attempts to map the concepts' properties and their relationships with other concepts into some concept descriptions. For example, the one who knows that 'horses breathe' (and as described: $\mathrm{Animal(horse)} \sqcap \exists \mathrm{hasTrait.Breath}$), could, respectively, know and understand that the individual rabbit (that is an animal) breathes as well. So, she/he could express that 'rabbits breathe' and, in fact, $\mathrm{Animal(rabbit)} \sqcap \exists \mathrm{hasTrait.Breath}$.

Conclusion. Relying on Predicate Logic (and DLs), the phenomenon of 'concept understanding' could be interpreted as:

- a binary predicate and
- a role of human beings on expressing some concept descriptions. Let me represent this role by 'understanding'.

3.2. CONCEPT UNDERSTANDING AS A CONCEPTUALISATION

The concept understanding could be interpreted to be the limit/type of conceptualisation. Accordingly, humans need to conceptualise concepts in order to understand them. According to (Badie, 2016a) and (Badie, 2016b), any concept understanding could be interpreted as a local manifestation of a global conceptualisation. Additionally, human beings' grasps of concepts could provide proper foundations for generating their own conceptualisations. I shall claim that 'concept understanding' could be acknowledged as a limited type of humans' concept constructions, when the concept constructions are supported by their own conceptualisations. Therefore, 'conceptualising' is a role of human beings. This conclusion, relying on DLs, could be represented by the 'role inclusion (or role subsumption)':

$$\mathrm{understanding} \sqsubseteq \mathrm{conceptualising}.$$

In other words, 'understanding a concept' has been acknowledged as the sub-role of 'conceptualising that concept'. On the other hand, 'it is not the case that all conceptualisations are understandings'. In fact, all the conceptualised concepts could not be understood.

3.3. CONCEPT UNDERSTANDING AS AN INTERPRETATION-BASED MODEL

Generally, an interpretation is the act of elucidation, explication and explanation, see (Simpson and Weiner, 1989). According to (Honderich, 2005) and through the lens of philosophy, "…in existential and hermeneutic philosophy, 'interpretation' becomes the most essential moment of human life: The human being is characterised by having an 'understanding' of itself, the world, and others. This understanding, to be sure, does not consist—as in classical ontology or epistemology—in universal features of universe or mind, but in subjective-relative and historically situated interpretations of the social. …". Regarding (Blackburn, 2016) and through the lens of logic, an 'interpretation' of a logical system assigns meanings (or semantic values) to the formulae and their elements. At this point, I shall take into consideration that the phenomenon of 'interpretation' could have a conjunction with the phenomenon of 'terminological interpretation' in formal languages. More specifically, the one who has engaged her/his interpretations to explicate [and justify] what [and why] she/he means by classifying a thing/phenomenon as an instance of a concept, needs to interpret the non-logical signifiers of different concept descriptions within her/his linguistic expressions.

Considering any set of non-logical symbols (that have no logical consequences) in a terminology, a terminological interpretation over humans' languages could be described to be constructed based on the tuple

$$\langle \text{Interpretation Domain, Interpretation Function} \rangle.$$

The interpretation domain (or the universe of interpretation) might be called 'universe of discourse'. As mentioned in previous section, the interpretation domain must be non-empty. This non-empty set forms the range of any variable that occurs in any of the concept descriptions within linguistic expressions. It's a fact that the collection of the rules and the processes that manage different terms and descriptions in linguistic expressions, cannot have any meaning until the non-logical signifiers and constructors are given terminological interpretations. The interpretations prepare humans for producing their personal meaningful [and understandable] concept descriptions. Hence, I have recognised all 'concept understandings' as 'concept interpretations'. This conclusion, relying on DLs, could be represented by the 'role inclusion':

$$\text{understanding} \sqsubseteq \text{interpreting}.$$

Therefore, 'concept understanding' has been expressed as the sub-role of 'concept interpreting'. But, on the other hand, not all interpretations (of concepts) imply understandings (of concepts). Equivalently, 'it is not the case that all interpretations are understandings'. In other words, all the interpreted concepts may not be understood. Accordingly, considering any interpretation as a function, 'concept understanding' is recognised as an 'interpretation function'.

From this point, I apply the function *UND* (as a limit of the interpretation function *I*) in my formalism. Then, C^{UND} represents 'Concept Understanding', where *C* stands for Concept. Consequently, considering *UND* as a kind of interpretation, there exists a tuple like $\langle D_U, C_{understood} \rangle$, where:

i. D_U represents the understanding domain (that consists of the variables that occur in any of the concept descriptions which are going to be understood), and
ii. $C_{understood}$ is the understood concept.

$C_{understood}$ is achievable based on the understanding function -^{UND}. Relying on the function -^{UND},

- $C^{UND} \subseteq C^I \subseteq \Delta^I$, and
- $D_U{}^{UND} \subseteq \Delta^I$.

It shall be stressed that $D_U{}^{UND}$ expresses 'understanding all concepts belonging to the understanding domain'. Note that -^{UND}, as a function, can provide a model for a terminological (and assertional) axiom. Therefore, the desired model

i. is a restricted form of a terminological (and interpretation-based) model, and
ii. can satisfy the semantics of the terminological and assertional axioms (read '$UND \vDash$ Axiom': *UND* satisfies the axiom), see Table 4.

Consequently:

- $C^{UND} \subseteq C^I \subseteq \Delta^I$, and
- $\text{-}^{UND} : C \rightarrow C^{UND}$

, where:

- $C^{UND} \subseteq D_U{}^{UND} \subseteq \Delta^I$.

I shall emphasise that we are not able to conclude that $C^I \subseteq D_U{}^{UND}$. On the other hand, we certainly know that $C^{UND} \subseteq \Delta^I$ (because $C^{UND} \subseteq C^I$ and $C^I \subseteq \Delta^I$). According to the analysed characteristics, the *UND* understanding model in my terminology is constructed based upon the tuple $\langle$*Understanding Domain, Understanding Function*$\rangle$ as:

$$UND = \langle D_U^{UND}, \cdot^{UND} \rangle.$$

Name	Description, Semantics
Understanding a Concept Inclusion	$UND \vDash (C \sqsubseteq D) \Rightarrow C^{UND} \subseteq D^{UND}$
Understanding a Role Inclusion	$UND \vDash (R \sqsubseteq S) \Rightarrow R^{UND} \subseteq S^{UND}$
Understanding a Concept Equality	$UND \vDash (C \equiv D) \Rightarrow C^{UND} = D^{UND}$
Understanding a Role Equality	$UND \vDash (R \equiv S) \Rightarrow R^{UND} = S^{UND}$
Understanding a Concept Assertion	$UND \vDash C(a) \Rightarrow a^{UND} \in C^{UND}$
Understanding a Role Assertion	$UND \vDash R(a,b) \Rightarrow (a^{UND}, b^{UND}) \in R^{UND}$

Table 4. *Understanding Model: Terminologies, World Descriptions and their Semantics*

Table 5 is based on Table 4 and itemises inductive concept descriptions and their semantics as the products of the understanding model.

Model Satisfies the Vocabulary	Semantics
$UND \vDash \top$	$\top^{UND} = \top$
$UND \vDash \bot$	$\bot^{UND} = \varnothing$
$UND \vDash \neg R$	$(\neg R)^{UND} = \top \setminus R^{UND}$
$UND \vDash \neg C$	$(\neg C)^{UND} = D_U^{UND} \setminus C^{UND}$
$UND \vDash (R \sqcap S)$	$(R \sqcap S)^{UND} = R^{UND} \cap S^{UND}$
$UND \vDash (C \sqcap D)$	$(C \sqcap D)^{UND} = C^{UND} \cap D^{UND}$

Table 5. *Understanding Inductive Concept Descriptions*

3.4. CONCEPT UNDERSTANDING AS A PRODUCT OF FUNCTIONAL ROLES

How could we employ DLs in order to describe an understanding function as a [functional] role of a human being? Let me interpret functional roles (features) as the roles that are existentially functions and, thus, they can express ‘functional’ actions, movements, procedures and manners of human beings. Let N_F be a set of functional roles and N_R be the set of role [descriptions]. Obviously: $N_F \subseteq N_R$ and informally, functional roles are some kinds of roles.

Lemma. The *UND* understanding model is—semantically—structured over:

a. the understanding domain (or D_U),
b. the understanding function (or $-^{UND}$), and
c. the set D_U^{UND} (or equivalently, the effect of the understanding function $-^{UND}$ on the Top concept) that represents understanding all atomic concepts (everything) in the understanding domain.

Analysis. The *UND* understanding model associates with each atomic concept a subset of D_U^{UND}, and with each ordinary atomic role a binary relation over $D_U^{UND} \times D_U^{UND}$. Note that any functional role can be recognised as a partial function. More specifically, considering $F = f_1 \circ \cdots \circ f_n$ (F is a chain of functional roles), the chain $f_1^{UND} \circ \cdots \circ f_n^{UND}$ represents the composition of n partial understanding functions. In fact, by employing *UND*, any f_i^{UND}—semantically—supports the [overall] functional role F^{UND}. Note that for all i in $(1,n)$, f_{i+1} produces the input of f_i. Therefore, the understanding of f_{i+1} (the output of f_{i+1}) provides the input of the understanding of f_i. In particular, any concept description could be understood over the subsets of D_U^{UND}. This characteristic is very useful in making a strong linkage between the terms ‘understanding’ and ‘chain of functional roles’. It supports my semantic model in:

- scheming and describing the ‘understanding’ as the ‘product of a chain of functional roles, where the functional roles are the partial understanding functions’.

You will see how it works.

3.5. HUMANS’ FUNCTIONAL ROLES THROUGH SOLO’S LEVELS

According to (Biggs and Collis, 2014), the Structure of Observed Learning Outcomes (SOLO) taxonomy is a proper model that can provide an organised framework for representing different levels of humans’ understandings. This model is concerned with various complexities of understanding on its different layers. According to SOLO and focusing on humans’ levels of knowledge with regard to a concept, we have:

- Pre-structured knowledge. Here humans’ knowledge of a concept is pre-structured (and is the product of their pre-conceptions).

- Uni-structured knowledge. Humans have a limited knowledge about a concept. They may know one or few isolated fact(s) about a concept.
- Multi-structured knowledge. They are getting to know a few facts relevant for a concept, but they are still unable to link and relate them together.
- Related Knowledge. They have started to move towards deeper levels of understanding of a concept. Here they are able to link different facts and to explain several conceptions of a concept.
- Extended Abstracts. This is the most complicated level. Humans are not only able to link lots of related conceptions [of a concept] together, but they can also link them to other specified and complicated conceptions. Now they are able to link multiple facts and explanations in order to produce more complicated extensions relevant for a concept.

Obviously, the extended abstracts are the products of deeper comprehensions of related structures. Related structures are the products of deeper comprehensions of multi-structures. The multi-structures are the products of deeper comprehensions of uni-structures, and the uni-structures are the products of deeper comprehensions of pre-structures.

Let me select a process (as a sample of humans' functional roles) from any of the SOLO's levels and formalise it. According to SOLO, *creation* (with regard to an understood concept) is an instance of 'extended abstracts', *justification* (with regard to an understood concept) is an instance of 'related structures', *description* (with regard to an understood concept) is an instance of 'multi-structures' and *identification* (with regard to an understood concept) is an instance of 'uni-structures'. Therefore, *Creation*, *Justification*, *Description*, and *Identification* are four processes which could be analysed as functions in the model. Any of these functions can support a functional role as a 'partial understanding function':

i. *Creation* has interrelatedness with creatingOf that is a functional role and extends the humans' mental abstracts.
ii. *Justification* has interrelatedness with the functional role justifyingOf that relates the lower structures.
iii. *Description* has correlation with the functional role describingOf that produces the multi-structures.
iv. *Identification* has correlation with the functional role identifyingOf that generates the uni-structures.

It shall be emphasised that identifyingOf, describingOf, justifyingOf, and creatingOf are only four examples of functional roles within SOLO's categories and, in fact, the SOLO's levels are not limited to these functions. For example, followingOf and namingOf are two other instances of uni-structures, combiningOf and enumeratingOf are two other instances of multi-structures, analysingOf and arguingOf are two other instances of related structures, and formulatingOf and theorisingOf are two other instances of extended abstracts.

As mentioned, the functional roles creatingOf, justifyingOf, describingOf, and identifyingOf represent the equivalent roles of the *creation*, *justification*, *description*, and *identification* functions respectively. Furthermore, these functions are the partial functions of the *understanding* function. Obviously, the understanding function (that is a process) could also be considered to be equivalent to a functional role like understandingOf. Employing the 'role inclusion' axiom we have:

1. creatingOf $\sqsubseteq$ understandingOf,
2. justifyingOf $\sqsubseteq$ understandingOf,
3. describingOf $\sqsubseteq$ understandingOf, and
4. identifyingOf $\sqsubseteq$ understandingOf.

Equivalently:

1) *creation* $\subseteq$ *understanding*,
2) *justification* $\subseteq$ *understanding*,
3) *description* $\subseteq$ *understanding*, and
4) *identification* $\subseteq$ *understanding*.

It shall be claimed that understandingOf, conceptually and logically, supports 'the *understanding* function based on the analysed understanding model (or *UND*)'. Similarly, we can define *CRN*, *JSN*, *DSN*, and *IDN* as sub-models of *UND* for representing *creation*, *justification*, *description* and *identification* respectively. Any of these models can—semantically—satisfy the terminologies and world descriptions in Table 4. Accordingly—relying on inductive rules—they can satisfy concept descriptions in Table 5.

Note that *CRN* (as a model) fulfils the desires of *UND* better (and more satisfying) than *JSN*, *DSN*, and *IDN*. Considering D_U as the understanding domain, we have:

$$D_U^{UND} \subseteq D_U^{CRN} \subseteq D_U^{JSN} \subseteq D_U^{DSN} \subseteq D_U^{IDN}.$$

More specifically:

- D_U^{CRN} represents the model of *creation* over the understanding domain. It consists of concepts which are (or could be) 'created' by human beings. Formally: $C^{CRN} \in D_U^{CRN}$.

- D_U^{JSN} represents the model of *justification* over the understanding domain. It consists of concepts which are (or could be) 'justified' by human beings. Formally: $C^{JSN} \in D_U^{JSN}$.

- D_U^{DSN} represents the model of *description* over the understanding domain. It consists of concepts which are (or could be) 'described' by human beings.

Formally: $C^{DSN} \in D_U^{DSN}$.

- D_U^{IDN} represents the model of *Identification* over the understanding domain. It consists of concepts which are (or could be) 'identified' by human beings. Formally: $C^{IDN} \in D_U^{IDN}$.

Proposition. The terminological axioms and the world descriptions (in Table 4) and inductive concept descriptions (in Table 5) are all valid and meaningful for *CRN*, *JSN*, *DSN*, and *IDN*. Therefore, inductive concept descriptions are also valid and meaningful over the concatenation of the *creation*, *justification*, *description* and *identification* functions that have supported these terminological models.

Proposition. All satisfactions based on *IDN* are already satisfied by *DSN*, *JSN* and *CRN* over D_U^{DSN}, D_U^{JSN}, and D_U^{CRN} respectively. Informally, if a human being is able to describe, justify and create with regard to her/his conception of a concept, so, she/he is already capable of identifying that concept. Furthermore, she/he might be able to identify something else with regard to her/his conception of that concept.

Formal Analysis. The semantics of the composite function '*creation* (*justification* (*description* (*identification* (C))))', that is the product of the chain of functional roles, supports the proposed semantic model on D_U^{UND}, which is the central domain of the understanding (central part of the understanding domain). Considering all the roles relevant for the concept C, we have:

1. $(\forall R_1.C)^{CRN} = \{\, a \in D_U^{CRN} \mid \forall b.(a,b) \in R_1^{CRN} \rightarrow b \in C^{CRN} \,\}$.

Therefore:

2. $(\forall R_2.C)^{JSN} = \{\, a \in D^{JSN} \mid \forall b.(a,b) \in R_2^{JSN} \rightarrow b \in C^{JSN} \,\}$.

Therefore:

3. $(\forall R_3.C)^{DSN} = \{\, a \in D_U^{DSN} \mid \forall b.(a,b) \in R_3^{DSN} \rightarrow b \in C^{DSN} \,\}$.

Therefore:

4. $(\forall R_4.C)^{IDN} = \{\, a \in D_U^{IDN} \mid \forall b.(a,b) \in R_4^{IDN} \rightarrow b \in C^{IDN} \,\}$.

In the afore-itemised formalism, R_1, R_2, R_3 and R_4 stand for creatingOf, justifyingOf, describingOf, and identifyingOf respectively. Consequently, *CRN*, *JSN*, *DSN*, and *IDN* have been observed as roles of human beings. Accordingly, it's possible to represent the chain of functional roles in the form of a collection of implications as following:

$$(\forall R_1.C)^{CRN} \Rightarrow (\forall R_2.C)^{JSN} \Rightarrow (\forall R_3.C)^{DSN} \Rightarrow (\forall R_4.C)^{IDN}.$$

It must be concluded that 'any role based on a conception of *C*' to the left of any of arrows makes a logical premise for 'other roles based on conceptions of *C*' to the right of that arrow. It shall be stressed that this is a very important terminological fact. The concluded logical relationship represents a flow of concept understanding from deeper layers to shallower layers.

4. AN ONTOLOGY FOR CONCEPT UNDERSTANDING

According to (Grimm et al., 2007; Staab and Studer, 2009), an ontology, from the philosophical point of view, is described as studying the science of being and existence. Ontologies must be capable of demonstrating the structure of the reality of a thing/phenomenon. They check multiple attributes, particularities and properties that belong to a thing/phenomenon because of its natural and structural existence. An ontology, from another perspective and through the lenses of information and computer sciences, is described as an explicit and formal specification of a shared conceptualisation in a domain of interest. However, in my opinion, there could be very strong relationship between these two descriptions of ontologies. In fact, ontologies in information sciences attempt to mirror the things'/phenomena's structures in virtual and artificial systems. The ontological descriptions in information sciences tackle to provide appropriate logical and formal descriptions of a phenomenon [and of its structure] considering various concepts relevant for that phenomenon. From this perspective, an ontology can be schemed and demonstrated by semantic networks and semantic representations. A semantic network is a graph whose nodes represent concepts (e.g., unary predicates) and whose arcs represent relations (e.g., binary/*n*-ary predicates) between the concepts. Accordingly, semantic networks provide structural representations of a thing/phenomenon. In Figure 1, I have designed a semantic network as an ontology for 'concept understanding'. This hierarchical semantic representation,

1. specifies the conceptual relationships between the most important ingredients of this research,
2. demonstrates the logical representation of concept understanding.

It shows how the proposed model attempts to represent concept understanding. This semantic representation can be interpreted as a specification of the shared conceptualisation of concept understanding within terminological systems. The proposed ontology can be reformulated and formalised in *ALC* in the form of a collection of fundamental terminologies as following:

A Formal Ontology for Concept Understanding

{
UnaryPredicate $\sqsubseteq$ Predicate,
BinaryPredicate $\sqsubseteq$ Predicate,
Concept $\sqsubseteq$ UnaryPredicate,
Concept $\sqsubseteq$ $\exists$hasInstance.Individual,

$BinaryPredicate \sqsubseteq (\exists hasNode.Individual \sqcap \exists hasNode.Individual)$,
$Role \sqsubseteq BinaryPredicate$,
$Relation \sqsubseteq BinaryPredicate$,
$Function \sqsubseteq Relation$,
$Interpretation \sqsubseteq Function$,
$Conceptualisation \sqsubseteq Function$,
$ConceptUnderstanding \sqsubseteq Interpretation$,
$ConceptUnderstanding \sqsubseteq Conceptualisation$,
$PartialFunction \sqsubseteq Function$,
$FunctionalRole \sqsubseteq Role$,
$FunctionalRole \sqsubseteq hasEquivalence.PartialFunction$,
$FunctionalRole \sqsubseteq Function$,
$SubModel \sqsubseteq Model$,
$SemanticModel \sqsubseteq Model$,
$InterpretationSemanticModel \sqsubseteq SemanticModel$,
$UnderstandingSemanticModel \sqsubseteq SemanticModel$,
$UnderstandingSemanticSubModel \sqsubseteq SubModel$,
$UnderstandingSemanticSubModel \sqsubseteq SemanticModel$,
$InterpretationSemanticModel \sqsubseteq \exists hasSupport.Interpretation$,
$UnderstandingSemanticModel \sqsubseteq \exists hasSupport.InterpretationSemanticModel$,
$UnderstandingSemanticModel \sqsubseteq \exists hasSupport.UnderstandingSemanticSubModel$,
$UnderstandingSemanticSubModel \sqsubseteq \exists hasSupport.FunctionalRole$
}

5. CONCLUSIONS

The readers of this article may ask “if the term ‘understanding’ in this research is related to the real human beings, or if this research’s domain is only information and computer sciences?” Actually, that’s why I have employed Description Logics. Under a plethora of names (among them terminological systems and Concept Languages), Description Logics (DLs) attempt to provide descriptive knowledge representation formalisms based on formal semantics to establish common [conceptual and logical] grounds and interrelationships between human beings and machines. Description Logics supported me in revealing some hidden conceptual assumptions that could support me in having a better understanding of ‘concept understanding’. DLs, by considering concepts as unary predicates and by applying terminological interpretations over them, have proposed a realisable logical description for explaining the humans’ concept understanding.

The central contribution of the article has been providing a formal semantics for logical analysis of concept understanding. According to the logical analysis, a background for terminological representation of concept understanding has been expressed. Consequently, a semantic representation [as an ontology and a specification of the shared conceptualisation of ‘concept understanding’] has been designed and formalised.

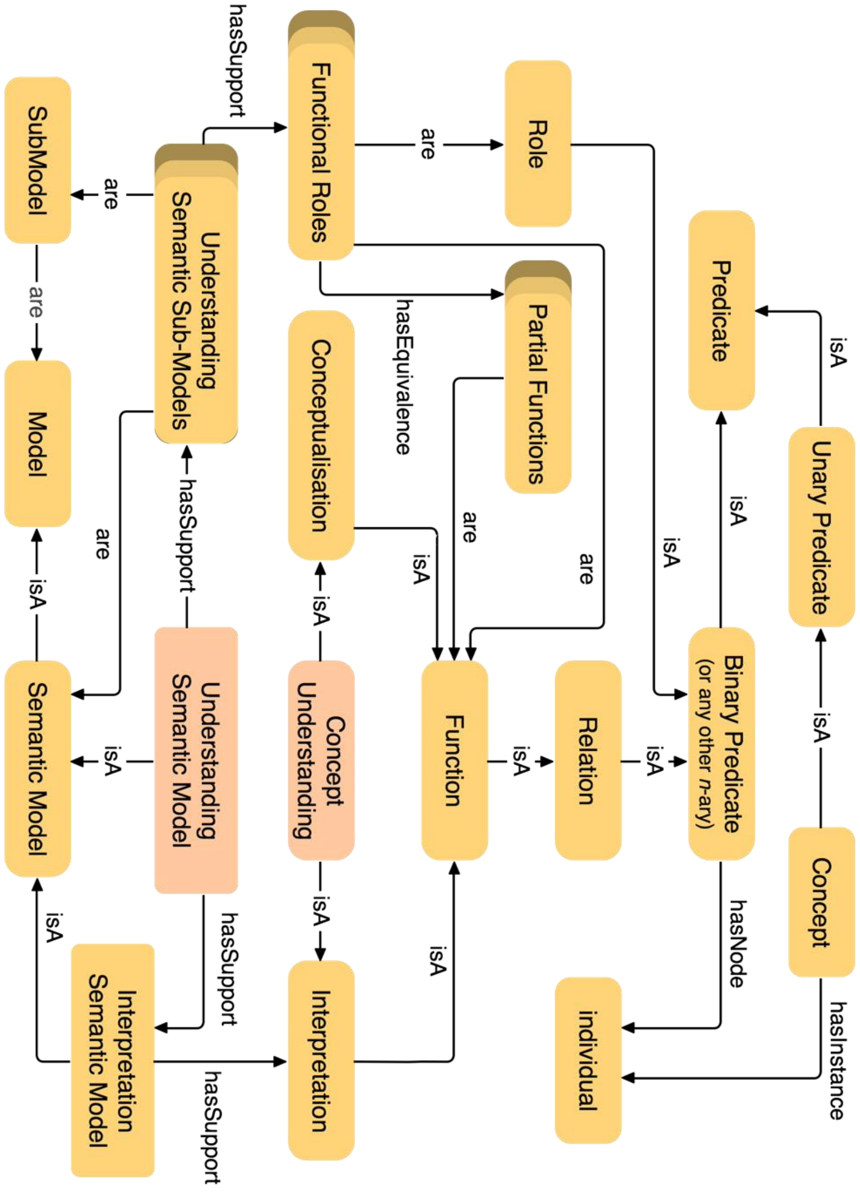

Figure 1: *An Ontology for Concept Understanding*

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PAPER N. ON LOGICAL CHARACTERISATION OF HUMAN CONCEPT LEARNING BASED ON TERMINOLOGICAL SYSTEMS

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ABSTRACT

The central focus of this article is the epistemological assumption that knowledge could be generated based on human beings' experiences and over their conceptions of the world. Logical characterisation of human inductive learning over their produced conceptions within terminological systems and providing a logical background for theorising over the Human Concept Learning Problem (HCLP) in terminological systems are the main contributions of this research. In order to make a linkage between 'Logic' and 'Cognition', Description Logics (DLs) will be employed to provide a logical description and analysis of actual human inductive reasoning (and learning). This research connects with the topics 'logic and learning', 'cognitive modelling' and 'terminological knowledge representation'.

1. INTRODUCTION

The point of departure is my special focus on the conceptualisation of 'learning'. In this research, learning will be seen as the process of construction, and thus, learning will be assumed to be supported by an epistemology which argues that knowledge is constructed based on human beings' experiences, and over their conceptions of the world, see (Piaget, 1967; Husén and Postlethwaite, 1989; Glasersfeld, 1989; Sjoberg, 2007; Sawyer, 2014).

According to (Bartlett, 1932), concepts might be understood as representations of (aspects of) reality in human beings' minds. Frederic Bartlett, in his studies in experimental psychology, arrived at the phenomenon of 'concept' with his focus on memory analysis. In memory studies subjects recalled details of stories that were not actually there. Considering concepts as mental representations, any concept could be recognised to be equivalent to a psychological entity, see (Peacocke, 1992; Zalta, 2001). Furthermore, concepts could be studied by the representational theory of the mind (and the theory of mental representation), see (Stich, 1992; Margolis and Laurence, 2007; Margolis and Laurence, 2010). In this research, the term 'concept' is suggested as following: "a concept could be said to be a linkage [and interconnection] between the mental representations of linguistic expressions and other mental images (e.g., aspects of representations of the world, and of inner experiences) that a human being has in her/his mind", see (Götzsche, 2013). Kindly observe that the ontological, existential and structural analysis of these linkages is beyond the scope of this paper, but it could be assumed that humans' conceptions are the outcomes and manifestations of their constructed concepts.

Let me begin with an example; Suppose that one (say John) has a visualisation of 'Book' in his mind. Regarding his mental image, he describes (and defines) 'Book' by "Set of written Sheets". Note that I am not interpreting the truth/falsity of John's description (definition) of 'Book', but I am just analysing the logical structure of his description (definition). First, I shall mention that Description Logics (see (Baader et al., 2010)) and concept languages recognise a definition of a concept as a definition of a new (and/or more specified) concept in terms of other previously definition concepts. More precisely, a definition could be defined as an equation between a concept and its description (based on one's conceptions). Obviously, John's conception of the concept 'Book' is transformable into a hypothesis like "Book is a Set of written Sheets". The most important word of this hypothesis is the distinct entity 'Book'. Therefore, John has established the correspondence "Set of written Sheets" to the entity 'Book'. More particularly, John has created a mental assignment from the description "Set of written Sheets" to the entity 'Book'. In fact, John, by means of "is a", has made a logical relation between the distinct entity 'Book' and the description "Set of written Sheets". More specifically, he has made two parallel relationships, i.e.,

i. a hyperonym-hyponym or SuperClass-SubClass relationship, and
ii. a hyponym-hyperonym or SubClass-SuperClass relationship

between a distinct entity and a description. Consequently, all characteristics, features and properties of 'Set of written Sheets' are assigned to 'Book', and vice-versa. From John's point of view (that has supported him in producing his pre-conception of 'Book'), all applications of 'Book' are determined and supported by his primary mental expression and definition "Book is a Set of written Sheets".

This article will—by considering concepts as the amalgamations of mental images and linguistic expressions—focus on analysing the logical characteristics of "humans' inductive[1] reasoning (and learning) over their conceptions". In fact, it will focus on logical analysis of how terminological knowledge could reasonably be assumed to be built based on an individual's conceptions of the world. It attempts to offer an explanatory framework to draw a linkage between logic and cognition. It aims at providing a logical background for providing a terminological scheme and for theorising over the Human Concept Learning Problem (HCLP) in terminological systems.

2. ON HUMAN CONCEPT LEARNING

In my opinion, one of the most fundamental characteristics of human concept learning is using background knowledge, which is formed and shaped over humans' pre-formed concepts and, respectively, over their pre-conceptions. So, this article relies on the idea that humans' pre-formed concepts form their background knowledge. In general, referring to (Matthews, 2014), the term 'background knowledge' represents the knowledge of the world in general, or the knowledge of the life in the specific society, the understanding of which people can be assumed to share as a framework for talking with each other. It shall, therefore, be claimed that any background knowledge could represent an individual's

[1] An inductive logic is a system of evidential support that extends deductive logic to less-than-certain inferences, see [8]. The premises of a strong inductive argument should be capable of providing some degree of support for the logical conclusion, where such support means that the truth of the premises indicates, with some degree of strength, that the conclusion is (could be) true.

i. universal knowledge of the world in general, or
ii. any local knowledge of a specific part of the world in particular.

Furthermore, in the learning sciences, the background knowledge can be defined as 'the knowledge that learners have (learned) both formally in the learning environments as well as informally through their life experiences', see (Marzano, 2004; Fisher and Frey, 2009). Therefore, the background knowledge of any individual can be constructed based on:

a. her/his descriptions of the world based on her/his pre-formed concepts (and pre-concept descriptions), and
b. her/his own conceptions which are generated with regard to the structures of different phenomena/things in her/his mind.

However, there are strong dependencies between

a. pre-concept descriptions (and, thus, pre-conceptions), and
b. the structural conceptions of different phenomena/things.

More specifically, any pre-concept description could be interpreted as an expression of a human's conception of a phenomenon's/things's structure. For example, pre-[concept-descriptions] could be produced based on humans' qualitative and phenomenographic realisations of different phenomena/things. Furthermore, human beings, within processes of concept learning, are concerned with their explanatory, structural, existential and comprehensive conceptions of different phenomena/objects. According to the mentioned characteristics of background knowledge, human concept learning could be suggested as following.

2.1. HUMAN CONCEPT LEARNING

Concept learning is an inductive learning theory that is supported by humans' inductive reasoning processes. Concept learning is logically shaped over a system of evidential support, which is structured over less-than-certain inferences of human beings. This theory is supported by humans' reasoning processes based on their constructed concepts (and their produced conceptions). Concept learning could be generated based on humans' background knowledge (and over their pre-formed concepts and preconceptions) and with regard to their conceptualisations of the characteristics and properties of concepts and through experiencing (e.g., observing, hearing, touching, reading about) various groups of examples of those concepts. Accordingly, humans could focus on hypothesis generation. It shall be concluded that humans become concerned with specification of the conceptualisation of their constructed concepts within their concept learning.

It is worth mentioning that some approaches (in educational and social sciences) have focused on applications of the analysed notion in inductive teaching and inductive learning strategies, see (Taba, 1971; McCracken, 1999; Du and Ling, 2011; Sell et al., 2014). As mentioned, humans could focus on hypothesis generation within their concept learning. The next section will focus on logical analysis of hypothesis generation. In this research, the main references to logic of induction, inductive reasoning and inductive concept learning paradigm are (Lavarac and Dzeroski, 1993; Lavarac and Dzeroski, 1994; Mitchell, 1997; Lehmann, 2009; Lehmann et al., 2011; Lehmann et al., 2014; Lehmann and Voelker, 2014). Also, (baader et al., 2010) is the main reference to Description Logics and Concept Languages. At this point, I shall state that Description Logics (DLs)—under a plethora of names (among them terminological systems and Concept Languages)—attempt to provide descriptive knowledge representation formalisms based on formal semantics. They can support us in revealing some hidden conceptual assumptions that could support us in having a better understanding of 'concept learning' and in proposing more realisable logical descriptions for it in terminological systems. That's why I have employed DLs in this research.

3. A TERMINOLOGICAL BASIS

A series of inductive learning approaches make humans concerned with three kinds of non-monotonic processes:

1. The process of producing their descriptions of more specified concepts regarding their descriptions of more general concepts. For example, Mary could describe her own constructed concept (and conception of) 'Red Dog' regarding her descriptions of her constructed concepts (and conceptions of) 'Red' and 'Dog'. Also, Ann could describe her own constructed concept (and conception of) 'Big Brown Horse' regarding her descriptions of her constructed concepts (and conceptions of) 'Big', 'Brown', and 'Horse'.

2. The processes of reforming and re-organising their conceptions of the same concepts with insights based on their acquired knowledge and new experiences. For example, James could describe his constructed concept (and conception of) 'Spring' by the term "a Season; when all the Trees are Green", or equivalently: $\mathsf{Season} \sqcap \forall \mathsf{hasTree.Green}$. Later on, he may reform his conception of 'Spring' and may produce the description "a season; when some (and, in fact, not all) trees are green", or equivalently: $\mathsf{Season} \sqcap \exists \mathsf{hasTree.Green}$.

3. The process of producing their more specific descriptions of the same concepts with insights based on acquired knowledge within their interactions with new experiences. For example, James could describe the concept 'Spring' by "a season; when some trees are green", or equivalently: $\mathsf{Season} \sqcap \exists \mathsf{hasTree.Green}$. Later on, he may produce the more specific description "a Season of the Moderate Weather; when some Trees are Green", or equivalently: $\mathsf{Season} \sqcap \exists \mathsf{hasTree.Green} \sqcap \exists \mathsf{hasWeather.Moderate}$.

Analysing the processes involved in human concept learning can be interpreted as a comprehensive study [of humans' minds] and as an explanatory and structural study [of humans' selves] that enable them to develop their own understanding of different phenomena's/things' realities within the world. It shall, therefore, be suggested that the phenomenon of 'human concept learning' has strong interrelationships with an existential and structural consciousness (that is related to a kind of ontology, see (Staab and Studer, 2004)). It might, then, be said that any human being actively deals with this ontology in her/his concept learning processes, and becomes concerned with its specification on various levels of her/his conceptualisation. Relying on this ontology, human beings focus on specification and categorisation of their conceptualisation, and accordingly, they induce and learn from the [more, and more] specific descriptions of their conceptions. It shall be stressed that this kind of ontology is shaped over the interrelationships between humans' mental images of the world and their linguistic expressions of the world. Here I focus on analysing terminological knowledge construction (and describing the world based on constructed terminological knowledge) over humans' conceptualisation. The analysis consists of two main categories:

I. The produced conceptions of human beings support them in modifying their terminologies. Suppose that Mary already knows about (and recognises) the concept 'Cyan'. The general concept 'Cyan' is the amalgamation of the mental word 'Cyan' and the mental image of Cyan. In this research, it's assumed that the word 'Cyan' belongs to Mary's terminologies and, respectively, to her terminological knowledge. The concept 'Cyan'

i. is already experienced and known by Mary,
ii. is existing in her terminological knowledge, and
iii. could be, terminologically, interpreted by her (in order to produce a meaningful comprehension for her).

Then, it can provide a terminological principle in her mind. For example, she could identify (and interpret) 'Cyan' as a kind of 'Colour' (i.e., $\mathsf{Cyan} \sqsubseteq \mathsf{Colour}$) in her terminology[2]. Consequently, Mary can apply the terminological principle $\mathsf{Cyan} \sqsubseteq \mathsf{Colour}$ in creating the new more specified world descriptions over the concept 'Cyan'. For example, she may describe a 'Cyan Stone' by "a Stone that has Cyan Colour", formally: $\mathsf{CyanStone} \doteq \mathsf{Stone} \sqcap \exists \mathsf{hasColour}.\mathsf{Cyan}$. This concept description (in the form of a concept definition) is expanded over her conception of the concept 'Cyan'. In Description Logics, a concept definition (represented by $\doteq$) is a definition of a new (or more specified) concept in terms of other previously defined concepts.

II. The produced conceptions of human beings support their developmental processes of terminological construction. Suppose that Simon does not know about the concept 'Cyan'. Then, he, as a basis for his reasoning (and inducing) processes, generates the general concept 'Cyan' and categorises it into his terminology T (let T be a set), i.e.,

[2] It is worth mentioning that the concept inclusion $\mathsf{Cyan} \sqsubseteq \mathsf{Colour}$ presupposes so-called 'colour realism'.

$\mathsf{Cyan} \sqsubseteq T$. In fact, he tries to provide a background for satisfying the concept 'Cyan' by his terminology. Relying on $\mathsf{Cyan} \sqsubseteq T$, T could satisfy Cyan (i.e., $T \vDash \mathsf{Cyan}$); It means that his developed terminology could provide a model for satisfying 'Cyan' based on his terminological interpretations. In particular, $\mathsf{Cyan} \sqsubseteq T$ (as a set membership) could be developed by designing different world descriptions (in the form of assertive principles). For example, Simon may construct assertions like Cyan(c) and Colour(d), where the individuals c and d are the instances of the concepts 'Cyan' and 'Colour' respectively. Note that producing Cyan(c) and Colour(d) are the products of his conceptions and based on his experiences. Therefore, he—mentally—satisfies the assertions Cyan(c) and Colour(d) by semantic models like

- Terminological Knowledge $\vDash$ Cyan(c), and
- Terminological Knowledge $\vDash$ Colour(d).

For example, according to the semantic model '$\vDash$ Cyan(c)', Simon

i. has experienced the individual c,
ii. has interpreted that c is an instance of his constructed concept Cyan, and
iii. has made a mental principle based on the assertion Cyan(c).

Consequently:

iv. Simon's terminological knowledge could satisfy Cyan(c).

Furthermore, he will be able to—inductively—subsume more specified concepts and their instances under his comprehension of Cyan. For example, he could induce that the Cyan Chairs are all Cyan (i.e., $\mathsf{CyanChair} \sqsubseteq \mathsf{Cyan}$).

As another example, Simon, by following his mental principle '$\mathsf{Cyan} \sqsubseteq \mathsf{Colour}$' and by considering the fact that 'all chairs are some kinds of furnitures', could induce that:

$$(\mathsf{CyanChair} \sqsubseteq \mathsf{Colour})$$
$$\&$$
$$(\mathsf{CyanChair} \sqsubseteq \mathsf{Furniture}).$$

This logical term means that all Cyan Chairs are Colourful and Furnitures. Accordingly, he can induce that:

$$\mathsf{CyanChair} \doteq$$
$$(\mathsf{Chair} \sqcap \exists \mathsf{hasColour.Cyan}) \equiv$$
$$(\mathsf{Furniture} \sqcap \exists \mathsf{hasColour.Cyan}).$$

3.1. HUMAN CONCEPT LEARNING AND HYPOTHESIS GENERATION

In concept learning, human beings, with regard to a set of experienced examples and over their background knowledge, focus on generating hypotheses. Subsequently, they focus on their generated hypotheses in order to adopt them in their world descriptions and for their reasoning processes. More specifically, they construct large numbers of hypotheses and record them in their minds. Accordingly, they search through the huge space of their hypotheses in order to find the most proper and useful ones and to describe and specify their constructed and developed knowledge based on them.

Assessed by mathematical logic, the search spaces are capable of archiving the reflexive and transitive relations. In fact, these spaces must be expressed by:

a. strictly-less-than (as well as less-than), and
b. proper-subset (as well as subset) relations.

Recording the relations:

- a is less than b (and b is greater than a), and
- A is the subset of B (and B is the superset of A)

are expressible over reflexive and transitive relations. Focusing on these characteristics, the search spaces could be represented by the binary quasi-order relation (Q , $\leqslant$), where Q is a set and '$\leqslant$' is a relation defined over Q. Any infinite sequence of the elements of Q could contain an increasing pair like (p_i , p_j), where $i \leqslant j$, see (Higman, 1952; Kruskal, 1972).

In my opinion, the most significant characteristic of a binary quasi-order relation is that 'if p and q are two elements of a binary quasi-order relation, then they will be comparable and, in fact, the relations $p \leqslant q$ and $q \leqslant p$ support the comparability'.

According to the mentioned features of search spaces, the binary relation (C , $\leqslant$) over the set of a human's constructed concepts (represented by C) is quasi-order and it is reflexive and transitive. The main reason for applying quasi-order relations is the fact that any concept learning relies on 'induction' and on 'comparability', and inductive reasoning is expressible over 'less-than' and 'subset' relations. More specifically, a quasi-order, as a well-founded induction, can be applied to the set of humans' constructed concepts (or C) in order to express concept subsumption (or concept inclusion). It shall be emphasised that concept subsumption is the most fundamental feature of hierarchical structures in terminological knowledge. Accordingly, the terminological principles based on concept subsumptions could be expressed.

Mechanism

In concept learning, human beings generate mental mappings (like L) from their primary constructed set of concepts (or C) into all its subsets (or 2^C) such that:

$$\forall p \in C,\ L(p) \subseteq \{\ q \in C \mid q \leqslant p\ \}.$$

L is a proper mental mapping if and only if for all constructed concepts A and B,

$$B \in L(A) \Rightarrow A \not\equiv B.$$

Going back to the example of Cyan, for human beings, it is conceptualisable and, thus, understandable that there is a subsumption relation between Cyan and CyanStone like:

$$\text{CyanStone} \sqsubseteq \text{Cyan}.$$

Furthermore, it's conceptualisable and understandable that:

$$\text{Cyan} \in \{\text{Cyan}, \text{Stone}\}.$$

Obviously, Cyan and CyanStone are not equivalent (they are not the same based on all possible terminological interpretations).

It shall be stressed that there are two kinds of mental mappings[3]:

1. Complete Mapping: Considering Cyan and CyanStone, humans could conceptualise, interpret and, respectively, understand that CyanStone $\sqsubseteq$ Cyan (i.e., all Cyan Stones are Cyan). For example, Maria can reach the concept CyanDoor from Cyan (regarding CyanStone $\sqsubseteq$ Cyan) by means of a complete mental mapping. Then, she can induce: CyanDoor $\sqsubseteq$ Cyan.

2. Weak Mapping: Regarding isA(Cyan, Colour) $\sqsubseteq \top$[4], Michael could induce the world description isA(Cyan, Paint) based on his own conception of 'the equivalence relation between the world descriptions isA(Cyan, Colour) and isA(Cyan, Paint)'. In fact, the world description isA(Cyan, Paint) could be induced from a Tautology by means of a weak mental mapping.

4. THE HUMAN CONCEPT LEARNING PROBLEM (HCLP)

Suppose that the function $\mathfrak{C}_K(C)$ describes that 'a human being has constructed ($\mathfrak{C}$) the concept C on a basis provided by her/his constructed knowledge (K)'. I shall draw your attention to the following components of knowledge:

➢ Component I (Terminologies)

T stands for humans' terminological knowledge, which is represented over constructed concepts (in humans' minds). The terminological component of

[3] I define a mental mapping as a mapping (function) from a concept into another concept.

[4] isA(Cyan, Colour) is a role assertion (= a world description over a role in Description Logics). It expresses that "Cyan is a Colour". This world description is a tautology (it is true for all possible interpretations), and thus, it belongs to the top concept (or $\top$).

knowledge is highly concerned with concept subsumption (concept inclusion) and concept equality.

A. **Concept Subsumption (Concept Inclusion).** Human beings, by interpreting concept subsumptions (like $E \sqsubseteq F$), produce their terminological models (like I) in order to [mentally] satisfy the concept subsumptions. Formally, they produce $I \vDash (E \sqsubseteq F)$. This model is semantically valid (and it is logically meaningful) if and only if the interpretation of E is the subset of the interpretation of F, or formally: $E^I \subseteq F^I$. Accordingly, human beings by limiting their terminological interpretations to their meaningful understandings could—terminologically—understand[5] that E is the sub-concept of (sub-concept description of) F, or formally: $E \sqsubseteq F$. In fact, humans focus on producing semantic models (like UND) in order to satisfy $E \sqsubseteq F$. Formally: $UND \vDash (E \sqsubseteq F)$. Subsequently, $E^{UND} \subseteq F^{UND}$. For example, a terminological interpretation like I could be produced in order to provide a terminological model and to support $\mathsf{Cyan}^I \subseteq \mathsf{Colour}^I$ (regarding concept subsumption $\mathsf{Cyan} \sqsubseteq \mathsf{Colour}$). Subsequently: $\mathsf{Cyan}^{UND} \subseteq \mathsf{Colour}^{UND}$. In fact, it has been understood that 'Cyan is a Colour'.

B. **Concept Equality.** Human beings, by interpreting concept equalities (like $C \equiv D$), produce their terminological models (like I) in order to satisfy the concept equalities. Formally, $I \vDash (C \equiv D)$. This model is semantically valid (and it is logically meaningful) if and only if the terminological interpretation of C is equal to the terminological interpretation of D, or formally: $C^I = D^I$. Subsequently, humans by limiting their terminological interpretations to their meaningful understandings could understand that C and D are equal ($C = D$). In fact, they produce understanding models (like UND) in order to satisfy $C \equiv D$. Formally: $UND \vDash (C \equiv D)$. Subsequently, $C^{UND} = D^{UND}$. For example, a terminological interpretation like I could be produced in order to provide a terminological model and to support $\mathsf{Colour}^I = \mathsf{Paint}^I$ (regarding $\mathsf{Colour} \equiv \mathsf{Paint}$). Subsequently, $\mathsf{Colour}^{UND} = \mathsf{Paint}^{UND}$. In fact, it has been understood that 'Colour and Paint are equivalent'.

➢ Component II (World Descriptions)

The symbol W stands for humans' World Descriptions over their constructed concepts. This component of knowledge is concerned with (i) instance assertion (or identifying a phenomenon/thing as a member of a constructed concept) and with (ii) relation assertion (or relating the instances of various concepts to each other).

A. Humans, by interpreting that the individual a is an instance of the constructed

[5] This 'understanding' is a limit of a 'terminological interpretation' based on own 'conceptualisation'. Therefore, it is—existentially—terminological and conceptual.

concept C, produce their terminological interpretations (like I) in order to satisfy the concept assertions $C(a)$. Formally, they produce semantic models like $I \vDash C(a)$. This model is logically meaningful if and only if the terminological interpretation interprets the individual a as an element of (and as a kind of) the concept C, or formally: $a^I \in C^I$. Humans by limiting their terminological interpretations to their meaningful understandings could understand that a is a kind of C. In fact, they produce understanding models (like UND) in order to satisfy $C(a)$ (ie. $UND \vDash C(a)$). Subsequently, $a^{UND} \in C^{UND}$. For example, Elizabeth can interpret and, respectively, understand that her personal computer (represented by pcez) is a Machine. Formally: $I \vDash$ Machine(pcez). So, $\text{pcez}^I \in \text{Machine}^I$. Therefore, $UND \vDash$ Machine(pcez) and, thus, $\text{pcez}^{UND} \in \text{Machine}^{UND}$. In fact, Elizabeth has understood that her personal computer is a machine.

B. Humans, by interpreting that the relation (a,b) is an instance of (is a kind of) the relationship R, produce their interpretation models (like I) in order to satisfy the role assertion $R(a,b)$. Formally, they produce semantic models like $I \vDash R(a,b)$. This semantic model is logically meaningful if and only if the interpretation of the tuple (a,b) belongs to the interpretation of R, or formally: $(a,b)^I \in R^I$. Humans by limiting their terminological interpretations to their meaningful understandings could understand that the individuals a and b are related by a relation like $R(a,b)$. In fact, they produce understanding models (like UND) in order to understand the relation $R(a,b)$. Formally: $UND \vDash R(a,b)$. Subsequently, $(a,b)^{UND} \in R^{UND}$. For example, Bob could interpret and, respectively, could understand that his magnet (represented by magb) attracts a pin (represented by pinb). Then: $I \vDash$ Attract(magb, pinb) and, thus, $(\text{magb, pinb})^I \in \text{Attract}^I$. Accordingly, $UND \vDash$ Attract(magb, pinb) and, thus, $(\text{magb, pinb})^{UND} \in \text{Attract}^{UND}$. In fact, Bob has understood that his magnet attracts his pin.

➢ Component III (Rules)

Suppose that the symbol R stands for Rules. First, I shall claim that any rule (in such a terminological system) is logically dependent on (and supported by) a logical implication. For example, considering 'Thirst' and 'Drinking' as two concepts, the logical term '$R \vDash$ (Thirst $\Rightarrow$ Drinking)' denotes that if one (say John) has been interpreted and, respectively, has been understood to be thirsty and be an instance of (and described by) the concept 'Thirst', then John is, also, an instance of (and described by) the concept 'Drinking'. Formally:

$$[\ (\text{john} \in \text{Thirst}) \sqcap (\text{Thirst} \sqsubseteq \text{Drinking})\ ]$$
$$\Rightarrow$$
$$\text{john} \in \text{Drinking}.$$

This logical term expresses that 'John is thirsty and the thirst will be followed by drinking, so, John is supposed to drink'.

4.1. KNOWLEDGE

Regarding the components I, II and III (terminologies, world descriptions, and rules), the triple $K = \langle T, W, R\rangle$ could represent the humans' constructed knowledge. However, it could be believed that the third component (R) is expressible based upon the components T and W. Let me be more specific:

a. Considering the logical term '$R \vDash (C \Rightarrow D)$', where R stands for a rule, and C and D stand for two concepts (concept descriptions), we can understand that there exists an individual (like c) for which, there is a logical implication between $C(c)$ and $D(c)$. Formally, $\exists c; R \vDash (C(c) \Rightarrow D(c))$. Obviously, R satisfies an implication [and indication] from a concept assertion (as a world description) into another concept assertion (as a world description). Therefore, the rule '$R \vDash (C(c) \Rightarrow D(c)$' has been split into two world descriptions and over a terminology (i.e., it has been expressed in the form of two world descriptions and based on a terminology).

b. Considering the logical term '$R \vDash (P \Rightarrow Q)$', where R stands for a rule, and P and Q stand for two roles (role descriptions), we can understand that there are two individuals (like a and b) for which a logical implication between $P(a,b)$ and $Q(a,b)$ is satisfied. Formally, $\exists a, b; R \vDash (P(a,b) \Rightarrow Q(a,b))$. This formalism expresses that the rule R satisfies "P implies Q" (where P and Q are two relations between a and b). Again, this implication has been described from a role assertion (as a world description) into another role assertion (as a world description). Therefore, such a rule has been split into two world descriptions and over a terminology (i.e., it has been expressed in the form of two world descriptions and based on a terminology).

Taking **a** and **b** into consideration, it shall be concluded that the constructed knowledge could be expressed in the form of world descriptions and based on terminologies. Therefore, the component R could be eliminated and, thus, it could be concluded that the tuple $K = \langle T, W\rangle$ represents the humans' constructed knowledge.

4.2. KNOWLEDGE CREATION – MECHANISM

The mechanism of knowledge construction must be checked over concepts and roles.

A. Construction over Concepts

In concept learning, human beings become concerned with a set of experienced phenomena/things in order to form new building blocks of their knowledge construction. Expressing new concept descriptions is highly dependent on

- a set of *Experienced Constructive* examples of *Concepts* (or Exp^{+}_{c}), and
- a set of *Experienced Non-Constructive* examples of *Concepts* (or Exp^{-}_{c})

over pre-formed concepts (and pre-conceptions). Accordingly, humans become concerned with a unifying set (like Exp_c) of their multiple constructive and non-constructive examples. Therefore:

$$Exp_c = \{ Exp^+_c , Exp^-_c \}, \text{ where:}$$

- $Exp^+_c = \{ a \in Exp_c(W) \mid C(a) \in W \} \subseteq \mathfrak{C}_K(C)$, and
- $Exp^-_c = \{ b \in Exp_c(W) \mid \neg C(b) \in W \} \subseteq \mathfrak{C}_K(\neg C)$.

In fact, any Exp^+_c consists of the individuals which could be described by humans' constructed concepts. Any member of Exp^+_c can be supported by humans' world descriptions (based on concept assertions). Also, Exp^-_c consists of the individuals which could not be described by humans' constructed concepts and cannot be supported by their world descriptions (based on concept assertions).

For example, considering Exp^+_{Dog} as the set of Martin's experienced constructive examples of 'dogs', Martin's German Shepherd belongs to Exp^+_{Dog}. Also, considering Exp^-_{Dog} as the set of Martin's experienced non-constructive examples of 'Dogs', a friend's rabbit belongs to Exp^-_{Dog}. Martin, by increasing the number of his experienced constructive and non-constructive examples of 'Dogs', could develop his knowledge of dogs over his own construction, conception, interpretation, and meaningful comprehension of 'Dogs'.

B. Construction over Roles

Describing more specified roles is dependent on

- a set of *Experienced Constructive* examples of *Roles* (or Exp^+_r), and
- a set of *Experienced Non-Constructive* examples of *Roles* (or Exp^-_r)

over pre-formed roles (as the relations between the instances of preformed concepts). Accordingly, humans become concerned with a unifying set of their multiple constructive and non-constructive examples with regard to their pre-constructions of roles. This unifying set is denoted by Exp_r. Then:

$$Exp_r = \{ Exp^+_r , Exp^-_r \}, \text{ where:}$$

- $Exp^+_r = \{ p, q \in Exp_r(W) \mid R(p,q) \in W \} \subseteq \mathfrak{C}_K(R)$, and
- $Exp^-_r = \{ p, s \in Exp_r(W) \mid \neg R(p,s) \in W \} \subseteq \mathfrak{C}_K(\neg R)$.

In fact, any Exp^+_r consists of the individuals which could be described by humans' constructed concepts and, respectively, by relating the instances of the constructed concepts. Accordingly, any member of Exp^+_r can be supported by their world descriptions (based on role assertions). Additionally, Exp^-_r consists of the individuals which could not be described by humans' constructed concepts and, respectively, cannot be described by relating the instances of the constructed concepts. Thus, they cannot be supported by humans' world descriptions (based on role assertions).

For example, consider $Exp^{+}_{marriedTo}$ as the set of David's experienced constructive examples of 'people who are married to each other'. David knows that Ronald and Susan are married. So, the relation (rolnad, susan) belongs to David's $Exp^{+}_{marriedTo}$. Now let $Exp^{-}_{marriedTo}$ be the set of David's experienced non-constructive examples of 'people who are married to each other'. David knows that Peter and Rebeca are not married. So, the relation (peter, rebeca) belongs to $Exp^{-}_{marriedTo}$. David, by increasing the number of his experienced constructive and non-constructive examples of 'people who are married to each other' (and by knowing more married and more non-married pairs of people), could develop his knowledge.

Conclusion

According to $K = \langle T, W \rangle$, human beings, by generating T and W, support their knowledge (= K) construction. Consequently:

- if a concept assertion (like $D(a)$) is satisfied by the constructed knowledge, then: $\forall a \in Exp^{+}_{c}(W); K \vDash D(a)$,
- if a concept assertion (like $D(b)$) is not satisfied by the constructed knowledge, then: $\forall b \in Exp^{-}_{c}(W); K \nvDash D(b)$,
- if a role assertion (like $R(a,b)$) is satisfied by the constructed knowledge, then: $\forall a, b \in Exp^{+}_{r}(W); K \vDash R(a,b)$, and
- if a role assertion (like $R(a,c)$) is not satisfied by the constructed knowledge, then: $\forall a, c \in Exp^{-}_{r}(W); K \nvDash R(a,c)$.

4.3. FROM CONCEPTIONS TO PREDICATES

According to my research in (Badie, 2016), there is a sort of reflector functions from humans' conceptions into predicates. Obviously, we could not directly move from 'concepts' to 'predicates' (and, respectively, to 'truth'), but, I shall stress that the central focus of this research is on a logical analysis of concept learning and hypothesis generation (and not on an ontological analysis of concepts). So, logics allow me to relate conceptions and predicates. My focus is on the logical fact that human beings can transform their conceptions [of their constructed concepts] into the [logically] equivalent ones in the form of predicates. Subsequently, the expressed predicates provide reasonable logical models that—semantically—can satisfy the collections of their generated hypotheses. Representing Concepts, Predicates and Hypotheses by C, P and H respectively, we formally have:

$$[Reflection: C \rightarrow P] \vDash H.$$

4.3.1. The Detailed Examination I

A. Experienced Concepts

The set of experienced concepts (or $Exp_{Concept}$) is equal to the union of the following sets:

Set 1: The set of experienced constructive examples of constructed concepts that could be represented by

$$Exp^{+}_{Concept} = \{ \text{ individual } \in Exp_{Concept}(W) \mid \text{Concept(individual)} \in W \}.$$

$Exp^{+}_{Concept}$ is subsumed under humans' constructed concepts. They could be generated over humans' constructed knowledge (or $\mathfrak{C}_K$ (Concept)). Consequently, the humans' constructed knowledge satisfies their own world descriptions based on their concept assertions and over their experienced constructive examples.

Set 2: The set of experienced non-constructive examples of constructed concepts that could be represented by

$$Exp^{-}_{Concept} = \{ \text{ individual } \in Exp_{Concept}(W) \mid \neg\text{Concept(individual)} \in W \}.$$

$Exp^{-}_{Concept}$ is subsumed under humans' non-constructed concepts. They could not be generated over humans' constructed knowledge (or $\mathfrak{C}_K$ (¬Concept)). Consequently, the humans' constructed knowledge doesn't satisfy their own world descriptions based on their concept assertions and over their experienced non-constructive examples.

B. Experienced Roles

The set of experienced roles (or Exp_{Role}) is equal to the union of the following sets:

Set 1: The set of experienced constructive examples of constructed roles that could be represented by:

$$Exp^{+}_{Role} = \{ \text{ individual}_1 \ \& \ \text{individual}_2 \in Exp_{Role}(W) \mid \text{Role}(\text{individual}_1,\text{individual}_2) \in W \}.$$

Exp^{+}_{Role} is subsumed under humans' constructed roles and, in fact, under humans' constructed concepts. They could be generated over humans' constructed knowledge (or $\mathfrak{C}_K$ (Role)). Consequently, the humans' constructed knowledge satisfies their own world descriptions based on their role assertions and over their experienced constructive examples.

Set 2: The set of experienced non-constructive examples of constructed roles that could be represented by:

$$Exp^{-}_{Role} = \{\ \text{individual}_1\ \&\ \text{individual}_2 \in Exp_{Role}(W) \mid \\ \neg\text{Role}(\text{individual}_1,\text{individual}_2) \in W\ \}.$$

Exp^{-}_{Role} is subsumed under humans' non-constructed roles. They could not be generated over humans' constructed knowledge (or $\mathfrak{C}_K$ (¬Role)). Consequently, the humans' constructed knowledge doesn't satisfy their own world descriptions based on their role assertions and over their experienced non-constructive examples.

4.3.2. The Detailed Examination II

Humans' conceptions [of their own constructed concepts] could be represented in the form of 'unary predicates' in order to be described and represented. Similarly, humans' conceptions [of their own constructed roles] could be represented in the form of 'binary (or any other *n*-ary) predicates' in order to be represented. A predicate could be interpreted to be an expression and assigner of concepts' different characteristics. Then, it could assign different characteristics to [and transmit them into] propositions (and statements) or even into truth-values. Consequently, regarding humans' constructed knowledge (K) and reflecting (mirroring) the conceptions in predicates, the tuple $\langle T, W\rangle$ could be expressed and analysed by predicate logic. Therefore, the terminological knowledge (T) could be structured over:

1. predicate symbols (e.g., unary, binary, …, *n*-ary),
2. variable symbols,
3. constant symbols, and
4. function symbols.

It is worth mentioning that these four kinds of symbols are identified as non-logical symbols in Predicate Logic, because they—independently—don't cause any logical consequence in logical descriptions. Also, all world descriptions are shaped by utilising multiple descriptions over the provided terminologies. Subsequently:

1. a predicate symbol denotes something which is a predication of the subject,
2. a variable symbol is what a human asserts the predicate to it, and
3. the constant symbols could be asserted to any variable.

Beside them,

4. function symbols operate the variable symbols.

Taking the translated terminological knowledge and, subsequently, the translated world descriptions into consideration, we could express the components of the detailed examination I as follows:

A. Experienced Unary Predicates

The set of humans' experiences could be represented in the form of unary predicates ($Exp_{Predicate}$) and be acknowledged as the union of the following sets:

Set 1: The set of humans' experienced constructive examples of their described unary predicates. It could be represented by:

$$Exp^{+}_{Predicate} = \{ \text{ constant } \in Exp_{Predicate}(W) \mid \text{Predicate(constant)} \in W \}$$

for expressing world descriptions based on unary predicates. The experienced constructive examples are expressed by humans' described predicates within their constructed knowledge (i.e., $\mathfrak{C}_K$ (Predicate)). Consequently, $\mathfrak{C}_K$ satisfies the humans' world descriptions based on their concept assertions and over their experienced constructive examples.

For example, consider the constant palm as an experienced constructive example of the concept Tree. Formally: palm $\in$ Tree within $\mathfrak{C}_K$. Note that this constructive example could be subsumed under the described binary predicate (relation) 'is a' within constructed knowledge. In fact, isA(palm, Tree) supports 'palm $\in$ Tree'.

Set 2: The set of humans' experienced non-constructive examples of their described unary predicates. It could be represented by:

$$Exp^{-}_{Predicate} = \{ \text{ constant } \in Exp_{Predicate}(W) \mid \neg\text{Predicate(constant)} \in W \}.$$

The experienced non-constructive examples could not be expressed by described predicates within constructed knowledge (i.e., $\mathfrak{C}_K$ (¬Predicate)). Consequently, $\mathfrak{C}_K$ doesn't satisfy the world descriptions over experienced non-constructive examples.

For example, consider the constant rose as an experienced non-constructive example of the concept Tree. Formally: rose $\notin$ Tree. This non-constructive example is not expressible and is not satisfied by the binary predicate (relation) "is a" within $\mathfrak{C}_K$. In fact, $\mathfrak{C}_K$ doesn't satisfy the world description "rose is a tree". Therefore, we have ¬isA(rose, tree) over the experienced non-constructive example rose $\notin$ Tree.

B. Experienced Binary Predicates

The set of humans' experiences could be represented in the form of binary predicates ($Exp_{Predicate}$) and be acknowledged as the union of the following sets:

Set 1: The set of humans' experienced constructive examples of their described binary predicates. It could be represented by:

$$Exp^{+}_{Role} = \{ \text{ constant}_1 \text{ \& constant}_2 \in Exp_{Predicate}(W) \mid \\ \text{Predicate(constant}_1\text{, constant}_2) \in W \}$$

for expressing world descriptions over binary predicates. The experienced constructive examples are expressed by humans' described predicates within their constructed knowledge (i.e., $\mathfrak{C}_K$ (Predicate)). Consequently, $\mathfrak{C}_K$ satisfies the humans' world descriptions based on their concept assertions and over their experienced constructive examples.

For example, the constants bob and mary could be the experienced constructive examples of the role fatherOf. Formally: (bob, mary) $\in$ fatherOf. This constructive example could be subsumed under a binary predicate within constructed knowledge. Equivalently, we have the world description fatherOf(bob, mary).

Set 2: The set of humans' experienced non-constructive examples of their described binary predicates. It could be described by:

$$Exp^{-}_{Predicate} = \{ \text{constant}_1 \ \& \ \text{constant}_2 \in Exp_{Role}(W) \mid \neg\text{Predicate}(\text{constant}_1, \text{constant}_2) \in W \}.$$

The experienced non-constructive examples could not be expressed by described predicates within constructed knowledge (i.e., $\mathfrak{C}_K$ (¬Predicate)). Consequently, $\mathfrak{C}_K$ doesn't satisfy the world descriptions based on role assertions and over experienced non-constructive examples.

For example, the constants bob and silvia could be the experienced non-constructive examples of the role fatherOf. Formally: (bob, silvia) $\notin$ fatherOf. We have the world description ¬fatherOf(bob,silvia). Thus, $\mathfrak{C}_K$ doesn't satisfy the world description 'Bob is the father of Silvia'.

4.4. SUMMARISATION

Let me go back to the example of Martin's conception of 'Dogs'. In this example, Exp_{Dog} is the set of Martin's experienced examples of 'Dogs' and it is equal to Exp^{+}_{Dog}. Relying on Exp_{Dog}, we have:

a. Exp^{+}_{Dog} = { i $\in$ Exp_{Dog} (*Martin's description of dogs*) | Dog(i) $\in$ *Martin's description of dogs* } $\subseteq$ *Martin's constructed knowledge of dogs*. Consequently, Martin's knowledge of dogs satisfies the concept assertion Dog(i), where the individual i belongs to his constructive examples of dogs within his own description of dogs.

b. Exp^{-}_{Dog} = { j $\in$ Exp_{Dog} (*Martin's description of dogs*) | ¬Dog(j) $\in$ *Martin's description of dogs* } $\subseteq$ *Martin's knowledge of NOT-dogs*. Consequently, Martin's knowledge of dogs doesn't satisfy the concept assertion Dog(j), where the individual j belongs to his non-constructive examples of dogs within his own description of dogs.

According to the logical term '[*Reflection*: $C \rightarrow P$] $\vDash$ H', a *Reflection* has been described as a function that mirrors the humans' conceptions [as the outcomes of their constructed concepts] in predicates. These predicates could be used in expressing humans' constructed concepts in terminological systems. A sort of reflector functions from human beings' constructed concepts and conceptions into predications (and described predicates) could [formal-]semantically satisfy the logical hypotheses.

It shall be claimed that the transmission of concepts from the detailed examination **I** to the detailed examination **II**, is a kind of logical reflection that semantically forms new hypotheses. The generated hypotheses could be used to describe humans' grasps of the world in terminological systems (and terminological knowledge representation systems).

5. CONCLUSIONS

In this article, the phenomenon of 'learning' has been assumed to be supported by an epistemology which argues that knowledge could be constructed from an interaction between human beings' experiences and over their conceptions of what they have experienced (e.g., studied, seen, heard, felt, touched). Accordingly, in this research, the term 'concept learning' has been expressed as "the developmental process of concept construction and specification of the constructed concepts". Note that I have not focused on an ontological analysis of concepts, but I have—by considering the theoretical idea that concepts might be said to be a linkage between the mental representations of linguistic expressions and the other mental images that a human being has in her/his mind—focused on conceptual and logical analysis of how terminological knowledge could reasonably be assumed to be built based on an individual's conceptions of the world and over her/his constructed terminological basis. Consequently, a logical background for theorising over the Human Concept Learning Problem (HCLP) has been provided. It has been concluded that the problem of human concept learning could be expressed in the form of a function that expresses a human who constructs a concept on a basis provided by her/his constructed knowledge. Also, her/his knowledge is constructed based upon her/his terminological basis and over her/his descriptions of the world. Accordingly, humans are concerned with a set of experienced information in order to form new building blocks of their knowledge. Then, the union of the:

- Experienced Constructive Examples of their Constructed Concepts and
- Experienced Non-Constructive Examples of their Constructed Concepts

have been considered as the set of Experienced Concepts that support the developmental processes of knowledge construction. Assessed by logics, a function (which I have called 'reflector') mirrors the humans' conceptions in predicates. The predicates could be used in expressing humans' constructed concepts within terminological systems. A sort of reflector functions from humans' 'constructed concepts and conceptions' into 'predications and described predicates' could semantically satisfy the collections of humans' hypotheses. The succession:

'Concept → Conception → Predication → Predicate'

represents a logical flow that attempts to satisfy the collection of logical hypotheses. More specifically, the transmission of concepts from the detailed examination **I** to the detailed examination **II**, has expressed a kind of logical reflection that semantically forms new hypotheses in order to describe humans' grasps of the world in terminological systems. It could make sense in terminological knowledge representation systems and terminological knowledge bases.

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PAPER O. LOGICAL FOUNDATION OF INDUCTIVE MEANING CONSTRUCTING IN CONSTRUCTIVIST INTERACTIONS

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This paper is submitted

ABSTRACT

In the framework of constructivism, learners with insights based on their pre-structured knowledge and pre-conceptions could actively participate in an interaction with their mentors. In such a framework, constructing and developing meanings and producing meaningful understandings of the world are the central objectives. This research, by employing Description Logics, will analyse inductive meaning constructing as a valuable product of constructivist interactions between mentors and learners. The main idea is based on a new scheme for interpretation based on semantics, induction and interaction.

1. INTRODUCTION

My point of departure is the special focus on Activity-based Communication Theory (see (Allwood, 2007; Allwood, 2013)) that argues that "any communication is a sharing of information, cognitive content and understanding with varying degrees of consciousness and intentionality". Correspondingly, an interaction between a learner and a mentor can be interpreted as their co-activation (consisting of their shared actions and transactions). More specifically, in an interaction between a learner and a mentor, they, both, become concerned with their co-activations (i.e., their collaborations, co-operations, and co-ordinations). In such a framework, mentor and learner exchange questions, answers, actions and transactions concerning their multiple descriptions, specifications, explanations and justifications.

The central focus of this article is on a constructivist theory (and model) of learning in the context of mentor-learner interactions. This model of learning could work as an explanatory, heuristic and developmental framework. For more information, see (Pask, 1975; Pask, 1980; Simpson, 1989; Scott, 2001; McIntyre Boyd, 2004). Relying on this model of learning, the learner-mentor interactions might be described as a constructivist account of their understandings. In addition, constructivist interactions could be described and specified over their conceptions and intentions. Such a framework explains how learner-mentor interactions could lead them to produce their own meaningful understandings based on their produced conceptions of the world. The produced local meaningful understanding (of any concept) give opportunities to the agents to produce their deeper personal understandings within their collaborative process of knowledge construction. For example, the learner is given an opportunity to produce her/his more proper understanding of applications of a mathematical formula, or the mentor is given an opportunity to produce a better understanding of the current problems of the learner concerning that mathematical formula's applications. I shall claim that:

- the negotiation of the learner's and the mentor's produced meanings,
- the reflection of their produced meanings on their background knowledge, and
- the affection of their personal developed knowledge on their universal knowledge

are the most significant products of constructivist interactions.

Let me be more specific on the expressions ‘constructivism’ and ‘constructivist interaction’. According to (Glasersfeld, 1989; Glasersfeld, 2001), constructivism was introduced in the modern era by Jean Piaget (see (Piaget and Cook, 1952)) as a way of thinking about cognition and knowledge. It is worth mentioning that Piaget’s developmental theory of learning argues that the constructivist model of learning is concerned with how the individual learner goes about the construction of knowledge in her/his own cognitive apparatus, see (Phillips, 1995). Furthermore, I shall stress that Vygotsky’s theories about humans’ interactions and humans’ minds in society (see (Vygotsky, 1978a, Vygotsky, 1978b)) have strongly supported the concept of ‘social constructivism’. Vygotsky believed that ‘social interaction’ plays a fundamental role in the process of humans’ cognitive development. According to Vygotsky’s theory, it’s believed that an individual who has stronger understandings and higher abilities in particular domains could be a so-called ‘mentor’. The concept of ‘mentor’ could be labelled by the abbreviation MKO (i.e., More Knowledgable Other). It shall be drawn into consideration that mentors are advanced learners who are always constructing and developing their personal knowledge through their constructivist interactions. Additionally, Vygotsky introduced ZPD (i.e., the Zone of Proximal Development) in order to express the concept of ‘learning’ by an individual learner (i) under MKO’s supervisions and/or (ii) with her/his collaborations with other individuals. It shall be concluded that learners can learn (could do ‘learning’ as their main task and role) in this zone.

This research will focus on a logical analysis of meaning construction within constructivist interactions relying on my own semantic framework. I will employ a Concept Language (Description Logics: DLs) in order to conceptualise my main logical ideas behind that framework. Accordingly, I will provide a logical and [formal-]semantic specification of ‘concepts’, ‘definitions’ and ‘meanings’. Subsequently, a conceptual and logical background for ‘semantic interpretation’ and ‘meaning construction’ will be provided. The main contribution of this research is that it provides a backbone for formal semantic analysis of meaning construction (that supports meaningful understanding production) within constructivist interactions.

2. CONCEPTS

From educationalists’ and educational psychologists’ perspectives, a concept (as a conceptual entity) can be identified by critical characteristics and properties shared by its examples, see (Parker, 2008; Parker, 2011). At this point, I need to focus on the concept of ‘meaning’ in order to be more clear on the concept of ‘concepts’. Section 6 will be more specific on logical analysis of meanings. In the framework of constructivism, meanings—to a large extent—influence any individual human’s knowledge constructions based on her/his background knowledge. In fact, meanings are some conceptual structures that are constructed based on conceptual entities. Thus, any conceptual entity can be interpreted to be a building block and a basic material of a conceptual structure. My theoretical model names these conceptual entities ‘concepts’. A conceptual entity, as a representation of a piece of reality/fiction in individual’s mind, can be interpreted to be a basic material of [to-be-constructed] meanings.

Human beings, with regard to their own concept constructions, can reason whether their different experienced phenomena could come under the label of their own constructed concepts. It shall be stressed that only an activated conceptual entity can be an instance of a concept and, thus, it shall be claimed that the learner transforms her/his experienced phenomena into some mental entities (and activates them in her/his mind) in order to consider them as the instances of concepts. Subsequently, individuals’ understandings and, also, their following acts and reacts materialise in virtue of their personal grasp of their constructed concepts. Regarding my theoretical model, human beings express their constructed concepts in the form of their conceptions.

From the point of view of logics, a concept (conceptual entity) could correspond to a distinct entity or to its essential features, see (Encyclopaedia Britannica, 2007). More specifically, in the framework of constructivism, the agents’ conceptions [of their constructed concepts] could be expressed in the form of predicates, see (Clapham, 2014; Blackburn, 2016). Predicates could be interpreted as assigners of conceptions’ characteristics. They can assign those characteristics to statements or even into truth-values. Relying on this assumption, conceptions and their interrelationships are hierarchically describable. Let me offer an example:

> Mentor: Who is a teacher?
> Sarah: Teacher is a person who has—at least—one student.

Relying on Predicate Logic, the predicate Teacher has been described as the specified form of the predicate Person. Translated into Description Logics (DLs), the concept Teacher is described by $\mathsf{Teacher} \doteq \mathsf{Person} \sqcap \exists \mathsf{hasStudent}.\mathsf{Person}$. Also, as you will see in next section, the concept Teacher can be described by $\mathsf{Teacher} \sqsubseteq \mathsf{Person} \sqcap \exists \mathsf{hasStudent}.\mathsf{Person}$ (considering the concept Teacher as a sub-concept of the concept Person). This description has provided a concept construction that supports Sarah’s specification of the conceptualisation of the concept Teacher.

It’s assumed that human beings’ mental images of concepts are displayed (in their minds) based on their mental structures. Also, the mental structures are constructed based on mental entities (or schemata).

3. DESCRIPTION LOGICS

My main reference to DLs is (Baader, 2010). DLs represent knowledge in terms of concepts, individuals (as instances of concepts) and roles (of concepts that relate two or more individuals). The basic step of any construction is provided by atomic concepts and atomic roles. In this research, we will see that the general conceptions of human beings could be represented by atomic concepts (that are equivalent to atomic literals in Predicate Logic).

The fundamental set of the main connectors in DLs is: {Conjunction ($\sqcap$), Disjunction ($\sqcup$), Negation ($\neg$), Existential Restriction ($\exists$), Universal Restriction ($\forall$)}. Atomic Concepts, Atomic Roles, Top concept ($\top$) and Bottom concept ($\bot$) are the other constructors of a basic DL.

Knowledge in DLs is mainly constructed based upon terminological axioms and world descriptions. The terminological axioms are introduced to make statements about how concepts and roles are related to each other, and world descriptions describe the world over concepts, roles and individuals. A terminological interpretation (like I) is called a 'model' of an axiom if it can satisfy the statement in the conclusions of the six fundamental axioms and world descriptions in Table 1. Any interpretation is a function that is extendable based on concept descriptions by inductive rules. Interpretations are employed to define formal semantics. I observe and specify the formal semantics through the usual and standard notion of the interpretations. The pair $(\Delta^I, .^I)$ represents the structure of a terminological interpretation, where the interpretation function $(.^I)$ assigns to each concept C a subset (like C^I) of Δ^I, and to each relation R a subset (like R^I) of $\Delta^I \times \Delta^I$.

Name	Syntax	Semantics
Concept Inclusion Axim	$C \sqsubseteq D$	$C^I \subseteq D^I$
Role Inclusion Axiom	$R \sqsubseteq S$	$R^I \subseteq S^I$
Concept Equality Axiom	$C \equiv D$	$C^I = D^I$
Role Equality Axiom	$R \equiv S$	$R^I = S^I$
Concept Assertion	$C(a)$	$a^I \in C^I$
Role Assertion	$R(a, b)$	$(a^I, b^I) \in R^I$

Table 1. *Terminological Axioms and World Descriptions in Description Logics*

4. CONCEPT CONSTRUCTION

It could be assumed that learning in the framework of constructivism and in the context of learner-mentor interactions is—heuristically—supported by:

1. factual, structural and existential questions (that are concerned with WhatNesses),
2. inferential questions (that are concerned with WhyNesses), and
3. methodological and technical questions (that are concerned with HowNesses).

Note that any of these questions invite the responder (the interlocutor) to search through a hierarchy of multiple related concepts in order to produce an appropriate answer. Any appropriate answer shapes a part of a construction which is—mutually and collaboratively—becoming constructed by the learner and mentor. Any agent, by asking a heuristic question, manages to guide her/his interlocutor to producing the heuristic answers or some updated heuristic questions.

Forming concepts [by the agents] over their mental structures is an initial step. Educationalists see concept formation as a process by which a person learns to sort specific experiences into general conceptions. More specifically, some educationalists and social scientists (like (Taba et al., 1971, Du and Ling, 2011)) define concept formation as "an inductive teaching and learning strategy that supports learners in learning something through studying a set of examples of that thing", see some approaches in (McCracken, 1999; Sell et al., 2014). The process of concept formation is a sub-process of a greater process which I have called 'concept construction', see Figure 1.

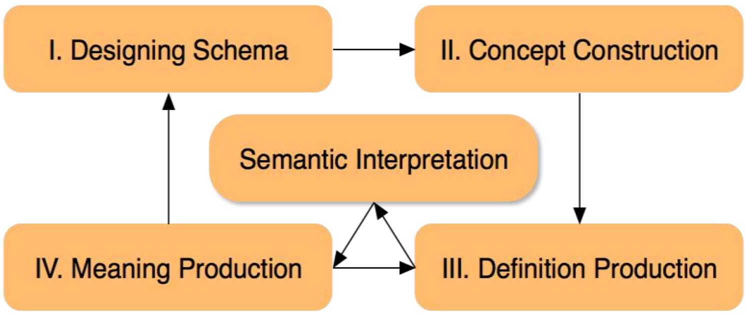

Figure 1. *Meaning Construction within Constructivist Interactions*

According to (Badie, 2015a; Badie, 2015b; Badie, 2016a; Badie, 2016b), any concept construction process is structured over the union (and disjunction form) of three sub-processes consisting of:

a. concept formation (that is intra-psychological),
b. concept transformation by, e.g., speaking, listening, hearing, touching, smelling, tasting (that is inter-psychological), and
c. concept reformation (that is intra-psychological).

Note that concept reformation happens either after transformation or at the more specific levels of conceptualisations of the formed concepts. According to the concept of 'concept construction' and its inductive essence, my approach considers the fact that learner and mentor generate multiple cases for their case-based reasoning and, respectively, multiple rules for their rule-based reasoning. Accordingly, they will be able to use their achievements for case-based and rule-based learning and mentoring.

Additionally, humans can interpret and make decision that whether their different experienced phenomena could come under the label of their own constructed concepts. Let me offer an example:

> John: Bats fly.
> Mentor: Do you know that bats are mammals?
> John: Really?
> Mentor: Yes!
> John: So, some mammals can fly.

In this conversational exchange John has formed the concepts Bat, Fly and Mammal. Also, these concepts have been transformed between John and his mentor and, correspondingly, John has reformed the concepts Bat and Mammal with regard to his mentor's question. Finally, John, based on his reasoning, has reformed the concepts Bat and Mammal, and has learned based on his conclusions. Additionally, he has generated the rule 'there exists a mammal that can fly' (formally: $\exists$isFlying.Mammal). I define the collection of characterised processes 'Concept Learning'.

5. CONCEPT REFORMATION

Suppose that James has a conception of 'Spring' (based on his constructed concept 'Spring'). Accordingly, he, with regard to his conception of 'Spring', may say something to his mentor, ask questions, and answer questions. It shall be emphasised that hearing different words from the mentor is conducive to developing his conception of 'Spring'. See the following conversational exchanges:

> Mentor: James, what could you tell me about spring?
> James: Spring is the season of the moderate weather; when all the trees are green.

James' statement can be translated into:

$$\text{Spring} \doteq \\ \text{Season} \sqcap \exists \text{hasWeather.Moderate} \sqcap \forall \text{hasTree.Green}.$$

This description is structured over the elements of the set:

$$C = \{\text{Season}, \exists \text{hasWeather.Moderate}, \forall \text{hasTree.Green}\}.$$

Also, C is structured over:

$$C' = \{\text{Season, Moderate, Weather, Green, Tree}\}.$$

These five concepts (in C´) has supported James' conception of the concept Spring. In fact, the mentor could focus on these concepts in order to conceptualise James' conception of Spring.

Note that in case the mentor and the learner look forward to achieving a satisfactory negotiation of a concept description, they will need to focus on the atomic concepts within that concept description. Accordingly, the mentor

1. conceptualises the learner's constructed atomic concepts,
2. conceptualises the learner's conceptions of those atomic concepts,
3. observes other related concepts from the perspective of that conception,
4. compares the results with her/his own conceptions, and
5. guides the learner to improve (and re-form) her/his descriptions.

6. DEFINITION

A definition is an equivalence (and, formal-semantically, an equation) between a concept and a description. Definitions assign concept descriptions to concepts. Inductive rules are employed in order to describe more specified concepts based on more general ones. A set of an agent's definitions must be explicit in order to be meaningful. Any agent may revise and re-organise her/his definitions during her/his interactions. This process could be named 'Definition Updating' or re-organising and reforming the proposed concept description. Consequently, the more organised concept descriptions support the agents in constructing more understandable meanings on higher levels of their interaction. Note that the levels of interaction are as follows:

Level 1: learner says/does something,
Level 2: mentor says/does something,
Level 3: learner says/does something,
…

or

Level 1: mentor says/does something,
Level 2: learner says/does something,
Level 3: mentor says/does something,
…

6.1. TERMINOLOGY GENERATION

A finite set of an agent's definitions generates a terminology if and only if no atomic concept has been defined more than once by her/him on the same level of interaction. The indexes '$_L$' and '$_M$' denote 'corresponded to learner' and 'corresponded to mentor' respectively. T_L and T_M represent the finite set of the learner's and the mentor's definitions respectively. For every atomic concept A_L (or A_M), there will be at most one axiom in T_L (or T_M). Accordingly, there will be [at most] one A_L (or A_M) to the left-hand side of any individual definition that shapes an axiom. The agents interchange any member (or any subset) of their own terminologies (T_L and T_M) during their interactions. Formally:

- Considering C_L as the product of the learner's conceptualisations of the concept C, the learner creates a terminological set like $D_{C_L} = \{D_1, D_2, \ldots, D_n\}$ for her/his multiple definitions of C. Accordingly, for $i \in [1,n]$, D_i denotes learner's definition of C on the ith interaction level.

- Considering C_M as the product of the mentor's conceptualisations of the concept C, the mentor creates a terminological set like $D'_{C_M} = \{D'_1, D'_2, \ldots, D'_n\}$ for her/his multiple definitions of C. Accordingly, for $i \in [1,n]$, D'_i denotes mentor's definition of C on the ith interaction level.

Therefore, the definitions D_i and D'_i are located on the same level of interaction based on learner's and mentor's conceptualisations respectively. Both agents move to one upper level. Consequently, the learner's definition becomes modified (from D_i to D_{i+1}) and the mentor's definition becomes modified (from D'_i to D'_{i+1}). Therefore, learner and mentor approach their more satisfactory agreements on their developed definitions. At any level of the interaction, the learner/mentor selects one element or a subset of D_{C_L} / D'_{C_M} in order to exchange it with the interlocutor. Thus:

A. For learner: $\forall i, j \in [1,n], \ \exists \ D_{C_L}^* = \{D_i, \ldots, D_j\} \subseteq \{D_1, D_2, \ldots, D_n\}$. Therefore, she/he interacts with mentor. Accordingly, there is a 'definition transformation' like t_L from the set of her/his selected definitions (from her/his terminology) into the mentor's set of definitions, and subsequently, into the mentor's terminology. Formally, T_L: $\{D_i, \ldots, D_j\} \rightarrow D'_{C_M}$.

B. For mentor : $\forall p, q \in [1,n], \ \exists \ D'^*_{C_M} = \{D_p, \ldots, D_q\} \subseteq \{D'_1, D'_2, \ldots, D'_n\}$. Therefore, she/he interacts with learner. Accordingly, there is a definition transformation like t'_M from the set of her/his selected definitions (from her/his terminology) into the learner's set of definitions, and subsequently, into the learner's terminology. Formally, T'_M: $\{D'_p, \ldots, D'_q\} \rightarrow D_{C_L}$.

Consequently, the multiple definition transformations—collectively—lead both agents to their more negotiable conceptions of atomic concepts and, subsequently, of concept descriptions over the atomic concepts.

Example. Suppose that Martin says/does something regarding concept 'Information System' and, accordingly, produces a definition transformation function like T_1(InformationSystem). T_1(InformationSystem) expresses the affect of Martin's first definition (that is an element of his terminology) on 'InformationSystem'. Subsequently, the mentor transacts and produces a transformation function like T'_1(InformationSystem). T'_1(InformationSystem) expresses the affect of Mentor's first description (that is an element of his terminology) on InformationSystem (and with regard to Martin's definition). This procedure is sequentially continued. In fact, any statement (and, of course, any question) will cause a transaction (and an answer). Ultimately, the concatenation of the multiple functions conduces the mentor and

Martin to their most negotiable understandings of the definition of the concept 'Information Systems'. Such an understanding is the product of the 'Composite Definition Transformation Function' (of the concept 'Information System') or *CDTF*(InformationSystem).

Formally, the Composite Definition Transformation Function of the concept *C* is equal to:

$$CDTF\,(C) =$$
$$[\, T'_i \circ T_i\,(C)\,] \;\circ\; [\, T'_{i\text{-}1} \circ T_{i\text{-}1}(C)\,] \;\circ\; \ldots \circ [\, T'_1 \circ T_1(C)\,] =$$
$$[\, T'_i \circ T_i \circ T'_{i\text{-}1} \circ T_{i\text{-}1} \circ \ldots \circ T'_1 \circ T_1\,]\,(C) =$$
$$T'_i\,(\,T_i\,(\,T'_{i\text{-}1}\,(\,T_{i\text{-}1}\,(\,\ldots\,(\,T'_1\,(\,T_1\,(C)\,)\,)\,\ldots\,)\,)\,)\,)\,.$$

This conclusion supports definition developments (and, in fact, conception developments) on higher levels of mentor-learner interactions. *CDTF*(*C*) prepares the agents for approaching a more satisfactory understanding of each other's conceptions of concept description *C*.

7. MEANING

In this approach, meanings are some conceptual structures that are shaped based on concepts. More specifically, a meaning is a 'Concept-update Function' (*CUF*). This function accepts a concept as an input, updates it, and returns the updated concept. It is worth mentioning that some approaches [in dynamic semantics] have considered meaning as the context-update function, see (Chierchia, 2009; Gabbay, 2010; Larsson, 2012).

Suppose that *C* stands for a concept and *M* represents a meaning function. Considering meaning as a concept-update function we have:

$$M(C) = C'.$$

Example. Suppose that an agent (say Maria) has initially produced a meaning for *C* in her mind. So, she has followed $M_1(C) = C'$. Maria, after more interactions with her mentor and on higher and more specified conceptual levels, might be able to produce the mental function $M_2(C') = C''$. Obviously:

$$M_2\,(M_1\,(C)\,) = M_2\,(C') = C''.$$

It shall be concluded that C'' is the most updated meaning of *C*. Such a procedure could be continued and, thus, the function *M*(*C*) develops itself during Maria-mentor interactions.

Note that the meaning function *M*(*C*) is inherently a 'Composite Concept-update Function'. Let me represent it by *CCUF*. Suppose that $M_i(C)$ is a *CCUF*, where *i* belongs to [1,n]. Then, $M_i(C)$ represents the 'meaning of C on level i' of the mentor-learner interaction. Formally:

$$M(C) =$$
$$(M_i \circ M_{i\text{-}1} \circ \ldots M_1)(C) =$$
$$M_i(\, M_{i\text{-}1}(\, \ldots (\, M_1(C)\,) \ldots)\,) =$$
$$M_i(\, \ldots (C) \ldots).$$

Subsequently, the agents on the ith level of their interactions achieve the most satisfactory agreement on the meaning of the concept description C (in their minds). In fact, $M_i(C)$ is more satisfactory than $M_{i\text{-}1}(C)$.

8. SEMANTIC INTERPRETATION

The interpretations assign meanings to the non-logical symbols [and words], see (Arthur N. Prior, 1955; Simpson, J. A., 1989). These symbols [and words] have no logical values. On the other hand, the logical ones (e.g., if, is, then, so, therefore, hence) have logical consequences. It shall be stressed that the proposed definitions are highly influenced by interpretations. In order to analyse the formal semantics, I need to focus on an interpretation (like I) that consists of:

a. a non-empty set D (as the domain of the interpretation), and
b. an interpretation function (like $.^I$).

Therefore, considering the individual a as an instance of the concept C, an interpretation assigns $C^I(a^I)$ to the concept description $C(a)$. It also assigns to every role description $R(p,q)$ (where p and q are the instances of two concepts), a binary relation like $R^I(p^I,q^I)$, which is a subset of $D^I \times D^I$.

Example. Suppose that Mary has proposed the definition:

$$\text{Spring} \doteq$$
$$\text{Season} \sqcap \exists \text{hasWeather.Moderate} \sqcap \forall \text{hasTree.Green}.$$

This definition is the product of her own interpretation of the concept 'Spring'. Subsequently:

i. the concept 'Season',
ii. the description '$\exists$hasWeather.Moderate', and
iii. the description '$\forall$hasTree.Green'

have been, logically, connected to each other to shape her definition of the concept 'Spring'. It shall be drawn into consideration that Mary has interpreted the concept 'Spring' over the elements of the set:

$$D = \{\text{Season, Moderate, Weather, Green, Tree}\},$$

and, in fact, she has focused on her semantic interpretation of all the members of D. Actually, she has—mentally—produced the set:

$$D^I = \{\text{Season}^I, \text{Moderate}^I, \text{Weather}^I, \text{Green}^I, \text{Tree}^I\}.$$

Note that if an interpretation could satisfy a definition, then that interpretation will be identified as a 'model' of that definition. It can also satisfy the semantics of the terminological axioms and world descriptions in Table 1. Actually, Mary—at least—attempts to design a mental model in order to validate and authenticate her definition of 'Spring'. Hence, the interpretability of her definition by her mentor's mental models determine the acceptability of that definition over the mentor's conceptions.

8.1. FORMAL ANALYSIS OF SEMANTIC INTERPRETATION IN THE CONTEXT OF MENTOR-LEARNER INTERACTIONS

According to the structure of the Composite Definition Transformation Function of the concept C or $CDTF(C)$ (which is presented in the 'Definition' section), the combination of the agents' terminologies could balance their personal sets of terminologies. The most optimistic result is approaching the unified terminology for both of the agents that supports them in re-organising and updating their definitions. In addition, the interpretation I could be employed in order to interpret the concept-update function (meaning function) as a 'functional role relevant for a concept'. The learner and the mentor at any level of their interactions aim at finding a proper [mental] model in order to:

a. follow the concept update-functions,
b. certify their constructed concept-update functions as some 'functional roles', and
c. satisfy the interlocutor's terminology.

Subsequently, the agents will be able to activate their interpretations in order to support the role inclusions (find them in Table 1). Accordingly, the meaning M, as a concept-update function $M(C) = C'$, could be represented by the functional role

$$M(C,C').$$

Then:

$$(C^I,C'^I) \in M^I.$$

This conclusion expresses that the agent's interpretations certify M as a functional role. Employing the Composite Concept-update Function ($CCUF$) and the role inclusion axiom, we have:

- Level 1 of interaction: $M_1(C, C') \;\Rightarrow\; (C^I, C'^I) \in M_1^{I1}$

- Level 2 of interaction: $M_2 (C', C'') \Rightarrow (C'^{I}, C''^{I}) \in M_2^{I2}$
  …
- Level n of interaction: $M_n (C^{(n\text{-}1)}, C^{(n)}) \Rightarrow (C^{(n\text{-}1)\,I}, C^{(n)\,I}) \in M_n^{In}$

, where:

a. '^{In}' represents the desired model of an agent on the nth level of her/his interaction with the interlocutor,
b. C stands for the concept, and also, represents the co-domain of functions M_1, M_2, …, M_n,
c. $C^{(n)}$ represents C after n times of being updated, and
d. $C^{(n)I}$ represents the interpretation of C after n times of being updated.

So, in fact, there is a meaning construction over the agent's interpretations on any level of her/his interaction.

Now I need to check how the collection of n interpretations (on n levels of interaction) works. Suppose that a learner and her/his mentor have participated in an interaction. I inductively focus on the conclusion of the following processes and propose the definition of 'Composite Interpretation Transformation' (or *CIT*):

- **The Process 1.** The learner utters her/his description of her/his conception. Subsequently the mentor interprets the learner's conception. This semantic process involves determining whether the mentor's understanding (or $M_{(1)}$) of that concept description could produce a concept inclusion with the learner's understanding (or $L_{(1)}$). Semantically: $M^I_{(1)} \sqsubseteq L^I_{(1)}$. In fact, the first domain of the mentor's interpretation must be the subset of the domain of the learner's interpretation.

- **The Process 2.** The mentor transacts and utters her/his own interpretation of the learner's conception. The learner has to interpret the mentor's uttered interpretation. Actually, the first semantic process is inversely becoming organised on the second level. This higher (and more specified) semantic process involves determining whether the learner's understanding of the mentor's interpretation (or $L_{(2)}$) could produce a concept inclusion with the individual understanding (or $M_{(2)}$). Semantically: $L^I_{(2)} \sqsubseteq M^I_{(2)}$. I shall emphasise that the second domain of the learner's interpretation must be the subset of the domain of the mentor's interpretation.

- **The Process i.** According to the logical characteristics of the first and the second processes and their interrelationships, we can, inductively, specify the 'Composite Interpretation Transformation' (*CIT*) as follows:

  - If i represents the learner's utterance, then: $M^I_{(i)} \sqsubseteq L^I_{(i)}$ & $L^I_{(i+1)} \sqsubseteq M^I_{(i+1).}$
  - If i represents the mentor's utterance, then: $L^I_{(i)} \sqsubseteq M^I_{(i)}$ & $M^I_{(i+1)} \sqsubseteq L^I_{(i+1).}$

Collectively, we can induce that:

$$(M^I_{(1)} \subseteq L^I_{(1)}) \ \cap \ (L^I_{(2)} \subseteq M^I_{(2)}) \ \cap$$
$$\dots \ \cap$$
$$(M^I_{(n)} \subseteq L^I_{(n)}) \ \cap \ (L^I_{(n+1)} \subseteq M^I_{(n+1)}). \ (*)$$

Proposition. An agent's interpretations at any lower level of her/his interactions with another agent is the subset of her/his own interpretations at the upper levels. Formally:

$$L^I_{(1)} \subseteq L^I_{(2)} \subseteq \dots \subseteq L^I_{(n)}$$
$$\&$$
$$M^I_{(1)} \subseteq M^I_{(2)} \subseteq \dots \subseteq M^I_{(n)}.$$

Relying on (*) and taking the afore-mentioned proposition into account, we have:

$$(M^I_{(1)} \subseteq M^I_{(n)}) \ \cap \ (M^I_{(n)} \subseteq L^I_{(n)})$$
$$\&$$
$$(L^I_{(1)} \subseteq L^I_{(n)}) \ \cap \ (L^I_{(n+1)} \subseteq M^I_{(n+1)}).$$

Therefore:

$$M^I_{(1)} \subseteq M^I_{(n)} \subseteq L^I_{(n)}$$
$$\&$$
$$(L^I_{(1)} \subseteq L^I_{(n)}) \cap (L^I_{(n+1)} \subseteq M^I_{(n+1)}).$$

In fact, both mentor and learner have optimised and restricted their individual interpretations by activating inclusions over their interpretations. Considering 'n $\rightarrow \infty$' we will have:

$$M^I_{(1)} \subseteq M^I_{(n)} \subseteq L^I_{(n)}$$
$$\&$$
$$L^I_{(1)} \subseteq L^I_{(n)} \subseteq M^I_{(n)}.$$

Therefore:

$$M^I_{(n)} = L^I_{(n)}.$$

This conclusion is the most valuable product of the frequent interpretation transformations in the context of mentor-learner interactions. $M^I_{(n)} = L^I_{(n)}$ encodes the fact that both agents have had the interlocutor's interpretation satisfied (by their own interpretation). This result is the most excellent valuable conclusion of the interpretations in the context of mentor-learner interactions.

9. CONCLUSIONS

This research has focused on conceptual and logical analysis of constructivist interactions between mentor and learner within an explanatory and developmental framework of meaning construction. The paper has focused on conceptual and logical description of how interactions lead agents to construct their own meaningful understandings based on their constructed meanings (with regard to their personal conceptions of their own constructed concepts). The main contribution has been to provide a logical support for the formal semantic analysis of meaning construction within constructivist interaction.

More specifically, it has been concerned with logical analysis of agents' conceptualisations, constructed concepts, produced conceptions, expressed concept descriptions, described definitions, generated terminologies, provided semantic interpretations (and mental models), constructed meanings and produced meaningful understandings.

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PAPER P. FROM CONCEPTS TO PREDICATES WITHIN CONSTRUCTIVIST EPISTEMOLOGY

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This paper is submitted

ABSTRACT

Constructivism is a philosophical approach that appears in a variety of guises, some of them pedagogical, some epistemological and some in complex combinations. This article is based on constructivist epistemology. More specifically, constructivist epistemology provides a ground for conceptual analysis of humans' concept constructions, conceptions and concept learning processes. It will focus on conceptual specification and logical description of a flow from concepts to predicates.

1. INTRODUCTION AND MOTIVATION

My point of departure is the special focus on the assumption that human beings aim at constructing knowledge using insights based on their background knowledge. I shall therefore claim that human beings construct their personal knowledge and produce their own understanding of the world through experiencing various phenomena and reflecting on their own experiences.

Constructivist epistemology (as a way of thinking about cognition and knowledge) focuses on the question of whether, and under which conditions, human beings may construct their own knowledge structures. In addition, it holds that humans can know [about] their personal built up constructions.

Jean Piaget was highly interested in expressing the HowNess of 'meaning making' (by human beings) with regard to their own experiences and ideas of the world and, subsequently, he became concerned with the concept of 'constructivism'. The central assumption of Piaget's constructivist theory of learning (based on his theory of Cognitive Development and developmental theory of learning) is that human beings produce their own understanding of the world primarily based on their background knowledge, over their experiences and through their interactions with their environment, see (Piaget, 1936; Piaget and Cook, 1952). Constructivism takes into consideration that the processes of knowledge acquisition and knowledge building involve the active (and dynamic) creation of mental structures, rather than the passive internalisation of information acquired from others. Note that the active creation of mental structures means that the mental structures could either change or tend to change. In addition, from the perspective of radical constructivism, knowledge is not a representation of objectives (e.g., objective facts, objective procedures), but a compendium of concepts, conceptual relationships, and rules that have proven useful in domesticating humans' experiential world, see (Foerster, 1981; Glasersfeld, 1984). One of the most significant objectives of the constructivist model of knowing (based on constructivist epistemology) is the study of the growth of knowledge and Genetic Epistemology, see (Berly, 1977). Genetic epistemology is concerned with 'the developmental theory of knowledge' and with 'how knowledge may reasonably be assumed to be built in an individual's cognitive systems'. It shall, therefore, be concluded that genetic epistemology has a strong connection with the expression 'conceptual change' within knowledge acquisition and knowledge building. In the context of cognitive developmental psychology, conceptual change is a kind of process that focuses on the conversion of humans' conceptions and the interrelationships between their old and new conceptions, see (Chi, 1992; Vosniadou and Brewer, 1992; Nersessian, 1992; Limon, 2002; Vosniadou and Verschaffel, 2004).

At this point, I feel the need to take into consideration Bruner's theoretical framework which could be believed to be one of the most modern constructivist theories of knowledge in education and psychology, see (Bruner et al., 1956; Bruner, 1974; Bruner, 1986; Bruner, 1990; Bruner and Kalmar, 1998). Jerome Bruner believed that "knowing is how human beings get beyond the information given" and, that "knowing, getting to know the world, is not just perceiving something; it's constructing it". In fact, this is how Bruner described the concept of 'constructivism'. It's possible to conclude that knowledge building, as an active process of construction, includes information 'selection' and information 'transformation', decision making, generating 'hypotheses', and making 'meaning' from information and experiences.

The core presupposition of my research is that the new conceptions (as the products of developed constructed concepts) always emerge from the old ones; albeit with the proviso that in this context I will say nothing about how first concepts in children are established. My central focus is on the assumption that we can—reasonably and logically—employ a constructivist model of knowing in order to describe 'knowledge construction' over concepts as a kind of 'conditional reasoning'. I will focus on conceptual analysis of Concepts, Concept Construction, Conceptions, and Concept Learning in order to make a logical linkage between conceptions and predications. Therefore, I shall claim that this research—relying on a constructivist model of knowing—will make an epistemological and logical linkage between concepts and predicates. Such a linkage could specify why logicians perceive concepts as predicates. I want to stress that I will not deal with the historical and basic epistemological treatment of concepts by, e.g., Locke or Kant. This is taken up by, e.g., (Gauker, 2011) including the modern relevance of the historical notions.

2. CONCEPTS

It is difficult to think of a foundational scientific concept about which there is more controversy among experts (in philosophy, linguistics, psychology, cognitive science and computer science) than the concept of a 'concept', see (Hampton and E. Moss, 2003; Margolis and Laurence, 2015). Furthermore, over the years, the concept of 'concepts' has not been used consistently, and it is not always transparent if:

a. what is meant by the expression 'concept' is some mental representation of phenomena in the world, for example as mental pictures of 'red dog', or
b. whether a concept always has to be bound up with some linguistic expression, e.g., the words 'dog' and 'red' in the concept 'red dog', or
c. concept refers to something understandable like, for instance, membership of sets and classes, for example as sets of animals.

So, it’s obvious that the concept of ‘concepts’ is vague and imprecise. Anyway, it seems to be plausible that concepts could be the primary units of knowledge—the basic materials, it is often said—out of which humans’ thoughts are built and developed. At this point, I take into consideration a few descriptions relevant for concepts and concept representations and, subsequently, I will focus on my understanding of the use of the expression ‘concept’ and I will assume its application in order to be comprehensible in my epistemological and conceptual analysis. I want to emphasise that the ontological analysis of concepts is beyond the scope of this research.

In Kant’s words, a concept is the “unity of the act of bringing various representations under one common representation”. Kant believed that “no concept is related to an object immediately, but only to some other representation of it”, see (Kant, 1781: 1924 edt.). Frederic Bartlett—in his studies in experimental psychology—arrived at the phenomenon of ‘concept’ with his focus on memory analysis, see (Bartlett, 1932). In memory studies, subjects recalled details of stories that were not actually there. Bartlett believed that concepts might be understood as “representations of [parts and pieces of] reality in mind”, see (Peacocke, 1992; Zalta, 2001). Concepts as the conceptual entities could be studied by the representational theory of mind and the theory of mental representation, see (Stich, 1992; Margolis and Laurence, 2007; Margolis and Laurence, 2010). According to (Götzsche, 2013), a concept might be said to be “a linkage between the mental representations of linguistic expressions and the other mental images (e.g., representations of the world, representations of inner experiences) that a human being has in her/his mind”.

In order to express my conception of ‘concepts’ I need to focus on the concept of ‘meaning’. It shall be taken into consideration that any meaning, in the framework of constructivism, is a ‘conceptual structure’ and, as such, meanings, to a large extent, influence any individual human’s constructions and developments of her/his individual experiential reality. Therefore, in the framework of constructivism, meanings could be interpreted as conceptual structures that are constructed over conceptual entities. Consequently, any conceptual entity can be interpreted to be a building block and a basic material of a conceptual structure. Note that we can have different perceptions of conceptual entities and, of course, there is no absolute schema for conceptual entities. In my opinion, conceptual entities might be labelled ‘concepts’ and, subsequently, a conceptual entity, as a representation of a piece of reality (or even fiction) in individual’s mind, can be interpreted to be a basic material of [to-be-constructed] meanings.

2.1. CONCEPT FORMATION

Referring to (Colman, 2016), concept formation is defined as “a process by which a concept is acquired or learnt, usually from exposure to examples of items that belong to the concept category and items that do not belong to it. Concept formation involves learning to distinguish and recognise the relevant attributes according to which items are classified and the rules governing the combination of relevant attributes”.

Furthermore, some educationalists and social scientists (like (Taba et al., 1971, Du and Ling, 2011)) define concept formation as “an inductive teaching and learning strategy that supports learners in learning something through studying a set of examples of that thing”, see some approaches in (McCracken, 1999; Sell et al., 2014).

According to (Colman, 2016), the labels ‘concept identification’ and ‘concept learning’ are equivalent to ‘concept formation’. In my opinion, ‘concept formation’ and ‘concept learning’ (especially in the framework of constructivism) are not equivalent. From my perspective, concept learning is a consequence of concept formation. In fact, the process of concept formation ‘might’ be followed by the process of concept learning. Let me elaborate and specify my conception of Concept Learning.

3. FROM CONCEPT CONSTRUCTION TO CONCEPT LEARNING

The process of concept formation is a sub-process of a greater process which I have called ‘concept construction’, see (Badie, 2015a, Badie, 2015b; Badie, 2016a; Badie, 2016b). Any concept construction process is structured over the union of three sub-processes consisting of:

i. concept formation,
ii. concept transformation, and
iii. concept reformation.

Note that concept reformation happens

- either after transformation,
- or at the more specific levels of conceptualisations (of the formed concepts) and as the outcome of conceptual change.

In fact, concept construction is equivalent to the union (and the disjunctive form) of i, ii and iii. For example, the concept ‘red dog’ could be constructed by Bob based on the following items:

1. Bob’s self-based and intra-psychological processes, e.g., thinking about red dogs, studying about red dogs, searching about red dogs on google, and comparing and analysing different breeds of red dogs. In fact, the concept ‘red dog’ could be reflected (i.e., epitomised and represented) in Bob’s mind in order to be developed, expanded, and promoted.

2. Bob’s interactions with other humans, nature, animals, dogs, and red dogs, e.g., in conversational exchanges with other humans about their conceptions of red dogs, watching red dogs, playing with red dogs, feeding red dogs, and walking red dogs. In conversational exchanges with other individuals, the concept ‘red dog’ could be transformed in order to be analysed, argued and criticised. Subsequently, the conclusions could be transformed back to Bob’s mind.

In fact, Bob—by getting from (1) to (2) and vice-versa and, by modifying his conception of 'red dog' over time—becomes concerned with the development of his mental construction of the concept 'red dog'. Bob can, at anytime, make inferences based on his most modified conception. Note that 'conception' is interpreted as the product of concept construction. Therefore, Bob's most modified conceptions are produced based on his most specified constructed concept 'red dog'. As concluded, moving from (1) to (2) and vice-versa and, in fact, making a logical connection between the concepts of 'concept formation' and 'concept transformation' is the most significant characteristic of concept construction processes. Relying on this characteristic, Bob develops the concept 'red dog' in his mind.

This conclusion of mine is in line with Vygotsky's theory of social constructivism. Lev Vygotsky was the founder of a theory of human cultural and bio-social development. It is in general taken for granted that he was one of the developers of the theory of constructivism. Vygotsky in (Vygotsky, 1978) stated that "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (inter-psychological) and then inside the child (intra-psychological)".

3.1. CLASSIFICATION

Bruner believed that "To perceive is to categorize, to conceptualize is to categorize, to learn is to form categories, to make decisions is to categorize". Obviously, Bruner believed that perceiving, conceptualising, learning and making decision are the consequences of classifications. In fact, regarding his idea,

i. there is a strong correlation between 'perceiving something' and 'classifying that thing', and
ii. conceptualising some phenomenon is strongly correlated with classifying that phenomenon and putting it into a class.

This conclusion (based on Bruner's statement) is strongly in line with my idea. In my opinion, this is where 'concepts (as conceptual entities and basic materials of meanings) become manifested within constructivist model of knowing'. Consequently, in such a framework, the concept of 'classification' can be interpreted as a process of 'constructing', which is dependent on 'representations'. Furthermore, any classification corresponds with an 'assignment' and, in fact, any individual human being is capable of classifying a phenomenon into one or multiple classes (with her/his determined and specified labels) on the basis of her/his own outlooks. Accordingly, it's possible to claim that humans form and produce multiple classes of different phenomena in order to construct knowledge over those phenomena. Note that classifications are strongly dependent on mental representations of any individual's personal (to-be-constructed) constructions based on her/his own conceptions (as the products of her/his constructed concepts). It is worth mentioning that "the term 'construction' also expresses the creation of an abstract entity", see (Oxford, 2017; Cambridge, 2017).

An abstract entity as a product of the activity of construction could be realised to be another linkage between 'concepts as classes' and 'constructivist model of knowing'. Taking the concepts of 'classification' and 'construction' into account, we could realise that there is a strong bi-conditional relationship between the following items:

i. Classification (= classifying a phenomenon into a class) regarding hierarchical viewpoints.
ii. Construction (= providing a mental construction and representation of a phenomenon).

3.2. CONCEPT LEARNING

The expression 'concept learning' can, by observing concepts as classes, be seen as the developmental process of concept construction and as the specification of the conceptualisation of the constructed concepts. Concept learning is activable with regard to humans' specification of the conceptualisations of the characteristics and properties of concepts and through experiencing various groups of examples of those concepts. As mentioned, regarding Bruner's statement, learning is a consequence of classification. So, again, I shall stress that his idea has been in line with mine and, in fact, I recognise 'concept learning' as a consequence of classification. Furthermore, I shall claim that—through concept learning—the phenomena of 'classification', 'representation', 'construction' and 'abstraction' are linked together. In addition, I want to stress that the conceptual interrelationships between 'concepts' (as basic materials of meanings) and 'meanings' (as conceptual structures) establish a semantics based upon humans' concept constructions within their concept learning process in the framework of constructivism.

4. SOME STRUCTURAL ANALYSIS OF CONCEPTS

As mentioned, a concept might be said to be "a linkage between the mental representations of linguistic expressions and the other mental images (e.g., representations of the world, representations of inner experiences) that a human being has in her/his mind", see (Götzsche, 2013). Suppose that the function $C(c)$ denotes concept construction (where the capital C stands for 'construction function' and the small c stands for 'concept'), and the relation (I , L) denotes the relationship between the mental images and the linguistic expressions. The relation (I , L) is constructed over the conjunctions of 'implying linguistic expressions from mental images' and 'implying mental images from linguistic expressions'. Therefore, (I , L) could be expressed by:

$$(I \rightarrow L) \ \wedge \ (L \rightarrow I)$$

and, equivalently, by:

$$I \Leftrightarrow L.$$

Consequently, there exists an inductive model (like $\vDash$), such that:

$$(I \Leftrightarrow L) \vDash C(c).$$

Informally, the constructed concept has been induced (and satisfied) by relating and matching 'linguistic expressions' and 'images of the world'. In fact, the constructed concept is semantically satisfied by '$I \Leftrightarrow L$', and this satisfaction is the source of meanings in the constructor's mind. In other words, meanings are constructed based upon these constructed concepts.

Representing Concept Formation, Concept Reformation and Concept Transformation functions by $F(c)$, $F'(c)$ and $T(c)$ respectively, we have:

$$(I \Leftrightarrow L) \vDash [\, F(c) \vee T(c) \vee F'(c) \,].$$

According to this formal semantic model,

a. the collections of the representations and illustrations of mental linguistic expressions in mental images, and
b. the epitomisations of mental images (that make an epitome of mental images) in the form of linguistic expressions,

could—semantically—satisfy the human's concept constructions. And, in fact, the disjunction form of 'concept formation', 'concept transformation' and 'concept reformation' is semantically satisfied by a one to one relation between 'linguistic expressions' and 'mental images'. It shall be concluded that any 'concept construction' is meaningful with regard to '$I \Leftrightarrow L$'. The formal semantic model '$(I \Leftrightarrow L) \vDash [\, F(c) \vee T(c) \vee F'(c) \,]$'—in broad sense—describes how the succession 'Concept … Conception' could be structured in humans' minds.

5. FROM CONCEPTIONS TO PREDICATES

In the framework of constructivism, knowledge can be interpreted to be constructed and developed by humans with their insights based on their constructed concepts. This research follows my central belief that the constructed concepts could be followed by the humans' produced conceptions and, in fact, I have claimed that the constructed concepts could be manifested in the form of conceptions for becoming expressed. But how could we draw a logical junction between humans' constructed concepts and their conceptions? In fact, we need a concept language (like Description Logics, see (Baader, 2010)) that could describe this logical junction properly.

By considering any conception as a class (and, in fact, as a mathematical set) and, subsequently, by representing that set in the form of a predicate, that conception can be expressed and stated. More specifically, a unary set and its superset make a class inclusion/subsumption relationship in the form of:

$$\text{SubClass} \subseteq \text{SuperClass}.$$

For example:

$$\text{Dog} \subseteq \text{Animal}$$
$$\&$$
$$\text{Red} \subseteq \text{Colour}.$$

Accordingly, that class inclusion relationship can be expressed in the form of a [Unary] Predicate inclusion/subsumption. For example, $\text{Dog} \subseteq \text{Animal}$ and $\text{Red} \subseteq \text{Colour}$ can—terminologically—be represented in the form of:

$$\text{Dog} \sqsubseteq \text{Animal}$$
$$\&$$
$$\text{Red} \sqsubseteq \text{Colour}$$

respectively. Subsequently, that terminological [Unary] Predicate inclusion can be expressed in the form of a predicate assertion. For example, $\text{Dog} \sqsubseteq \text{Animal}$ and $\text{Red} \sqsubseteq \text{Colour}$ can be represented in the form of:

$$\text{Animal(Dog)}$$
$$\&$$
$$\text{Colour(Red)}$$

respectively. Note that the relation Animal(Dog) expresses that the class Animal covers the class Dog and, in fact, all Dogs are Animals. Therefore, the class Dog is subsumed under the class Animal. Furthermore, relying on inductive rules, the class inclusions $\text{Dog} \sqsubseteq \text{Animal}$ and $\text{Red} \sqsubseteq \text{Colour}$ can be merged in order to produce:

$$\text{RedDog} \sqsubseteq \text{ColouredAnimal}$$

and, respectively:

$$\text{ColouredAnimal(RedDog)}.$$

Let me focus on the terminological description ‘$\text{Dog} \sqsubseteq \text{Animal}$’ and its equivalent assertional description (or ‘Animal(Dog)’). Representing the logical description ‘$\text{Dog} \sqsubseteq \text{Animal}$’ is strongly dependent on the following items:

a. Constructing Dog and Animal in one’s mind and, respectively, producing a conception of them.

b. Scheming some individual dogs and animals (as the instances of Dog and Animal respectively) in one's mind and, accordingly, producing a conception of them.
c. Constructing logical relationships between Dog and Animal in one's mind and, subsequently, producing a conception of that.

It shall be emphasised that any semantic interpretation could be given sense over the collection of (a), (b), and (c). Logically, a triple like ⟨a, b, c⟩ is equivalent to:

⟨
classes,
inclusions and memberships in classes,
the interrelationships between the members of various classes
⟩.

5.1. PREDICATES

There are different perspectives from which predicates are observable and interpretable. Assessed by mathematical and formal logics, a predicate is an expression that makes a kind of 'assignment'. It could be believed that humans—by their semantic interpretations—assign their own conceptions and the interrelationships between their conceptions to values. In particular, [Clapham, 2014] describes a predicate as:

- an expression, which ascribes a property to one or more subjects.

Furthermore, according to (Blackburn, 2016),

- a predicate might be any expression that is capable of connecting with one or more singular terms to make a proposition.

Also:

- predicates express the conditions that the entities referred to may satisfy, and in the case the conditions are satisfied the resulting sentence will be true.

In fact, semantic interpretations map the conceptions and their interrelationships into true/false values. In more proper words, predicates, by employing semantic interpretations (i.e., generated interpretation functions from words and symbols into truth values), transmit the characteristics and properties (of conceptions) into statements or into truth-values. By taking these characteristics into consideration, unary predicates could stand in the place of conceptions and *n*-ary predicates could stand in the place of conceptions' interrelationships.

It can therefore be concluded that a predicate is an assignment function from characteristics, features and properties of a conception (and respectively, of a constructed concept) into subjects. Furthermore, "a subject is something which is—in a situation/setting—the conceptual entity (i.e., a configuration) of the act of linguistic communication (i.e., transfer of information) or cognition (i.e., transformation of information) of the interlocutor uttering the statement. Therefore, subjection is an assertive predication", see (Götzsche, 2013: 90). Figure 1 represents the analysed conceptual relationship between concepts and predicates.

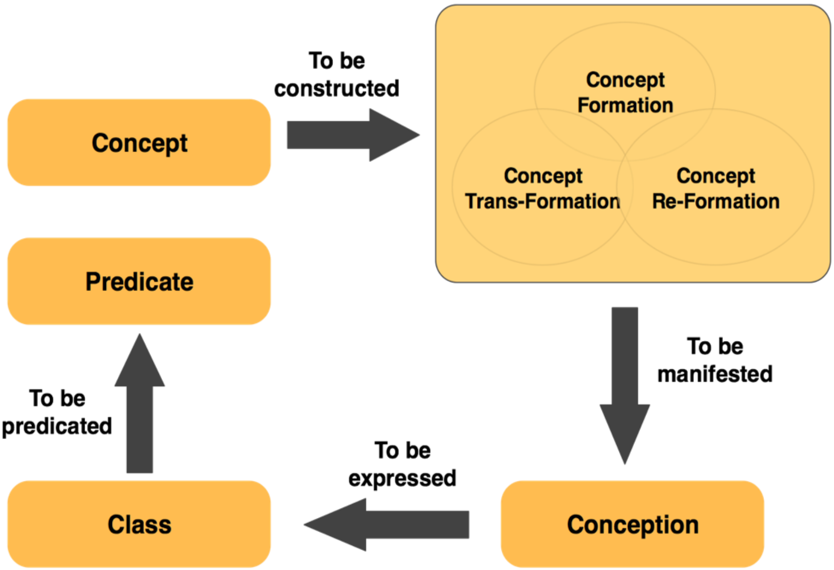

Figure1. *From Concepts to Predicates in the Framework of Constructivism*

6. CONCLUDING REMARKS

Predication (see Link a) of a conception (or, equivalently: a to-be-created class) is concerned with the question 'what is it to state something about that conception?' and, thus, a predication tackles to find an answer for describing and expressing the question 'what is there and what does exist relevant for a constructed concept and a produced conception?'. Heuristically, the latter question focuses on the existence of a constructed concept. This question is concerned with ontological descriptions of a constructed concept, while the first question is concerned with a structural description of that constructed concept. I shall acknowledge that there is a strong correlation between predication of a conception and that concept's ontology. Relying on constructivist ontology, any individual human being has her/his personal constructed concepts, and hence, it is reasonable to assume that the constructed concepts are valid and existing.

It could be said that a predication, indirectly, focuses on a kind of ontological underpinning of a conception and, respectively, of a constructed concept.

Additionally, in the framework of constructivist epistemology, knowledge is recognised to become constructed over the background knowledge. It must be assumed that the background knowledge has strong interrelatedness with humans' ontological conceptions that are generated with regard to the nature and structure of the pieces of reality in their minds. And, in fact, there are strong correlations between 'pre-concept descriptions and pre-conceptions' and 'ontological conceptions'. In fact, the pre-concept descriptions and pre-conceptions could be realised as the outcomes of ontological conceptions. Furthermore, the supportive processes involved in human concept learning can be seen as an explanatory ontology [of mind] and as a comprehensive ontology [of selves]. Obviously, we can see that there is a triangle covering:

a. ontological realisation of a concept,
b. concept construction, and
c. predication of a conception.

Accordingly, in my opinion, the realisation of characteristics, features and properties[1] of concepts, tackle to deal with their ontologies. Therefore, the predication functions are from those characteristics [, features and properties] into subjects. Based on these understandings, I propose the following definitions:

- Philosophy of constructivism is a kind of comprehensive and explanatory ontology of human beings, and the constructivist epistemology provides a model of knowing over this ontology. The central focus of constructivist epistemology is the origin of an individual's constructed knowledge.

Subsequently,

- the pure grasp of a concept in constructivist epistemology describes an individual's comprehension of the unity of the act of bringing various representations under one common representation in the framework of constructivism.

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[1] I shall draw your consideration to the fact that the properties of concepts denote the interconnections of concepts with themselves and with other concepts. Therefore, unary, binary, …, n-ary predications are generated in order to express the properties of concepts and in order to reflect those concepts in multiple subjects.

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Links

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SECTION III

TERMINOLOGY

TERM	DESCRIPTION
Classification	*Classification* is the process of constructing classes. There is a bi-conditional relationship between ‘learning’ and ‘constructing classes’. Cognitive activities of human beings involve their constructed classes. Classification is highly correlated with representation and assignment. Human beings can classify different phenomena/things as belonging to multiple classes and under specific labels. Classification is strongly tied to conceptualisation.
Complete Mental Mapping	If there is a mental mapping (like *M*) from a concept (like *C*) into another concept (like *D*), then it could be conceptualised, interpreted, and understood that *M* is also valid from a sub-concept of *C* into *D*. Such a mental mapping is *Complete*.
Composite Concept-update Function	Meanings as concept-update functions develop and organise themselves during human-human interactions. The *Composite Concept-Update Function* (*CCUF*) expresses that the constructed meanings are always in the process of developing themselves. The previous situations are the domains of this function. Therefore, *CCUF* becomes constructed based on the concatenation of the constructed meanings.
Composite Definition Transformation	The concatenation of the multiple ‘definition transformations functions’ between two (ore more) agents conduce them to their most negotiable comprehensions of the definition of the concept *C*. This comprehension is the product of the *Composite Definition Transformation Function* (of the concept *C*) or *CDTF*(*C*).
Composite Interpretation Transformation	The following conceptual and logical process is valid at any level of an interaction between two human beings: “Once an agent utters her/his description of her/his conception, the interlocutor interprets that conception. This semantic process involves determining whether the interlocutor’s understanding of that concept description could produce a concept inclusion with the agent’s understanding. Thus, the domain of the interlocutor’s interpretation must come under the set (i.e., be its subset) of the domain of the agent’s interpretation”. The concatenation of all these transformations, from agent into interlocutor and inverse, is called a *Composite Interpretation Transformation*. According to that, an agent’s interpretations

	at any lower level of her/his interactions with another agent is the subset of her/his own interpretations at the upper levels of interaction.
Concept	A *Concept* is a conceptual entity that represents a part as well as parts of reality/fiction in a human being's mind. Any concept might be said to be a linkage between the mental representations of linguistic expressions and the other mental images (e.g., representations of the world, representations of inner experiences) that a human has in her/his mind. In the framework of constructivist concept learning, concepts are conceptual entities and can be interpreted as the basic materials of conceptual structures (meanings). Subsequently, in the framework of constructivism, concepts could be recognised as the basic materials of meaningful understandings.
Concept-update Function	A *Concept-update Function* (*CUF*) accepts a constructed concept as an input, updates it, and returns the updated concept. *CUF*s develop and organise themselves over time. There is a strong connection between one's mental concept-update functions and her/his conceptual changes.
Concept Construction	*Concept Construction* processes are structured over the union of three sub-processes consisting of (i) the intra-psychological process of 'concept formation', (ii) the inter-psychological process of concept transformation, and (iii) the intra-psychological process of concept reformation.
Concept Definition	Any *Concept Definition* is a Definition. In Description Logics, a concept definition (represented by $\doteq$) is a kind of equation. It expresses the definition of a new, as well as more specified, concept in terms of other previously defined concepts.
Concept Description	A *Concept Description* is structurally constructed based on a definition. A concept description attempts, inductively and based on logical connectors, to describe a concept. Logical connectors are logical symbols. Concept descriptions are the most common ways of expressing conceptions by human beings.
Concept Equivalence	If C and D stand for two concepts, the logical description '$C \equiv D$' is called a *Concept Equivalence*. Such a relationship means, semantically, that C and D are equal (i.e., $C = D$). The

	expression '$C = D$' represents a so-called '*Concept Equality*'.
Concept Inclusion	See *Concept Subsumption*.
Concept Learning	*Concept Learning* is a learning theory that is supported by humans' inductive reasoning processes. Concept learning is logically shaped over a system of evidential support which is structured over humans' less-than-certain inferences. This theory is supported by humans' reasoning processes based on their constructed concepts as well as their produced conceptions. Concept learning could be generated based on humans' background knowledge and over their pre-formed concepts and pre-conceptions. Also, concept learning is generated with regard to human's conceptualisations of the characteristics and properties of concepts. Furthermore, concept learning could be structured through experiencing (e.g., observing, hearing, touching, reading about) various groups of examples of concepts. Accordingly, humans could focus on hypothesis generation. Human beings become concerned with specification of the conceptualisation of their constructed concepts within their concept learning processes.
Concept Learning Problem	The *Concept Learning Problem* is expressible in the form of a function that describes a human being who constructs a concept on a basis provided by her/his constructed knowledge. Also, her/his knowledge is constructed based upon her/his terminological basis and over her/his descriptions of the world.
Concept Subsumption	The logical term '$C \sqsubseteq D$', where C and D stand for two concepts, expresses a *Concept Subsumption*. In equivalent words, C is subsumed under D. Therefore, C is the sub-concept of D. More specifically, the idea of 'being a concept and becoming subsumed under another concept' is called a Concept Subsumption. Through the lenses of formal semantics, C is the sub-class as well as subset of D.
Concept Understanding	*Concept Understanding* is the product of (i) a conceptualisation and (ii) a semantic interpretation. The characteristics and properties of a concept by means of the concept understanding function become mapped into concept descriptions. In fact, a concept understanding can relate the characteristics of a concept to a description (concept description). Concept understanding could logically

	be acknowledged to be subsumed under concept constructions.
Concept Understanding Semantic Model	A *Concept Understanding Semantic Model* is supported by interpretation-based semantic models. Also, any concept understanding semantic model supports and covers its sub-models. *Concept Understanding Semantic Sub-Models* are some functional roles of human beings at different layers (with different complexities) of realisations as well as comprehensions of the world.
Conception	Human beings manifest their constructed concepts in the form of their *Conceptions* in order to express them. Conceptions are individual-centered, central-organising, and generalised across experiences and direct observations. They require re-organisable pre-conceptions. Conceptions make sense to communities by becoming shared. The most significant logical necessity for conceptions to be expressed is that any of them needs to be considered a unary predicate. Additionally, any interrelationship between two as well as more conceptions needs to be considered a binary as well as n-ary ($n \geq 3$) predicate.
Conceptual Change	*Conceptual Change* is a psychological process that focuses on the conversion of humans' conceptions (of their constructed concepts) and on the interrelationships between their old and new conceptions.
Conceptual Knowledge	Regarding the revised version of Bloom's taxonomy, knowledge of theories, models, and structures are recognised as *Conceptual Knowledge*. Conceptual knowledge deals with 'the interrelationships between the constituents and the basic materials of constructed structures'. This means that learners, by their conceptual knowledge, can be able to do functionalising.
Conceptual Representation	*Conceptual Representation* is one's epitomisation and, respectively, mental representation of her/his constructed concepts and the conceptual and logical interrelationships between the constructed concepts.
Conceptualisation	Humans' grasps of their constructed concepts (in the form of their conceptions) are based on their own conceptualisations. Relying on such a conceptual and logical process, humans can find out that an individual thing/phenomenon is an instance of that concept.

	An understanding expresses a local manifestation of a global/universal conceptualisation. In addition, any understanding (based on a concept) could be interpreted as a local manifestation of a global conceptualisation (of that concept). Therefore, it is not the case that all conceptualisations are understandings of concepts.
Conformation Functions (from Machines' Knowledge Bases into Minds)	Suppose that (i) I_n denotes the *n*-component linear relational model for humans' ideas (of their constructed concepts), (ii) P_n denotes the *n*-component linear relational model for their expressed predicates, and (iii) H_n denotes the *n*-component linear relational model for the generated hypotheses in machines' knowledge bases. Accordingly, there are *Conformation Functions* like $\boldsymbol{C}_i$s from predicates into humans' ideas (and in fact, into their constructed concepts). Representing the the set of $\boldsymbol{C}_i$s by $\boldsymbol{C}$, we will have '$\boldsymbol{C}: P_n \rightarrow I_n$' and, semantically: '$H_n \vDash (\boldsymbol{C}: P_n \rightarrow I_n)$'.
Constant Symbol	In Predicate Logic, a *Constant Symbol* is a non-logical symbol and can be considered an instance of any variable symbol.
Constructed World	Mental *Constructed World* is the product of one's 'constructed meanings'. Such a world becomes reflected in the constructor's 'conceptual knowledge'. Additionally, it shall be emphasised that, in parallel to that reflection, the produced conceptual knowledge (with regard to any discussion with other agents) becomes adaptated in conceptual knowledge.
Constructivism	*Constructivism* is a style of thinking about knowledge. Constructivism, as an epistemology as well as a model of knowing, supports constructivist models of learning/mentoring and constructivist learning/mentoring theories. Constructivist models of learning/mentoring asses the phenomena of 'learning' and 'mentoring' the activities of construction. The constructivist models of learning don't assess the phenomenon of 'learning' as an outcome of a development. They do, however, recognise 'learning' as a development.
Constructivist Discussion	According to Badie's designed conceptual framework for meaning construction in the context of constructivist interactions (between two agents), the *Constructivist Discussions* appear between the agents' constructed conceptual knowledge.

Constructivist Interaction	A *Constructivist Interaction* between two or more aware, as well as intentional, agents could lead them to producing their own meaningful understandings (i) based on their produced conceptions of the world, (ii) through their collaborative meaning construction, and (iii) through the collaborative process of knowledge construction. Additionally, regarding Badie's designed conceptual framework for meaning construction, the constructivist interactions relate the agents' constructed worlds to each other.
Constructivist Machine Training	Training machines, by human beings, based on their personal mental images and linguistic expressions of the real world in the framework of constructivism and in the context of their interactions with machines, could be seen as *Constructivist Machine Training*. In such a framework, 'training' could be done by (i) giving examples (and expanding them) to machines' knowledge bases and (ii) clarifying a set of tasks for a machine. Relying on (i) and (ii), machines generate hypotheses in order to focus on machine class expression learning.
Definition	A *Definition* is an isochronism of two logical implications in one's mind: (1) Sub-Class implies Super-Class, (2) Super-Class implies Sub-Class. Consequently, based on one's interpretations, it would be an equivalence relation between a predicate and a sentence (where the predicate implies the sentence and the sentence implies the predicate). In Formal Semantics, such a logical concurrency generates an equation between a predicate and a sentence as well as a description. It's worth mentioning that this logical concurrency is structured based on two concurrent assignments. In the framework of constructivist concept learning, produced meanings are heavily supported by one's definitions of the world.
Definition Transformation	A *Definition Transformation* is a function/mapping from the set of a human's selected definitions (from her/his terminology) into another agent's set of definitions and, subsequently, into that agent's terminology.
Definition Updating	Any agent may revise and re-organise her/his definitions during her/his interactions with other agents as well as the environment. This process is identified as *Definition Updating*. Definition updatings express the reformations as well as reorganisations of one's proposed concept

	descriptions over time.
Experienced Constructive Examples of Concepts	In concept learning, human beings are concerned with a set of experienced phenomena/things in order to develop their knowledge constructions. Expressing new concept descriptions is dependent on *Experienced Constructive Examples of* humans' constructed *Concepts* (or Exp^{+}_{c}). Exp^{+}_{c} consists of the examples (= positive examples) of humans' constructed concepts.
Experienced Non-Constructive Examples of Concepts	In concept learning, human beings are concerned with a set of experienced phenomena/things in order to develop their knowledge constructions. Expressing new concept descriptions is dependent on *Experienced Non-Constructive Examples of* humans' constructed *Concepts* (or Exp^{-}_{c}). Exp^{-}_{c} consists of the non-examples of humans' constructed concepts.
Factual Knowledge	Regarding the revised version of Bloom's taxonomy, the knowledge of terminologies and knowledge of specific details and elements are seen as *Factual Knowledge*.
Function Symbol	In classical logics, *Function Symbols* are non-logical symbols that express the concept of 'functionality' (and the HowNess of becoming able to serve a purpose). Function symbols operate variable symbols.
Functional Role	*Functional Roles* (*Features*) are the roles that are inherently functions and, thus, they can express functional actions, movements, procedures, and manners of human beings. Let N_F be a set of functional roles and N_R be the set of role descriptions in Description Logics. Obviously: $N_F \subseteq N_R$. Informally, functional roles are some kinds of roles.
Hypothesis	A *Hypothesis* is a primary logical description of a class/concept in either humans' minds or machines' knowledge bases. A hypothesis arises and, accordingly, becomes more expanded and specified during concept learning and class expression processes. Hypotheses can describe different theories based on terminologies and world descriptions. They support inductive reasoning processes in order to satisfy different conditions for definitions of truth.
Individual Symbol	In Description Logics, an *Individual Symbol* is a non-logical symbol and can be asserted to any variable. Individual symbols are equivalent to constant symbols in Predicate

	Logic.
Induction	An *Induction* provides humans with a mental system of evidential support that extends deduction to less-than-certain inferences. Human beings, by constructing logical premises for their own inductive inferences, become prepared to be capable of providing various degrees of support for their own logical conclusions.
Interaction Level	The levels of interaction are defined as follows: Level 1: Agent 1 says/does something, Level 2: Agent 2 says/does something, Level 3: Agent 1 says/does something, …
Interpretation	An *Interpretation* is seen as the act of elucidation, explication, and explanation. This PhD thesis has provided the following conceptions of interpretations: **(A)** The interpretations make bridges between (i) a human's 'expressions and explanations' and (ii) the phenomena of 'semantics' and 'meanings'. **(B)** An interpretation could be known as the continually adjusted relationship between (i) the human's intention behind her/his conceptions and her/his actual mental universe of her/his conceptions that are based on her/his accumulated experiences. Subsequently, all understandings are interpreted as limits of interpretations. On the other hand, though, all interpretations are not necessarily understandings. **(C)** Through the lenses of logics, an interpretation is a mapping from a meaning into a non-logical symbol/word.
Interpretation Function	*Interpretation Functions* (like $.^I$) are the most significant producers of terminological as well as semantic interpretations. Considering the individual *a*, the concept *C*, and the Role *R*, an interpretation function can generate a^I, C^I and R^I in order to prepare *a*, *C*, and *R*, respectively, for becoming, terminologically and semantically, interpreted.
Knowledge Construction	*Knowledge Construction* is an active process of producing knowledge. In the framework of constructivism, the phenomena of 'learning' and 'mentoring' are interpreted as active and dynamic processes of knowledge construction. Knowledge construction is strongly dependent on interpretation, explanation, and arrangement.

Knowledge Development	*Knowledge Development* is an active process of developing the constructed knowledge. In the framework of constructivist concept learning, knowledge development is equivalent to developing the constructed collections (like *K*) of terminologies and world descriptions. Therefore, through concept learning processes and based on logical inferences, *K* can be expanded as well as developed over concept constructions.
Learner	A *Learner* is an individual who attempts to be concerned with (as well be involved in) the process of learning. Therefore, she/he tackles to do 'learning' as her/his most general task/role.
Learning	*Learning* is the involvement of a self, by any individual, in increasing knowledge about a phenomenon as well as objects, processes, and events. This PhD dissertation analyses the phenomenon of 'learning' as follows: **(A)** Learning is a process of acquiring knowledge and constructing knowledge (over experiences) that could cause long-lasting or permanent changes at one's level of knowledge as well as depth of understanding. **(B)** In the framework of constructivist concept learning, the phenomenon of 'learning' can be interpreted to be structured based on a collection of transformations from one's personal constructed concepts into her/his own constructed knowledge.
Logical Symbol	*Logical Words/Symbols* have logical consequences in natural and formal languages. We cannot, semantically, interpret some terms like, e.g., 'or', 'and', 'since', 'then', 'therefore', 'so', 'but', 'not'. This means that they have the same logical consequences as well as meanings in all logical structures.
Machine Learning	A machine learning approach attempts to develop strong algorithms that allow machines to improve the productivity of their performances on a given goal. A machine program is said to learn from an experience if there is a set of tasks and a performance measure for it and, also, if its performance at those tasks, as measured, improves with its given experiences.
Machine Inductive Concept Learning	Machine Inductive Concept Learning (*ICL*) is a specified Inductive Learning paradigm. It attempts to describe, logically, concepts and their relationships. It employs the members (instances) and non-members of a class. A

(Machine Class Expression Learning)	characteristic feature of most inductive learning approaches is the use of background knowledge. In concept learning with background knowledge, machines with regard to the given sets of training examples and background knowledge find different hypotheses.
Meaning	In the framework of constructivist concept learning, *Meanings* are some conceptual structures that are constructed based on concepts (conceptual entities). Then, concepts are the basic materials of meanings. More specifically, in the framework of constructivism and in the context of interactions, meanings are concept-update functions.
Meaning Balancing	Through the lenses of formal semantics, interpretation functions can be defined as mappings from 'meanings' into 'non-logical symbols/words'. This means that a meaning is considered as the product of the inverse of interpretation function. Human beings can, by following their mental interpretation functions and interpretations' inverse functions (in multiple repetitive loops), be able to balance and adjust their initial meanings (in their minds) over time. *Meaning Balancing* is quite supportive in balancing personal definitions and vice versa.
Meaning Construction	*Meaning Construction* is the most salient product of constructivist models of knowing as well as constructivist models of learning. Meaning construction has strong connections with humans' abilities of interpretation and construing. Humans, by constructing meanings, provide supportive backgrounds for their meaningful understandings. Sense Making is a valuable outcome of meaning construction.
Meaning Formulation	Humans formulate their balanced meanings (see *Meaning Balancing*) and do *Meaning Formulation* based on the balanced definitions of their personal constructed concepts. There is an appropriate relationship between formulated meanings and balanced definitions. This is based on the supposition that meanings would be given better shapes (in minds) with regard to balanced definitions.
Meaningful Conceptual Structuring	The formulated meanings (see *Meaning Formulation*) reorganise and reinforce humans' mental structures. The formulated meanings are applicable prerequisites for *Meaningful Conceptual Structuring* based on personal

	formulated meanings as well as personal constructed concepts.
Meaningful Understanding	*Meaningful Understandings* are the conclusions of the constructed meanings. Also, the constructed meanings are supported by humans' constructed knowledge as well as their models of knowings. It shall be emphasised that one's constructed meanings could be seen as an identification of the way which she/he moves on, in order to produce her/his meaningful understanding.
Mental Mapping	A *Mental Mapping* is a mapping/function from a concept into another concept. This means that this function is definable from a conceptual entity into another conceptual entity. Considering M as a mental mapping and C and D as two concepts, the function M: $C \rightarrow D$ represents a mental mapping.
Mental Model	Any human being attempts to design a *Mental Model* in order to validate and authenticate her/his descriptions. Note that the interpretability of one's definitions by her/his interlocutors' mental models determine the acceptability of those definitions over the interlocutors' conceptions (Also, see *Model*).
Mentor	A *Mentor*, as a More Knowledgable Other (MKO), is an individual who has a better understanding as well as a higher ability level than the learner with respect to a particular task, process, or any other concept. Mentors can open the world to the learners and open the learners to the world. In the framework of constructivism, mentors are perceived as advanced learners.
Model (in Logics)	In Predicate Logic, in case an interpretation could assign the value True to a sentence (and satisfy it), that interpretation would be a *Model* of that sentence. Similarly, in Description Logics, in case a given [terminological] interpretation could assign the value True to a concept description, that interpretation is called a model of that description. Consequently, a terminological interpretation can be a model of a terminological (and, respectively, of an assertional description) if and only if it can satisfy them semantically.
Overall Understanding Functional Role	According to Badie's designed semantic model for concept understanding, any partial understanding function of a human being like f_i^{UND} (see *Partial Understanding Functions*), semantically, supports her/his *Overall*

	Understanding Functional Role (like F^{UND}). This means that F^{UND} is equal to the chain of n partial understanding functions. Formally: $F^{UND} = f_1{}^{UND} \circ \cdots \circ f_n{}^{UND}$.
Partial Understanding Functions	According to Badie's designed semantic model for concept understanding, the function $f_i{}^{UND}, \forall i \in [1,n]$, is a *Partial Understanding Function*, where *UND* stands for an understanding model (see *Understanding Model*). Any partial understanding function of a human being, semantically, supports her/his overall understanding functional role (see *Overall Understanding Functional Role*).
Predicate	A *Predicate* is an assignment function from characteristics, features, and properties of a conception and respectively, of a constructed concept into subjects. Also, predicates are non-logical symbols in Predicate Logic. In Predicate Logic, predicates express something about the variables and, respectively, about the constants.
Predication	A *Predication* is a transformation/mapping from a conception (of a constructed concept) into a predicate. Formally, *Predication*: *Conception* → *Predicate*.
Procedural Knowledge	Regarding the revised version of Bloom's taxonomy, the knowledge of methods, algorithms, and techniques that are all strongly dependent on humans' skills, are recognised as *Procedural Knowledge*.
RFC	*RFC* is an abbreviation for *Relation-Function-Constant.* According to logical analysis of mentoring-learning relationships, any formal semantics within the relationships between two agents (as mentor and learner) is establishable over a '(*Relation* ↔ (*Function* ↔ *Constant*))'.
Reflection Functions (from Minds into Machines' Knowledge Bases)	Suppose that (i) I_n denotes the n-component linear relational model for humans' ideas (of their constructed concepts), (ii) P_n denotes the n-component linear relational model for their expressed predicates, and (iii) H_n denotes the n-component linear relational model for the generated hypotheses in machines' knowledge bases. Accordingly, there are *Reflection Functions* like $\boldsymbol{R}_i$s from human being's ideas into predicates. Representing the the set of $\boldsymbol{R}_i$s by $\boldsymbol{R}$, we will have '$\boldsymbol{R}: I_n \rightarrow P_n$' and, semantically, '$(\boldsymbol{R}: I_n \rightarrow P_n) \vDash H_n$'.

Role Equivalence	If R and S stand for two roles (role symbols), then the logical description '$R \equiv S$' is called a *Role Equivalence*. Such a relationship, semantically, means that R and S are equal (i.e., $R = S$). The expression '$R = S$' represents a *Role Equality*.
Role Inclusion	See *Role Subsumption*.
Role Subsumption	The logical term '$R \sqsubseteq S$', where R and S stand for two roles (role symbols), expresses that R is subsumed under S. More specifically, the concept of 'being a role and becoming subsumed under another role' is called a *Role Subsumption*. Equivalently, R is the sub-role of S.
Role Symbol	In Description Logics, *Roles* are non-logical symbols. They are equivalent to i-ary (for $i \in [2,n]$) predicates in Predicate Logic. Roles relate and connect the individuals (individual symbols) to each other.
Rules	*Rules* are structured based on logical implications. They directly/indirectly express 'if X, then Y', where X and Y stand for two symbols, propositions, predicates as well as concepts.
Search for Meaning	In the framework of constructivism, human beings *Search for* (i) the [initiative] *Meanings* of the class/classes of their personal constructed concepts and for (ii) their significant relationships in order to make a background for their semantic interpretations.
Semantic Interpretation	*Semantic interpretations* assign meanings to the non-logical symbols and words. A semantic interpretation (like I) must satisfy the terminological and, respectively, the assertional axioms (i.e., $I \vDash$ Axiom) in humans' minds in order to assign meaning to the non-logical words/symbols within their natural/formal languages.
Schema	*Schemata* or *Mental Entities* are the constituents and the basic materials of mental structures. Accordingly, they are the ingredients of concepts (conceptual entities). According to constructivist models of learning, all learning and knowledge construction is mediated by the construction of schemata.
Signature	The collection of various conceptions (of the constructed concepts) and their interrelationships makes a *Signature*. More specifically, considering N_C, N_R, and N_O as the sets of

	atomic concepts, atomic roles, and individuals respectively, the ordered triple (N_C, N_R, N_O) represents a signature. Any formal semantics is definable and analysable based on 'terminological interpretations over signatures'.
Smart Constructivism	*Smart Constructivism* is a modern learning theory introduced and conceptualised by Farshad Badie (see Article L). A smart constructivist model of learning is theorised (i) based on traditional constructivist theory of learning and (ii) by considering new requirements of learners in the digital age. This theory can support (a) the modern developments of smart learning strategies and (b) the renewed qualitative developments of knowledge building and understanding production within smart learning environments.
Social Constructivism	The concept of '*Social Constructivism*' is conceptualisable in the framework of constructivism and in the context of social interactions. Social interactions between aware, as well as intentional, agents play fundamental roles in (i) the processes of their cognitive development and in (ii) how any of them goes about the construction as well as the development of her/his personal knowledge. Regarding social constructivism, conceptions (of the individuals) make sense to communities by becoming shared. Accordingly, individuals in communities can, collaboratively, focus on constructing meanings and producing meaningful understandings together.
Subject	A *Subject* is the concept which is, in a situation/setting, the conceptual (mental and cognitive) entity that is the object (i.e. the starting point) of a line of thought in transformation of information on the part of an interlocutor uttering a statement.
Subjection	*Subjection* is an assertive Predication.
Supervised Machine Learning	In *Supervised Machine Learning* method, the pairs (*input*,*output*) as the training examples are supplied by the trainer (who is a human being). Accordingly, the learner (which is a machine) searches for function mappings from the inputs into the outputs.
Tautology [Model]	In Predicate Logic, if a sentence can be assigned the value 'True' for all possible interpretations, it would be a *Tautology Model*. Similarly, in Description Logics, if a concept description is assigned the value 'True' for all

	possible terminological interpretations, it would be a tautology model.
Terminology	In the framework of constructivist concept learning, a finite set of a human's descriptions (as well as definitions) generates a *Terminology*, if no atomic concept (literal) has been defined more than once by her/him at the same moment. Terminologies must be explicit in order to be meaningful.
Terminological Axiom (Logic)	In a logical-terminological system, the *Terminological Axioms* are introduced to make logical statements about (i) how concepts and roles are related to each other as well as (ii) how concepts and roles, logically, support each other.
Terminological Interpretation	*Terminological Interpretations* are generated based on (i) a non-empty set *D* that is the domain of the interpretation and (ii) an interpretation function (like $.^{I}$). Terminological interpretations are linkages between syntax and formal semantics. More specifically, they are the constructors of formal semantics.
Terminological Knowledge	Human beings, by constructing their personal *Terminological Knowledge*, attempt to satisfy their terminological axioms [mainly] based on concept inclusion, concept equality, role inclusion, and role equality. Accordingly, they can satisfy their world descriptions based on their assertional axioms.
Understanding	*Understanding* or the ability to comprehend something is producible based on one's personal knowings, knowledge constructions (ultimately, over the extended abstracts), interpretations of the world, constructed meanings of the world, and senses made of world. It shall be emphasised that the abilities of self awareness as well as interpreting the reality of the world could be interpreted as the most valuable products of the phenomenon of 'understanding'.
Understanding Function	According to Badie's designed semantic model for concept understanding, an *Understanding Function* is a function (like -^{UND}) that is the most significant constructor of understanding models (See *Understanding Model*).
Understanding Model	According to Badie's designed semantic model for concept understanding, an *Understanding Model* (like *UND*) is constructed based upon the tuple (*Understanding Domain*, *Understanding Function*). Then: $UND = (D_U{}^{UND}, \text{-}^{UND})$. Any

	UND (*i*) is a restricted form of a terminological interpretation-based model, (*ii*) must be able to satisfy the semantics of the terminological and assertional axioms (i.e., $UND \vDash$ Axiom) in humans' minds, and (*iii*) associates with each atomic concept a subset of D_U^{UND}, and with each ordinary atomic role a binary relation over $D_U^{UND} \times D_U^{UND}$.
Unified Conceptual Knowledge	According to Badie's designed conceptual framework for meaning construction in the context of constructivist interactions, the *Unified Conceptual Knowledge* is the product of the constructivist discussions between agents' constructed conceptual knowledge. In the context of constructivist interactions, the unified conceptual knowledge always becomes extended and expanded.
Variable Symbol	In classical logics, *Variables* are non-logical symbols. In Predicate Logic, predicates are assigned to variable symbols. Any variable symbol can accept an infinite number of constant symbols. Similarly, in Description Logics, concepts are assigned to variable symbols and any variable symbol can accept an infinite number of individual symbols.
Weak Mental Mapping	If there is a mental mapping (like *M*) from a concept (like *C*) into another concept (like *D*), then it could be conceptualised, interpreted, and understood that *M* is also valid from a concept like *C´* (that is equivalent to *C*) into *D*. Such a mental mapping is *Weak*.
World Description (Assertion)	*World Descriptions* express different facts based on accepted axioms and defined fundamental terminologies. In Description Logics, any world description (assertion) describes the world over concepts, roles, and individuals and in fact, over signatures. World descriptions are strongly tied to terminological axioms.

APPENDICES

APPENDIX I (CLARIFICATIONS)

I. IDEA

In articles A, D, E, F, G, H, a 'concept' has been said to be interpretable as an 'idea'. Accordingly, it's assumed that an idea can be expressed in the form of a 'distinct entity' or 'a class of entities' as well as its/their 'essential features and attributes'. Also, it's mentioned that ideas can determine the application of a term, especially in the form of predicates. This means that an idea can play a fundamental part in the use of reason or language.

Description Logics are a family of semi-formal languages and represent the interrelationships between nominals, logical, and non-logical symbols in order to focus on terminological knowledge. Description Logics are logically structured based on Predicate Logic. They have descriptive features and that's why they are interpreted as a family of descriptive logical languages. According to Description Logics, a concept can be interpreted a class as well as an idea. Subsequently, such an idea can be described within a hypothesis. Consequently, the hypothesis makes an interrelationship between the idea and a distinct entity (as well as a class of entities) or to its (their) essential features.

According to the afore-mentioned descriptions relevant to ideas, what this dissertation has brought under the label of 'idea', could be expanded to 'idea of a concept'. In later publications (after H), I preferred to use the term 'conception' instead of 'idea' in order to express my assumption that human beings are able to ideate their constructed concepts (and in fact, to ideate their formed, transformed, and reformed concepts) in the appearance of their conceptions in order

- to represent them in the form of logical hypotheses, and subsequently,
- to utilise them in their reasoning and learning processes.

I shall propose the following Figure in order to support what I have suggested.

Figure a. *Concepts and Ideas*

II. SCHEMATA

In articles A, B, C, E, F, H, I, J, P, the concept of 'Schema' has been taken into account. It shall be emphasised that the term 'schema' is very imprecise in cognitive and psychological sciences as well as learning sciences. Note that my theoretical model has attempted to focus on the concept of 'schema' and to assume the schema's existentialities (as basic materials of concepts) in order to reach the phenomenon of 'concept' and, subsequently, the phenomenon of 'meaning'.

My theoretical model has adopted Piaget's idea and notion of 'schema' and has expanded it in order to arrive at the phenomena of 'concept construction' and 'concept learning' in the framework of constructivism. In addition, taking into account some standard descriptions of 'schemata', I have assumed that one's designed schemata (i) provide backgrounds for her/his concept construction processes, (ii) could describe different theories based on terminological axioms and assertions, (iii) could give sufficient and satisfying conditions for definitions of truth, and (iv) could specify her/his inferences and reasoning processes based on personal constructed concepts. Let me be more specific.

Regarding Piaget's conception of schema, the expression 'schemata first emerge as concrete actions and then gradually develop into more abstract (general) concepts' can be accepted as a fundamental expression in my model. Since then, I have attempted to expand it logically. More specifically, I have concluded that human beings, mentally, design schemata and, then, gradually develop and divide them into more general concepts (conceptual entities). A proposed (designed) schema describes a pattern of the person's thought. As I have summarised in section I (Constructivism – Constructivism and Meaning Construction), "meanings (as conceptual structures) are constructed based on conceptual entities (concepts). More specifically, meanings are shaped based on an undetermined number of the developments and updatings of the linked collection(s) of conceptual entities. Furthermore, conceptual entities are made of mental entities". According to that, my theoretical model assumes that human beings' mental images of conceptual entities are, mentally, designed and visualised based on their mental structures. Also, the mental structures are constructed based on mental entities. These mental entities are what I can bring under the label of 'schemata'. Therefore, what Piaget has expressed as mental objects which learning is mediatable based on them, would be labelled 'mental entities' in my semantic model. Consequently, in my model, conceptual entities and mental entities are not equivalent, but mental entities are the constituents of the conceptual entities (concepts).

APPENDIX II (REVISIONS)

I. CONCEPT UNDERSTANDING AND CONCEPTUALISATION

In the main versions of papers E and I, it was expressed that:

"*a conceptualisation is a uniform specification of separated understandings; a conceptualisation provides a global manifestation of local understandings in the context of a human's thoughts. Additionally, a human's grasp of concepts provides a proper foundation for generating her/his own conceptualisations*".

Instead, in the current versions of papers E and I (in this dissertation), I have decided to substitute it by:

"*an understanding expresses a local manifestation of global conceptualisations*" and "*any understanding (based on a concept) could be interpreted as a local manifestation of a global conceptualisation (of that concept). It shall be claimed that human beings' grasps of concepts could provide proper foundations for generating their own conceptualisations*".

Additionally, in the main version of paper M, I had stated that:

"*human beings can find out that an individual thing/phenomenon is an instance of a formed concept and, thus, their individual grasp of that concept (in the form of their conceptions) provide foundations for producing their own conceptualisations*".

Instead, in the current version of paper M (in this dissertation), I have decided to substitute it by:

"*human beings' grasps of their constructed concepts (in the form of their conceptions) provide foundations for producing their own conceptualisations. Accordingly, they can find out that an individual thing/phenomenon is an instance of that concept*".

II. CONCEPT UNDERSTANDING AS A RELATION

In the main version of paper I, I had described that:

"*[Concept] understanding is a type of relation between person and concept. Therefore, this relation transforms the characteristics, attributes and qualities of that concept into the person's mind. It also transforms the properties of that concept and its relationships with other concepts into mind*".

Later on, I just interpreted that concept understanding is, definitely, a relation, but not between a human being and a concept. More clearly, I interpreted a 'concept understanding' as a relation between (i) characteristics of a concept and (ii) a description, see Figure b. Hence, in the current version of paper I as well as paper M (where I focused on formal semantic analysis of concept understanding and proposing an ontology for concept understanding), I have stated that:

"*Concept understanding, as a relation, could relate 'the characteristics and attributes of a concept' with 'a description'. More specifically, understanding is a function (mapping) from a concept into some propositions (and statements) which could be interpreted as 'concept descriptions'. In fact, the characteristics and properties of a concept by means of the understanding function could become mapped into concept descriptions*".

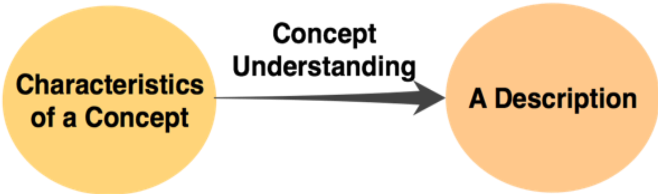

Figure b. *Concept Understanding as a Relation*

III. FORMALISM

In section 3 of the main version of paper B, there were two typing mistakes:

1. The formalism 'Spring ≡ Season ⊓ ∃hasModerate.Weather ⊓ ∀hasGreen.Tree' has changed to 'Spring ≡ Season ⊓ ∃hasWeather.Moderate ⊓ ∀hasTree.Green' in the current version.

2. The same problem goes to 'Spring ≡ Season ⊓ ∃hasModerate.Weather ⊓ ∀hasGreen.Tree ⊓ ∃hasApril.Month'. The right formalism, Spring ≡ Season ⊓ ∃hasWeather.Moderate ⊓ ∀hasTreeGreen ⊓ ∃hasMonth.April, is considered for the current version.

In addition, in section 4 of the main version of paper B, it was stated that:

"*considering R as a binary relation between two concepts C and D, there is a R^I from D into $D^I \times D^I$*",

but, in the current version, I have changed it to:

"*considering R as a binary relation between two concepts C and D, there is a I, such that, I: $D \rightarrow D^I \times D^I$*".

SUMMARY

The central focus of this Ph.D. research is on ‘Logic and Cognition’ and, more specifically, this research covers the quintuple (Logic and Logical Philosophy, Philosophy of Education, Educational Psychology, Cognitive Science, Computer Science). The most significant contributions of this Ph.D. dissertation are conceptual, logical, terminological, and semantic analysis of Constructivist Concept Learning (specifically, in the context of humans’ interactions with their environment and with other agents). This dissertation is concerned with the specification of the conceptualisation of the phenomena of ‘learning’, ‘mentoring’, and ‘knowledge’ within learning and knowledge acquisition systems. Constructivism as an epistemology and as a model of knowing and, respectively as a theoretical model of learning builds up the central framework of this research.

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